EE 341 – Homework Chapter 1

1.5 Determine if the following CT signals are periodic. If yes, calculate the fundamental period T_0:

a) $x2(t) = |\sin(-\frac{5\pi t}{8} + \frac{\pi}{2})|$
b) $x4(t) = \exp(j\left(5t + \frac{\pi}{4}\right))$
c) $x6(t) = 2\cos\left(\frac{4\pi t}{5}\right) * \sin^2(\frac{16t}{3})$

a) $x2(t) = |\sin(-\frac{5\pi t}{8} + \frac{\pi}{2})|$

$x2(t) = |\sin(-\frac{5\pi t}{8} + \frac{\pi}{2})| = |\cos(\frac{5\pi t}{8})|$

So

$|\cos(\frac{5\pi t}{8} + \frac{5\pi T}{8})| = x2(t)$ if 5πT/8 or if T=8/5

b) $x4(t) = \exp(j\left(5t + \frac{\pi}{4}\right))$

All CT complex exponentials are periodic

x4(t) is periodic with fundamental period T=2π/5

c) $x6(t) = 2\cos\left(\frac{4\pi t}{5}\right) * \sin^2(\frac{16t}{3})$

$$x6(t) = 2\cos\left(\frac{4\pi t}{5}\right) * \sin^2\left(\frac{16t}{3}\right) = 2\cos\left(\frac{4\pi t}{5}\right) * \left(\frac{1}{2}\right)\left(1 - \cos\left(\frac{32t}{3}\right)\right)$$
$$= \cos\left(\frac{4\pi t}{5}\right) - \cos\left(\frac{4\pi t}{5}\right)\cos\left(\frac{32t}{3}\right)$$
$$= \cos\left(\frac{4\pi t}{5}\right) - \left(\frac{1}{2}\right)[\cos\left(\frac{4\pi}{5} - \frac{32}{3}\right)t + \cos\left(\frac{4\pi}{5} + \frac{32t}{3}\right)t]$$
$$= \cos\left(\frac{4\pi t}{5}\right) - \left(\frac{1}{2}\right)[\cos\left(\frac{12\pi - 160}{15}t\right) - (\frac{1}{2})\cos\left(\frac{12\pi + 160}{15}t\right)$$

Periods T1=5/2, T2=(30π)/(12π-160), T3==(30π)/(12π+160)

x6(t) is periodic if all combinations T1/T2, T1/T3, T2/T3 are rational numbers.

Since T1/T2 (5/2)×((12π-160)/(30π)) is not a rational number, then x6(t) is not periodic.

1.9 Show that the average power of the CT period signal $x(t) = Asin(\omega 0t + \theta)$ with real-valued coefficient A, is given by $A^2/2$.

The signal has a period T0=2π/ω0.

$$Px = \left(\frac{1}{T0}\right)\int_0^{T0} sin^2(\omega 0t + \theta)dt = \left(\frac{A^2}{2T0}\right)\int_0^{T0}[1 - cos(2\omega 0t + 2\theta)]dt$$

$= \left(\frac{A^2}{2T0}\right)[t]_0^{T0} - \left(\frac{A^2}{4T0\omega0}\right)[\sin(2\omega0t + 2\theta)]_0^{T0}$ sinceT0=2π/ ω0 then we will have:

$$= \left(\frac{A^2}{2}\right) - \left(\frac{A^2}{4T0\omega0}\right)[\sin(2\theta + 4\pi) - \sin(2\theta)]$$

$$= \left(\frac{A^2}{2}\right) - \left(\frac{A^2}{4T0\omega0}\right)[\sin(2\theta) - \sin(2\theta)] = \left(\frac{A^2}{2}\right)$$

1.16 Consider the following signal:

$$x(t) = 3\sin\left(\frac{2\pi(t-T)}{5}\right)$$

Determine the values of T for which the resulting signal is (a) an even function, and (b) an odd function of the independent variable t.

a) For x(t) to be an even function then x(t)=x(-t) so T=Te,

$$3\sin\left(\frac{2\pi t}{5} - \frac{2\pi Te}{5}\right) = 3\sin\left(-\frac{2\pi t}{5} - \frac{2\pi Te}{5}\right) = -3\sin\left(\frac{2\pi t}{5} + \frac{2\pi Te}{5}\right)$$

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x(t) x(-t)

$$3\sin\left(\frac{2\pi t}{5} - \frac{2\pi Te}{5}\right) = 3\sin\left(\frac{2\pi t}{5} + \frac{2\pi Te}{5} + (2m+1)\pi\right)$$

By matching terms we will have:

$-\frac{2\pi Te}{5} = \frac{2\pi Te}{5} + (2m+1)\pi$ so Te=5(2m+1)/4

b) For x(t) to be an odd function then x(t)=-x(-t) so T=To

$$3\sin\left(\frac{2\pi t}{5} - \frac{2\pi To}{5}\right) = -3\sin\left(-\frac{2\pi t}{5} - \frac{2\pi To}{5}\right) = 3\sin\left(\frac{2\pi t}{5} + \frac{2\pi To}{5}\right)$$

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x(t) -x(-t)

$$3\sin\left(\frac{2\pi t}{5} - \frac{2\pi To}{5}\right) = 3\sin\left(\frac{2\pi t}{5} + \frac{2\pi To}{5} - (2m)\pi\right)$$

By matching terms we will have:

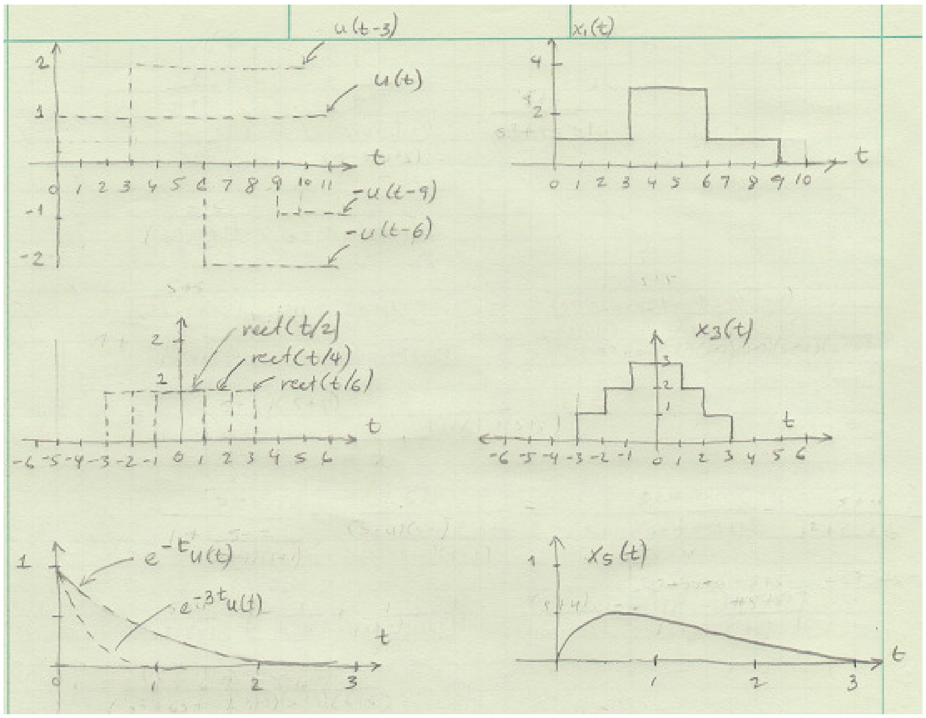
$-\frac{2\pi To}{5} = \frac{2\pi To}{5} - (2m)\pi$ so To=5m/2

1.18 Sketch the following CT signals:

a) $x1(t) = u(t) + 2u(t-3) - 2u(t-6) - u(t-9);$

b) $x3(t) = rect\left(\frac{t}{6}\right) + rect\left(\frac{t}{4}\right) + rect\left(\frac{t}{2}\right);$

c) $x5(t) = (\exp(-t) - \exp(-3t))u(t)$

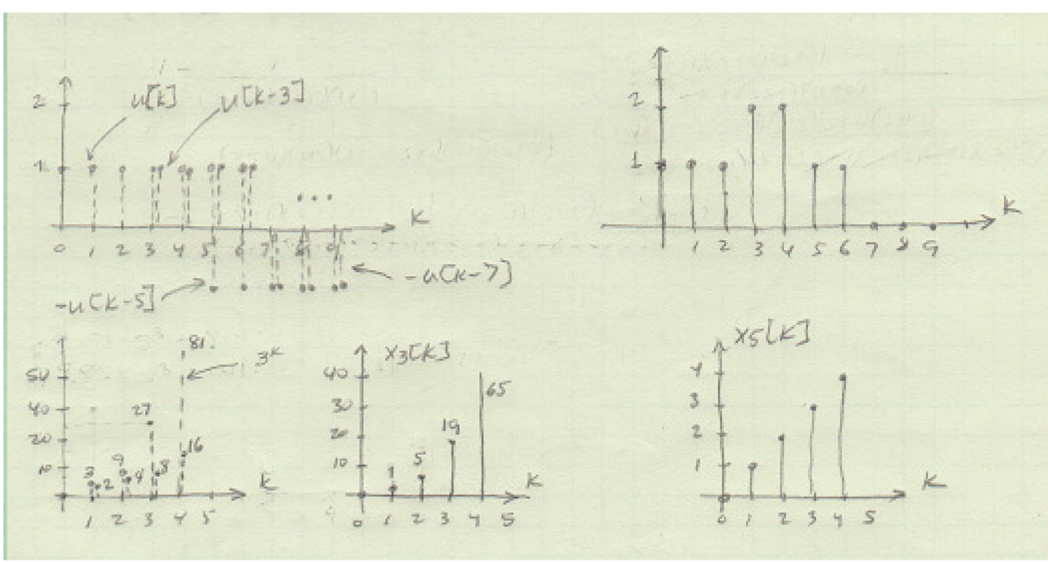


1.20 Sketch the following DT signals:

a) $x1[k] = u[k] + u[k-3] - u[k-5] - u[k-7]$

b) $x3[k] = (3^k - 2^k)u[k]$

c) $x5[k] = ku[k]$

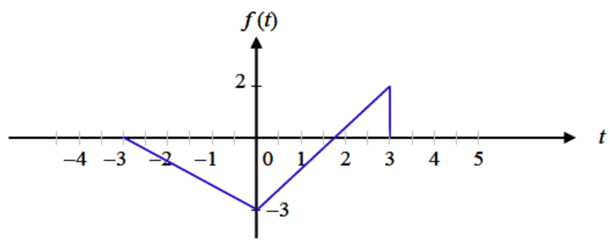


1.25 Consider the function f(t) shown in Fig. P1.25.

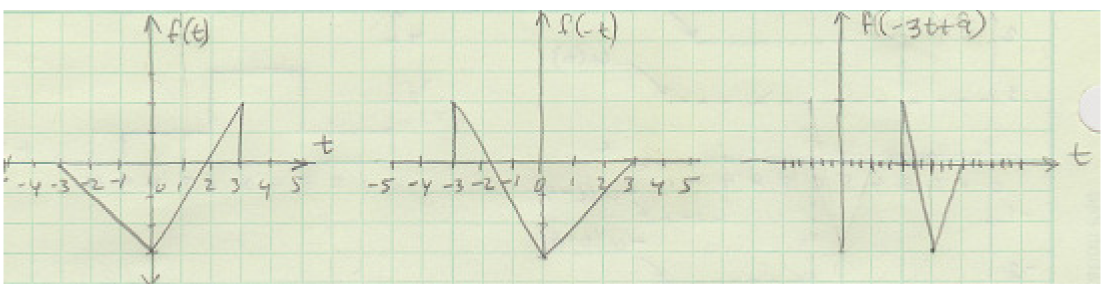
i) Sketch the function g(t)=f(-3t+9)

ii) Calculate the energy and power of the signal f(t). Is it a power signal or an energy signal?
The f(t) is a finite signal so it is an energy signal. The average power of f(t)=0.
The total energy is:

iii) Repeat (ii) for g(t).



(i) Sketch the function g(t)=f(-3t+9)



(ii) Calculate the energy and power of the signal f(t). Is it a power signal or an energy signal?
The f(t) is a finite signal so it is an energy signal. The average power of f(t)=0.
The total energy is:

$$E = \int_{-\infty}^{\infty} f^2(t)dt = \int_{-3}^{0} (-t-3)^2 dt + \int_{0}^{3} (\frac{5}{3}t - 3)^2 dt$$

$$E = \int_{-3}^{0} (t^2 + 6t + 9)dt + \int_{0}^{3} (\frac{25t^2}{9} - 10t + 9)dt = 16$$

(iii) Repeat (ii) for g(t).

The function g(t) can be represented as g(t)= $\begin{cases} -5t+12 & 2 \le t \le 3 \\ 3t-12 & 3 \le t \le 4 \end{cases}$

Since g(t) is a finite signal, it is an energy signal. The average power of g(t)=0.
The total energy is:

$$E = \int_{-\infty}^{\infty} g^2(t)dt = \int_{2}^{3} (-5t+12)^2 dt + \int_{3}^{4} (3t-12)^2 dt$$

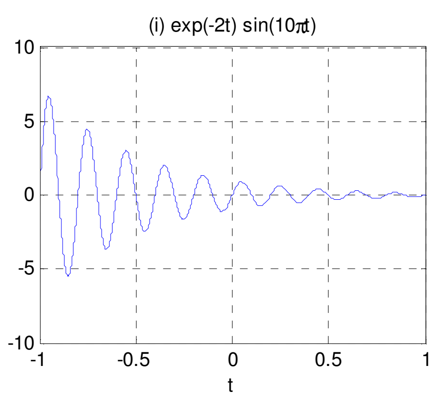
$$E = \int_{-3}^{0} (25t^2 - 120t + 144)dt + \int_{0}^{3} (9t^2 - 72t + 144)dt = 16/3$$

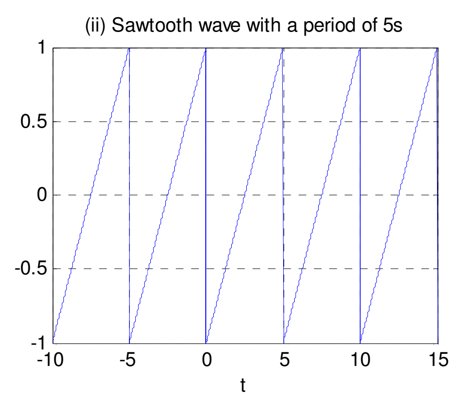
1.31 (MATLAB exercise) Write a set of MATLAB functions that compute and plot the following CT signals. In each case, use a sampling interval of 0.001s.
(i) x(t)=exp(-2t)sin(10πt) for|t|≤1.

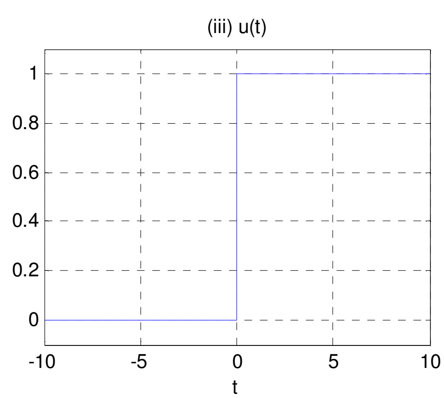
(ii) A periodic signal x(t) with fundamental period T=5. The value over one period is given by

x(t)=5t 0≤t<5

Use the sawtooth function available in MATLAB to plot five periods of x(t) over the range -10≤t<15.
(iii) The unit step function u(t) over [-10,10] using the sign function available in MATLAB.





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% part (i)
t = -1:0.001:1; x = exp(-2*t).*sin(10*pi*t);
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subplot(221); plot(t,x); xlabel('t');
title('(i) exp(-2t) sin(10\pit)');
grid;

% part (ii)
t = -10:0.001:15; x = sawtooth(2*pi*t/5);
subplot(222); plot(t,x); xlabel('t');
title('(ii) Sawtooth wave with a period of 5s');
grid;

% part (iii)
t = -10:0.001:10; x = 0.5*(1 + sign(t));
subplot(223); plot(t,x); xlabel('t');
title('(iii) u(t)');
grid; axis([-10 10 -0.1 1.1]);
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