- Decimal, Hexadecimal and Binary Numbers
- Binary numbers are a code, and represent what the programmer intends for the code
- Convert binary and hex numbers to unsigned decimal
- Convert unsigned decimal to hex
- Unsigned number line and wheel
- Signed number line and wheel
- Binary, Hex, Signed and Unsigned Decimal
- Signed number representation --- 2's Complement form
- Using the 1 's complement table to find 2 's complements of hex numbers
- Overflow and Carry
- Addition and subtraction of binary and hexadecimal numbers
- The Condition Code Register (CCR): N, Z, V and C bits

| Binary | Hex | Decimal |
| :---: | :---: | :---: |
| 0000 | 0 | 0 |
| 0001 | 1 | 1 |
| 0010 | 2 | 2 |
| 0011 | 3 | 3 |
| 0100 | 4 | 4 |
| 0101 | 5 | 5 |
| 0110 | 6 | 6 |
| 0111 | 7 | 7 |
| 1000 | 8 | 8 |
| 1001 | 9 | 9 |
| 1010 | A | 10 |
| 1011 | B | 11 |
| 1100 | C | 12 |
| 1101 | D | 13 |
| 1110 | E | 14 |
| 1111 | F | 15 |

What does a number represent?
Binary numbers are a code, and represent what the programmer intends for the code.
0x72 Some possible codes:
'r' (ASCII)
INC MEM (hh l1) (HC12 instruction)
2.26 V (Input from A/D converter)

114 ${ }_{10}$ (Unsigned number)
$114_{10}$ (Signed number)
Set temperature in room to 69 F
Set cruise control speed to 120 mph

## Binary to Unsigned Decimal:

Convert Binary to Unsigned Decimal
$1111011_{2}$
$1 \times 2^{6}+1 \times 2^{5}+1 \times 2^{4}+1 \times 2^{3}+0 \times 2^{2}+1 \times 2^{1}+1 \times 2^{0}$
$1 \times 64+1 \times 32+1 \times 16+1 \times 8+0 \times 4+1 \times 2+1 \times 1$
$123{ }_{10}$

## Hex to Unsigned Decimal

Convert Hex to Unsigned Decimal 82D6 16
$8 \times 16^{3}+2 \times 16^{2}+13 \times 16^{1}+6 \times 16^{0}$
$8 \times 4096+2 \times 256+13 \times 16+6 \times 1$
$33494{ }_{10}$

## Unsigned Decimal to Hex

Convert Unsigned Decimal to Hex

| Division | Q | R |  |
| :---: | :---: | :---: | :---: |
|  |  | Decimal | Hex |
| $721 / 16$ | 45 | 1 | 1 |
| $45 / 16$ | 2 | 13 | D |
| $2 / 16$ | 0 | 2 | 2 |

$$
721_{10}=2 \mathrm{D} 1_{16}
$$

Unsigned Number Line: Numbers go from 0 to $\infty$


Unsigned Number Wheel: Numbers go from 0 to $2 \mathrm{~N}-1$


Signed Number Line: Numbers go from $-\infty$ to $\infty$


Number Wheel: What to do about $100_{2}$


Number Wheel: Numbers go from $-2^{(\mathbb{N}-1)}$ to $2^{(\mathrm{N}-1)}-1$


## Number Wheel: Carry and Overflow

- Carry applies to unsigned numbers - when adding or subtracting, result is incorrect.
- Overflow applies to signed numbers-when adding or subtracting, result is incorrect.


Blue: Unsigned Numbers
Red: Signed Numbers
Binary, Hex and Decimal (Signed \& Unsigned) Numbers (4-bit representation)

| Binary | Hex | Decimal |  |
| :---: | :---: | :---: | :---: |
|  |  | Unsigned | Signed |
| 0000 | 0 | 0 | 0 |
| 0001 | 1 | 1 | 1 |
| 0010 | 2 | 2 | 2 |
| 0011 | 3 | 3 | 3 |
| 0100 | 4 | 4 | 4 |
| 0101 | 5 | 5 | 5 |
| 0110 | 6 | 6 | 6 |
| 0111 | 7 | 7 | 7 |
| 1000 | 8 | 8 | -8 |
| 1001 | 9 | 9 | -7 |
| 1010 | A | 10 | -6 |
| 1011 | B | 11 | -5 |
| 1100 | C | 12 | -4 |
| 1101 | D | 13 | -3 |
| 1110 | E | 14 | -2 |
| 1111 | F | 15 | -1 |

## Signed Number Representation in 2's Complement Form:

If most significant bit is 0 (most significant hex digit $0-7$ ), number is positive. Get decimal equivalent by converting number to decimal, and using + sign.

Example for 8-bit number:
$3 \mathrm{~A}_{16}->+\left(3 \times 16^{1}+10 \times 16^{0}\right)_{10}$
$+(3 \times 16+10 \times 1)_{10}$
$+58{ }_{10}$
If most significant bit is 1 (most significant hex digit $8-\mathrm{F}$ ), number is negative. Get decimal equivalent by taking 2 's complement of number, converting to decimal, and using - sign.

Example for 8-bit number:
$\mathrm{A} 3_{16} \rightarrow-(5 \mathrm{D})_{16}$
$-\left(5 \times 16^{1}+13 \times 16^{0}\right)_{10}$

- $(5 \times 16+13 \times 1)_{10}$
- $93{ }_{10}$

One's Complement Table Makes It Simple To Find 2's Complements

| 0 | F |
| :---: | :---: |
| 1 | E |
| 2 | D |
| 3 | C |
| 4 | B |
| 5 | A |
| 6 | 9 |
| 7 | 8 |

To take two's complement, add one to one's complement.
Take two's complement of D0C3 :
$2 \mathrm{~F} 3 \mathrm{C}+1=2 \mathrm{~F} 3 \mathrm{D}$

- Overflow and Carry assume you have a fixed word size
- A carry is generated when you add two unsigned numbers together, and the result is too large to fit in the fixed word size.
- A carry is generated when you subtract two unsigned numbers, and the result should be negative.
- An overflow is generated when you add or subtract two signed numbers, and the fixed-length answer has the wrong sign.
Addition and Subtraction of Binary and Hexadecimal Numbers

1) Limit number of digits to specified word size.

4-bit word:

| 1101 |
| ---: |
| $+\quad 1011$ |
| 11000 |

Keep only 4 bits in answer
2) Does not matter if numbers are signed or unsigned - mechanics the same Do the operation, then determine if carry and/or overflow bits are set.

4-bit word:

$$
\begin{array}{r}
1101 \mathrm{Neg} \\
+1011 \mathrm{Neg} \\
\hline 11000 \mathrm{Neg}
\end{array}
$$

Carry is set, overflow is clear

## Condition Code Register (CCR) Gives Information On Result Of Last Operation

| S | X | H | I | N | Z | V | C |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Condition Code Register - 8 FFs

C - Carry $\quad: 1 \rightarrow$ last operation generated a carry
$\mathrm{V}-$ Overflow : $1 \rightarrow>$ last operation generated an overflow
Z - Zero $\quad: 1->$ result zero, $0->$ result not zero
N - Negative : most significant bit of result
I - Interrupt mask
H - Half carry
X - Interrupt mask
S - Stop disable

Note: Not all HC12 instructions change CCR bits. A bit in the CCR is the result of the last executed instruction which affects that bit. For example, consider the following instruction sequence:

| aba | ; Add B to A |
| :--- | :--- |
| staa $\$ 0900$ | ; Store A in address $\$ 0900$ |

The ABA instruction will change the $\mathrm{H}, \mathrm{N}, \mathrm{Z}, \mathrm{V}$ and C bits of the CCR. The STAA instruction will change the N and Z bit, and clear the V bit. After the two instructions, the H and C bits will reflect the result of the ABA instruction; the N and Z bits will reflect the result of the STAA instruction (was the number stored negative or zero), and the V bit will be 0 .

## Overflow occurs only under certain addition and subtraction operations.

- If you add a positive and a negative number, on overflow never occurs.
- If you subtract two positive numbers, an overflow never occurs.
- If you subtract two negative numbers, and overflow never occurs.

