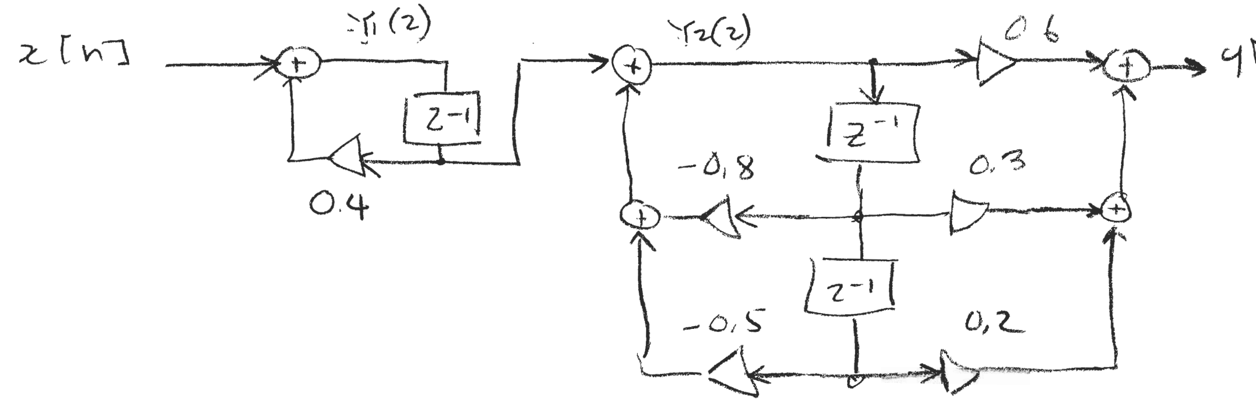
7 (c)(d) Analize the block diagrams of figure P2.1 and develop the relation $y[n]$ and $x[n]$

c)



$$Y_1(z) = X(z) + 0.4z^{-1}Y_1(z)$$

$$Y_1(z)[1-0.4z^{-1}] = X(z) \qquad Y_1(z) = \frac{1}{1-0.4z^{-1}}X(z)$$

$$Y_2(z) = Y_1(z) - 0.8z^{-1}Y_2(z) - 0.5z^{-2}Y_2(z)$$

$$Y_2(z)[1+0.8z^{-1}+0.5z^{-2}] = Y_1(z)$$

$$Y_2(z) = \frac{1}{1+0.8z^{-1}+0.5z^{-2}}z^{-1}Y_1(z)$$

$$Y(z) = 0.6Y_2(z) + 0.3z^{-1}Y_2(z) + 0.2z^{-2}Y_2(z)$$

$$Y(z) = (0.6+0.3z^{-1}+0.2z^{-2})Y_2(z)$$

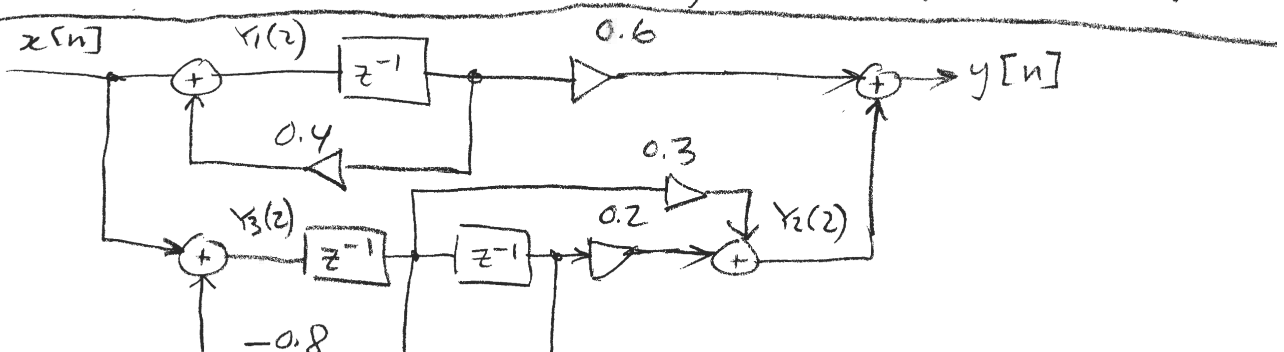
$$Y(z) = (0.6+0.3z^{-1}+0.2z^{-2})\left(\frac{1}{1+0.8z^{-1}+0.5z^{-2}}\right)\left(\frac{z^{-1}}{1-0.4z^{-1}}\right)X(z)$$

$$Y(z) = \frac{0.6z^{-1}+0.3z^{-2}+0.2z^{-3}}{1+0.8z^{-1}+0.5z^{-2}-0.4z^{-1}-0.32z^{-2}-0.20z^{-3}}X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{0.6+0.3z^{-1}+0.2z^{-2}}{1+0.4z^{-1}+0.18z^{-2}-0.2z^{-3}}$$

$y(n) = 0.6x(n) + 0.3x(n-1) + 0.2x(n-2) - 0.4y(n-1) - 0.18y(n-2) + 0.2y(n-3)$

d)



$$Y_1(z) = X(z) + 0.4z^{-1}Y_1(z)$$

$$Y_1(z) = \frac{1}{1-0.4z^{-1}} X(z)$$

$$Y(z) = 0.6z^{-1}Y_1(z) + Y_2(z)$$

$$Y_2(z) = 0.3z^{-1}Y_3(z) + 0.2z^{-2}Y_3(z), \quad Y_2(z) = (0.3z^{-1} + 0.2z^{-2})Y_3$$

$$Y_3(z) = X(z) - 0.8z^{-1}Y_3(z) - 0.5z^{-2}Y_3(z)$$

$$Y_3(z) = \frac{1}{1+0.8z^{-1}+0.5z^{-2}} X(z)$$

$$Y(z) = \left[0.6\left(\frac{z^{-1}}{1-0.4z^{-1}}\right) + (0.3z^{-1}+0.2z^{-2})\left(\frac{1}{1+0.8z^{-1}+0.5z^{-2}}\right)\right]X(z)$$

$$Y(z) = \left[\frac{0.6z^{-1}}{1-0.4z^{-1}} + \frac{0.3z^{-1}+0.2z^{-2}}{1+0.8z^{-1}+0.5z^{-2}}\right]X(z)$$

$$Y(z) = \frac{0.6z^{-1} + 0.48z^{-2} + 0.30z^{-3} + 0.3z^{-1} + 0.2z^{-2} - 0.12z^{-2} - 0.08z^{-3}}{(1-0.4z^{-1})(1+0.8z^{-1}+0.5z^{-2})} \Big] X(z)$$

$$= \frac{0.9z^{-1} + 0.56z^{-2} + 0.22}{1 - 0.4z^{-1} + 0.8z^{-1} - 0.32z^{-2} + 0.5z^{-2} - 0.20z^{-3}} X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{0.9z^{-1} + 0.56z^{-2} + 0.22}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

$$y(n) = 0.9x(n-1) + 0.56x(n-2) + 0.22x(n) - 0.4y(n-1) - 0.18y(n-$$

$$+ 0.2y(n-3$$

2.4 Express the length-4 sequence $x[n] = \{1\ 3\ {-2}\ 4\}$, $0 \le n \le 3$ in terms of a unit step sequence $\mu[n]$.

Recall $\mu[n] - \mu[n-1] = \delta[n]$

$$x[n] = \delta[n] + 3\delta[n-1] - 2\delta[n-2] + 4\delta[n-3]$$

$$= \mu[n] - \mu[n-1] + 3[\mu[n-1] - \mu[n-2]] - 2[\mu[n-2] - \mu[n-3]] + 4[\mu[n-3] - \mu[n-4]]$$

$$= \mu[n] + 2\mu[n-1] - 5\mu[n-2] + 6\mu[n-3] - 4\mu[n-4]$$

2.12 Let $x_{ev}[n]$ and $x_{od}[n]$ be even and odd sequences, respectively. Which one is an even or odd sequence?

(a) $g[n] = x_{ev}[n]\, x_{ev}[n]$

$g[-n] = x_{ev}[-n]\, x_{ev}[-n] = x_{ev}[n]\, x_{ev}[n] = g[n] \rightarrow$ $g[n]$ even sequence

(b) $u[n] = x_{ev}[n]\, x_{od}[n]$. Thus $u[-n] = x_{ev}[-n]\, x_{od}[-n]$

$= x_{ev}[-n]\,(-x_{od}[n]) = -u[n] \longrightarrow$ $u[n]$ odd sequence

(c) $v[n] = x_{od}[n]\, x_{od}[n]$. Thus $v[-n] = x_{od}[-n]\, x_{od}[-n]$

$= (-x_{od}[n])(-x_{od}[n]) = x_{od}[n]\, x_{od}[n] = v[n]$

— $v[n]$ even sequence

2.15 Which sequence is bounded?

(a) $x[n] = A\alpha^n$ A, α complex, $|\alpha| < 1$

for $n < 0$, $|\alpha|^n$ becomes arbitrarily large. $\{x[n]\}$ not bounded

(b) $y[n] = A\alpha^n \mu[n]$ A, α complex, $|\alpha| < 1$

2.23 Compute the energy of the length-N sequence $x[n] = \sin(2\pi kn/N)$ $0 \le n \le N-1$

$$x[n] = \sin(2\pi kn/N), \quad 0 \le n < N-1$$

$$\mathcal{E}_x = \sum_{n=0}^{N-1} [x[n]]^2 = \sum_{n=0}^{N-1} \sin^2(2\pi kn/N)$$

$$= \frac{1}{2}\sum_{n=0}^{N-1}(1 - \cos(4\pi kn/N)) = \frac{N}{2} - \frac{1}{2}\cancel{\sum_{n=0}^{N-1}\cos(4\pi kn/N)}^{\to 0}$$

$$= \underline{\underline{\frac{N}{2}}}$$