



Chapter 3

Vectors and Arrays

Outline



3.1 Concept: Using Built-in Functions

3.2 Concept: Data Collections

3.3 Vectors

3.4 Engineering Example—Forces and Moments

3.5 Arrays

3.1 Concept: Using Built-in Functions



MATLAB comes equipped with hundreds of functions that abstract out details of how to calculate their answers.

MATLAB **help** and **Appendix A** provide details of how to use each one.

For example, we can validate the relationship between $\sin(\theta)$ and $\cos(\theta)$ by running this script:

```
th = 27 * pi / 180
result = cos(th)^2 + sin(th) ^2
```

which should return the value 1***.

*** Since computers are not completely accurate in representing fractions, this might be 0.99999.

3.2 Concept: Data Collections



Vectors and arrays are simple examples of data abstraction wherein we group a number (0 or more) of things together and give them a name.

- “all temperature readings for May”
- “all the purchases from Wal-Mart”

Different collections have different rules about what type of data the collection may contain and how the collection as a whole can be manipulated.

3.3 Vectors



A vector is a linear, homogeneous collection of data, either numbers or logical values for now, each of which is called an element. Each element is located at a position, or an **index**

An example of a vector :

$$\text{vec} = [1, 2, 3, 4, 5];$$

Experienced programmers should note that due to its FORTRAN roots, indices in MATLAB language start from 1 and not 0. So

$\text{Vec}(1)=1, \text{vec}(2)=2, \dots$

Creating a Vector



1. **Direct Entry**: Just type it in....

$X = [1\ 2\ 3\ 4\ 5];$

2. **Colon Operator**: Here you specify a starting point, a step value (which is automatically ONE if you do not specify), and an ending point, and separate them with colons “:”

$x = 0 : 2 : 10; \rightarrow X = [0, 2, 4, 6, 8, 10];$

The colon operator is inclusive of your *start* point, not necessarily your endpoint.

**Also, please *TAKE NOTE*:

$x = 3 : 2 : -8 \rightarrow$ **this will return an empty vector! []**

Building Vectors from Function Calls



- **X = linspace(a, b, n)** → linspace is a function that will create a vector of length 'n' that starts at 'a' and ends at 'b'. For example:
$$X = \text{linspace}(1, 2, 6) \rightarrow X = [1, 1.2, 1.4, 1.6, 1.8, 2];$$
- **X = zeros(r, c) OR ones(r, c)**: zeros() and ones() will create a vector of all 0's or 1's, not too tricky 😊 however these functions also create arrays, so your inputs are 'r' the number of rows (1 if you want a vector) and 'c' the number of columns (or the number of elements you want in your vector. For example:
$$X = \text{zeros}(1, 3) \rightarrow X = [0 \ 0 \ 0]; \text{ ((one row, 3 cols))}$$
- **X = rand(r, c)**: rand is the same as 'ones' or 'zeros' except it generates a vector/array of 'r' rows and 'c' columns of random numbers between 0 and 1. There is a formula as follows for generating a vector of random numbers of a range, say you want 5 random numbers between 4 and 7:
$$X = 4 + (7-4) * \text{rand}(1, 5); \text{ OR vector} = \text{min} + (\text{max} - \text{min}) * \text{rand}(r, c)$$

Building a Vector by Concatenation



- What does that word mean? → Concatenation is a fancy way of saying add one, or append one, onto the end of the other as follows:

– **HORIZONTAL** concatenation:

$X = [1 \ 2 \ 3]; \quad Y = [4 \ 5 \ 6]$

$XY = [X, Y];$ → I used a comma, I didn't HAVE to, I could just do $[X \ Y]$

$XY = [1 \ 2 \ 3 \ 4 \ 5 \ 6];$ (see what happened?)

– **VERTICAL** concatenation: **you must have the same number of elements in your vectors to do this!!! This will create an array, or stack them one on top of the other.

$Vert = [X; Y]$ → instead of a comma, I used a semi colon to get $vert = [1 \ 2 \ 3;$
 $4 \ 5 \ 6];$

Accessing and Indexing!



Indexing:

An 'index' refers to a 'spot' in a vector or a particular element. i.e.

```
vec = [3 5 7]
```

Here $\text{vec}(3) = 7$; $\text{vec}(2) = 5$; and $\text{vec}(1) = 3$;

You can also set elements in such a manner:

```
vec(4) = 9;
```

Now, $\text{vec} = [3\ 5\ 7\ 9]$;

If you attempt to add an element past the length of the vector it will fill all the empty slots leading to it with zeros:

```
vec(7) = 11 → vec = [3 5 7 9 0 0 11];
```

Indices in MATLAB start at 1, there is NO index of ZERO!

Accessing Multiple Items:

You can access/manipulate several items at once as follows:

```
vec([1 2 3]) = 0;
```

This would result in :

```
vec = [0 0 0 9 0 0 11];
```

Notice the first three indices have now been switched to zeros. If we wanted to change the first three indices to 6, 4, and 8:

```
vec([1 2 3]) = [6, 4, 8];
```

Let's say we want to access the ODD indices only, and set them to zero:

```
vec(1:2:end) = 0;
```

Arithmetic Operations



- When **adding/subtracting** vectors, MATLAB will do so element by element automatically, however, either the vectors **MUST** be the **same length** or one of them must be length 1. Or else you will get another dimension mismatch.
e.g. $A = 1:4$; $B = [2\ 4\ 6\ 3]$;
 $C = A + B \rightarrow [3\ 6\ 9\ 7]$

Multiplication and Division



- When **multiplying / dividing** two vectors you must use the dot operators `.*`, `./`, `.^`:
e.g $D = A .* B \rightarrow [2 \ 8 \ 18 \ 12]$
- What if we tried to execute the instruction $A*B$? \rightarrow this would give us a dimension mismatch error because this is referring to matrix multiplication, which we will learn more about later! You need to be able to recognize these errors when they occur!

Logical Indexing



There are 5 classes of variables in MATLAB that we deal with MOST:

1. Logical
2. Char
3. Double
4. Cell
5. uint8

Logicals: true/false values represented by 1's and 0's in MATLAB. Please note the very big difference between $X=0$; and $X=false$:

if $vec = [1\ 2\ 3\ 1\ 2\ 1]$;

typing $vec==1$, or $vec < 2$ into the command window will return : $ans = [1, 0, 0, 1, 0, 1]$

where 0 stands for false, and 1 stands for true.

Typing $vec([1\ 0\ 0\ 1\ 0\ 1])$ will return an error. Why? because there is no index of ZERO!

$x = vec([true\ false\ false\ true\ false\ true])$ would return $x = [1\ 1\ 1]$ of class logical where the 1 stands for true.

This is because...



When you say `vec = x > 2;` you are asking MATLAB to return to you a list of each index in your vector with either a true or false by it stating whether it is greater than 2 (true) , or whether it is not greater than 2 (false).

So in return you get a vector of 1's and zero's (Which are just MATLAB representation of true and false) that specifies if the value at that index is greater than 2.

The following are operators to know for logical indexing:

> greater than

< less than

== equal to (not the same as =)

>= greater than or equal to

<= less than or equal to

Shortening a Vector



- You can shorten a vector by replacing one of its elements with the empty vector, []

e.g $D = [2\ 8\ 18\ 12]$

$D(3) = [] \Rightarrow D = [2\ 8\ 12]$

- A better way to accomplish the same goal is to use indexing to specify the values you want to keep

e.g $D = D([1\ 2\ 4])$

Common Functions Used on Vectors



- sum(vec) → gives the sum of all the elements in vec
- mean(vec) → gives the average of the elements in vec
- prod(vec) → gives the product of the elements in vec
- sort(vec): the sort function will sort the vector in ascending order unless specified otherwise. It has two outputs:
[sorted_vec, indices] = sort(vec);
- The second output is the indices that each element in your sorted vector occupied in the original vector.

More Functions



find(logical) → will return the indices in `vec` where the element is true.

e.g: `vec = [7 3 9 5];`
 `x = find(vec <= 7);` → `x = [1 2 4]`

mod(vec, n) → returns the modulus (remainder) of each element in `vec` divided by `n`.

e.g: `vec = [4 5 6];`
 `mod(vec, 4)` → `[0 1 2]`

This function can be used to find even/odd elements in a vector:

`x = vec(mod(vec, 2) ~= 0);` → finds odds, finds elements in `vec` that when divided by 2 do NOT have a remainder of zero! How?

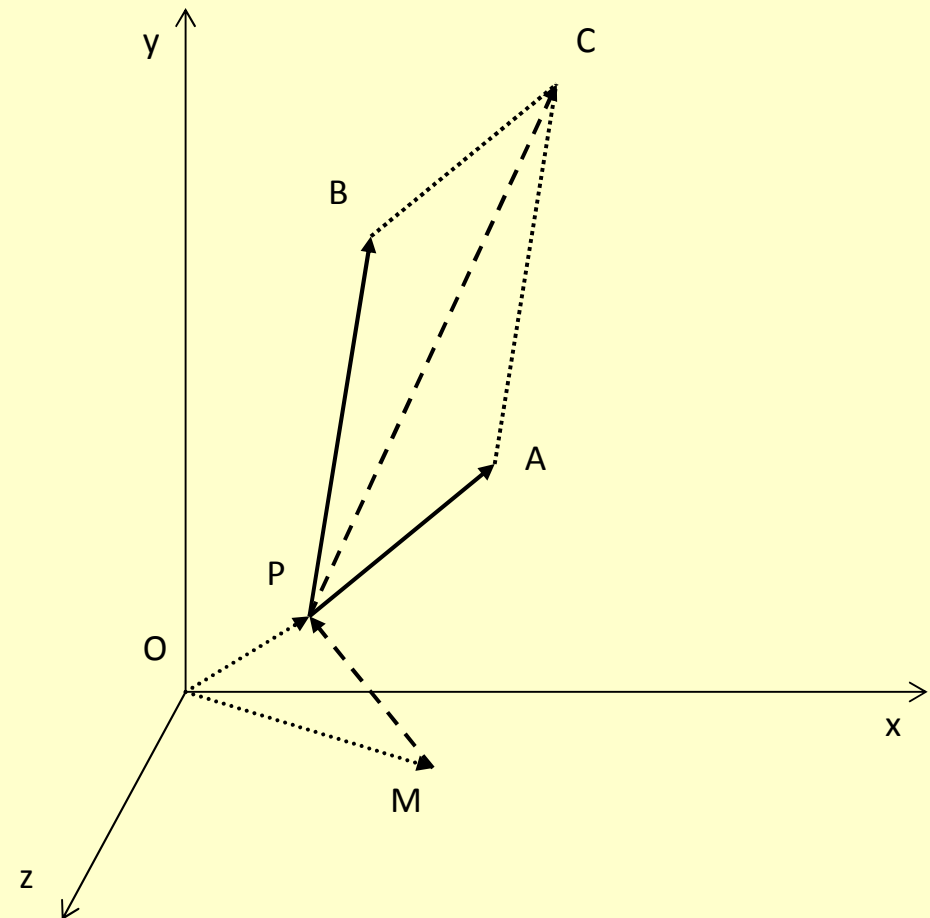
`mod(vec, 2) ~= 0` gives us a logical vector `[false, true, false]`, then we used that to index.... `vec([false, true, false])`.... so it returned the 5 only. The only odd number in our vector. What if we want the index of the odd values?

→ `find(mod(vec, 2) ~= 0)`

3.4 Engineering Example—Forces and Moments



- Vectors are ideal representations of the concept of a vector used in physics.
- Consider two forces acting on an object at point P , as shown in the figure.
- Calculate the resultant force at P , the unit vector in the direction of the resultant, and the moment of the force about the point M .



3.4 Engineering Example—Forces and Moments



```
1 - PA = [0 1 1]; PB = [1 1 0];
2 - P = [2 1 1]; M = [4 0 1];
3 - % find the resultant of PA and PB
4 - PC = PA + PB
5 - % find the unit vector in the direction of PC
6 - mag = sqrt(sum(PC.^2));
7 - unit_vector = PC/mag
8 - % find the moment of the force PC about M
9 - % this is the cross product of MP and PC
10 - MP = P - M;
11 - moment = cross( MP, PC )
```

Command Window

PC =

1 2 1

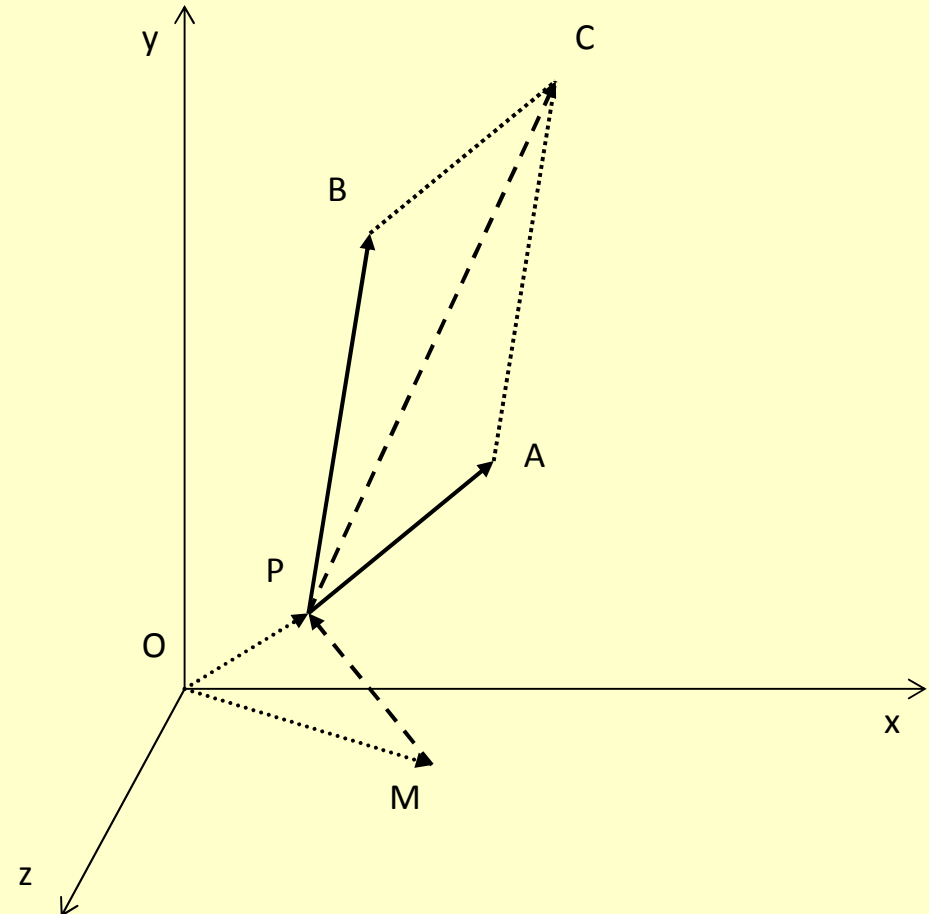
unit_vector =

0.4082 0.8165 0.4082

moment =

1 2 -5

fx >>



Questions?



3.5 Arrays



An array is a rectangular homogeneous collection of data, either numbers or logical values for now, each of which is called an element. Each element is located at a row and column of the array

An example of an array:

$$A = \begin{bmatrix} 1, & 2, & 3 \\ 4, & 5, & 6 \end{bmatrix};$$

The array above will be referred to as $A_{2 \times 3}$

3.5 Arrays



Properties of an Array

Individual element in an array are referred to as its **elements**

The function `sz=size(...)` will return a vector of length `n` containing the size of each dimension in an array.

The function `length(...)` returns the maximum dimension of an array.

The transpose of an $m \times n$ array, indicated by apostrophe character (`'`), returns an array of $n \times m$ dimensions with the rows and column interchanged.

Creating an Array



1. **Direct Entry**: Just type it in....

$X = [1 \ 2 \ 3; 4 \ 5 \ 6];$ must be rectangular

Or $X = [1 \ 2 \ 3$
 $4 \ 5 \ 6];$

2. **X = zeros(r, c) OR ones(r, c)**: the inputs are 'r' the number of rows and 'c' the number of columns

$X = \text{zeros}(2, 3) \rightarrow X = [0 \ 0 \ 0$
 $0 \ 0 \ 0];$

3. **X = rand(r, c)**: generates an array of 'r' rows and 'c' columns of random numbers between 0 and 1. If you want a 3*5 array random numbers between 4 and 7:

$X = 4 + (7-4) * \text{rand}(3, 5)$

Building an Array by Concatenation



– **HORIZONTAL** concatenation:

$X = [1; 2; 3];$ $Y = [4; 5; 6]$

$XY = [X, Y];$ → I used a comma, I didn't HAVE to, I
could just do $[X \ Y]$

$XY = [1 \ 4$
 $2 \ 5$
 $3 \ 6];$

– **VERTICAL** concatenation.

$Vert = [X; Y] \rightarrow vert = [1$
 2
 3
 4
 5
 $6];$

Accessing and Indexing!



Indexing:

An 'index' refers to a 'spot' in an array or a particular element. i.e.

```
ar = [3 5; 2 7]
```

Here $ar(2, 2) = 7$; $ar(1, 2) = 5$; and $ar(1, 1) = 3$;

You can also set elements in such a manner:

```
ar(2,2) = 9;
```

Now, $ar = [3 \ 5; 2 \ 9]$;

If you attempt to add an element past the length of the array it will fill all the empty slots leading to it with zeros:

```
ar(3,4) = 11 → ar = [3 5 0 0  
                    2 9 0 0  
                    0 0 0 11];
```

Indices in MATLAB start at 1, there is NO index of ZERO!

Accessing Multiple Items:

You can access/manipulate several items at once as follows:

```
vec(1,[1 2]) = 0;
```

This would result in :

```
vec = [0 0 0 0  
       9 0 0 11];
```

Notice the first two elements have now been switched to zeros. If we wanted to change them to 6, 4:

```
vec(1, [1 2]) = [6, 4];
```

Let's say we want to access the ODD columns of the EVEN rows only, and set them to 99:

```
vec(2:2:end, 1:2:end) = 99;
```


Arithmetic Operations



- When **adding/subtracting** arrays, MATLAB will do so element by element automatically, however, either the arrays **MUST** have the **same dimensions (rows and columns)** or one of them must be length 1. Otherwise, you will get another dimension mismatch.

e.g. $A = [1 \ 2; 3 \ 4]; B = [2 \ 4; 6 \ 3];$

$C = B - A \rightarrow [1 \ 2; 3 \ -1]$

Multiplication and Division



- When **multiplying / dividing** two arrays you must use the dot operators `.*`, `./`, `.^`:
e.g $D = A .* B \rightarrow [2 \ 8; 18 \ 12]$
- What if we tried to execute the instruction $A*B$? \rightarrow this would result in a matrix multiplication resulting in:
$$\text{ans} = [14 \ 10 \\ 30 \ 24]!$$
- We will learn about matrix operations in a later chapter.

Logical Indexing



if arr = [1 2; 3 1; 2 1];

typing `vec==1`, or `vec < 2` into the command window will return

: ans = [1, 0; 0, 1; 0, 1]

where 0 stands for false, and 1 stands for true.

What is really odd is the result when you use `find` on a logical array:

`find(ans) -> [1 4 6]!`

`[rows cols] = find(ans) -> rows = [1 2 3], cols = [1 2 2]`

Shortening an Array



- You can shorten an array by replacing a complete row or column with the empty vector, []
e.g $D = [2 \ 8; 1 \ 2; 18 \ 12]$
 $D(2, :) = [] \Rightarrow D = [2 \ 8; 18 \ 12]$
- A better way to accomplish the same goal is to use indexing to specify the values you want to keep
e.g $D = D([1 \ 3], :)$

Common Functions Used on Arrays



- **sum(arr)** → gives a vector containing the sum of the elements in each column of arr
- **mean(arr)** → gives a vector containing the average of the elements in each column of arr
- **prod(arr)** → gives a vector containing the product of the elements in each column of arr
- **find(logical)** → will return the indices in a column vector that is a reshape of the original logical array where the element is true.

e.g: `vec = [7 3; 9 5];`
 `x = find(vec <= 7);` → `x = [1 2 4]!!!`

Let's Write some Code ...



3.6 Engineering Example Computing Soil Volume

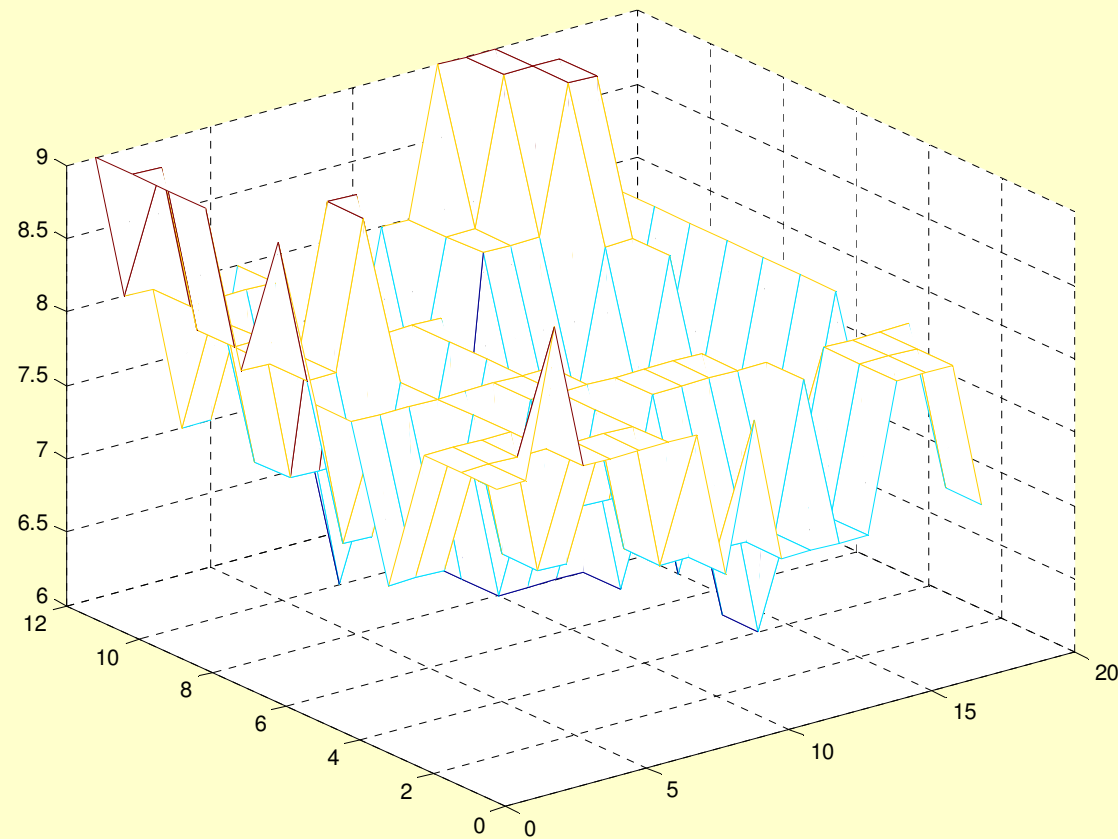


When digging for foundations for a building, it is necessary to estimate the amount of soil that must be removed. The first step is to survey the land on which the building is to be built, which results in a rectangular grid defining the height of each grid point

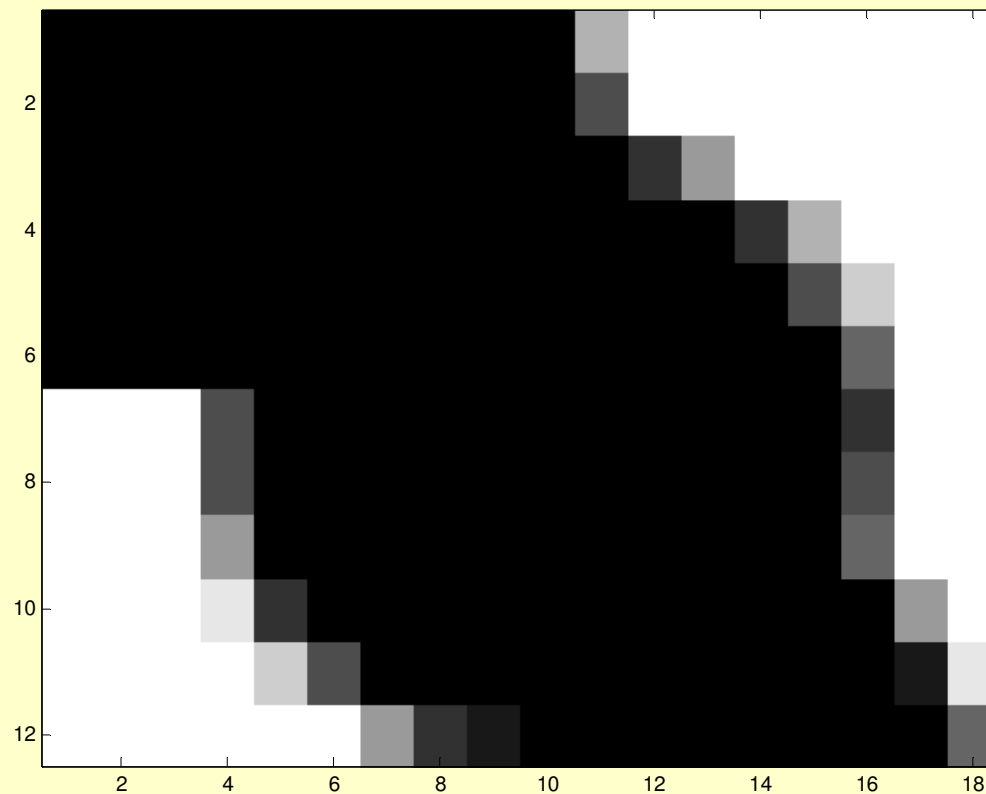
The next step is to consider an architectural drawing of the basement of the building.

The total amount of soil to move is then the sum of the individual grid point depths multiplied by the area in each square to be removed.

3.6 Engineering Example Computing Soil Volume



3.6 Engineering Example Computing Soil Volume



3.6 Engineering Example Computing Soil Volume



```
1      %soil depth data for each square produced by the survey
2 -    dpth=[8 8 9 8 8 8 8 8 7 8 7 7 7 7 8 8 8 7; 8 8 8 8 8 8 8 7 7 7 7 7 8 7 8 8 8 7
3          8 8 8 8 7 7 8 7 8 8 8 8 8 7 8 8 8 8; 7 7 7 8 7 8 8 8 8 8 8 8 7 6 7 7 7 7
4          8 8 8 8 8 8 8 8 7 7 7 7 7 6 6 7 7 8; 8 7 7 8 7 7 8 7 7 7 7 7 7 7 7 7 7 8
5          9 8 8 9 8 7 8 7 7 7 7 7 6 7 6 7 7 8; 8 8 8 9 9 8 8 8 7 6 6 6 6 7 7 8 7 8
6          9 8 8 7 7 7 7 7 6 6 7 7 7 8 8 7 8; 9 8 8 7 7 7 6 7 7 6 6 8 8 8 9 9 7 8
7          9 9 8 8 8 8 7 7 7 7 7 8 8 9 9 9 8 8; 9 8 8 7 7 8 7 7 7 7 8 8 9 9 9 8 7 8];
8
9      %estimated proportion of each square that should be excavated
10 -   area=[1 1 1 1 1 1 1 1 1 1 1 .3 0 0 0 0 0 0
11         1 1 1 1 1 1 1 1 1 1 1 .7 0 0 0 0 0 0
12         1 1 1 1 1 1 1 1 1 1 1 1 .8 .4 0 0 0 0
13         1 1 1 1 1 1 1 1 1 1 1 1 1 .8 .3 0 0 0
14         1 1 1 1 1 1 1 1 1 1 1 1 1 1 .7 .2 0 0
15         1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 .6 0 0
16         0 0 0 .7 1 1 1 1 1 1 1 1 1 1 1 .8 0 0
17         0 0 0 .7 1 1 1 1 1 1 1 1 1 1 1 .7 0 0
18         0 0 0 .4 1 1 1 1 1 1 1 1 1 1 1 .6 0 0
19         0 0 0 .1 .8 1 1 1 1 1 1 1 1 1 1 1 .4 0
20         0 0 0 0 .2 .7 1 1 1 1 1 1 1 1 1 1 .9 .1
21         0 0 0 0 0 0 .4 .8 .9 1 1 1 1 1 1 1 .6];
22
23 -   sq_vol = dpth.*area;
24 -   tot_soil = sum(sum(sq_vol))
```

3.6 Engineering Example Computing Soil Volume



The code in Listing 3.4 produces an answer $\sim 1,120$ and we should ask whether this is reasonable.

```
tot_soil =  
  
    1117.50  
fx >>
```

There are 12×18 squares, each with area 1 unit, about 80% of which are excavated, giving a surface area of about 170 square units. The average depth of soil is about 7 units, so the answer ought to be about $170 \times 7 = 1,190$ cubic units.

This is reasonably close to the computed result.



Homework on Chapter 3 is posted on the website:

http://www.ee.nmt.edu/~erives/289_F12/EE289.html

Homework is due in a week