EE 341 – Homework Chapter 6

6.4 The Laplace transforms of two CT signals x1(t) and x2(t) are given by the following expressions:

$$x1(t) \leftrightarrow \frac{s}{s^2+5s+6} \; with\, ROC(R1): \Re(s) > -2$$

and

$$x2(t) \leftrightarrow \frac{1}{s^2+5s+6} \; with\, ROC(R2): \Re(s) > -2$$

Determine the Laplace transform and the associated ROC R of the combined signal x1(t) +2x2(t). Explain how the ROC R of the combined signal exceeds the intersection (R1∩R2) of the individual ROCs R1 and R2

6.12 Using the initial- and final-value theorems, calculate the initial and final values of the causal CT functions with the following unilateral Laplace transforms. In each case, first determine the ROC to see if the initial value exists. Hint: use the Property 2 which states: For a right sided (causal) function, the ROC takes the form Re{s} > σ0 and consists of the right side of the complex s-plane.

(a) $X(s) = \frac{s}{s^2+7s+1}$

(b) $X(s) = \frac{s}{s^2+5s-4}$

(c) $X(s) = \frac{s^2+9}{s^2-25}$

6.15 Sketch the locations of the poles and zeros for the following transfer functions, and determine if the corresponding causal systems are stable, unstable, or marginally stable:

(a) $H(s) = \frac{s^2+1}{s^2+2s+1}$

(b) $H(s) = \frac{2s+5}{s^2+s-6}$

(c) $H(s) = \frac{3s+10}{s^2+9s+18}$

6.21 Consider the RLC series circuit shown in Fig. 3.1. The relationship between the input voltage x(t) and the output voltage w(t) is given by the following differential equation:

$$\frac{d^2w}{dt^2}+\frac{R}{L}\frac{dw}{dt}+\frac{1}{L}w(t)=\frac{1}{LC}x(t)$$

By determining the locations of the poles of the T.F. describing the RLC series circuit, show that the causal implementation of the RLC circuit is always stable for positive values (R>0, L>0, and C>0) of the passive components.

6.25 Determine the overall T.F. for the three interconnected systems shown in Fig. P6.25.