39) A discrete time system is characterized by $y[n] = x^2[n] - x[n-1]x[n+1]$

Is it linear? time-invariant? causal?

$y_1[n] = x_1^2[n] - x_1[n-1]x_1[n+1]$

$y_2[n] = x_2^2[n] - x_2[n-1]x_2[n+1]$

then if $x[n] = \alpha x_1[n] + \beta x_2[n]$

$$y[n] = (\alpha x_1[n] + \beta x_2[n])^2 - (\alpha x_1[n-1] + \beta x_2[n-1])(\alpha x_1[n+1] + \beta x_2[n+1])$$

$$\neq \alpha y_1[n] + \beta y_2[n]$$ **NONLINEAR**

Let $y_1[n] = x_1^2[n] - x_1[n-1]x_1[n+1]$

if $x_1 = x[n-n_0]$ then $y_1[n] = x_1^2[n-n_0] - x_1[n-1-n_0]\,x[n+1-n_0]$

$\equiv y[n-n_0]$ **TIME-INVARIANT**

The impulse response $h[n] = \delta^2[n] - \delta[n-1]\delta[n+1]$

$= \delta[n]$

since $h[n] = 0$ $n<0$ NON-**CAUSAL**

6) Develop a closed-loops expression for

(a) $\alpha^n u[n] \circledast u[n]$

This one's ok.

$$\alpha^n u[n] \circledast u[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k]\, u[n-k] = \sum_{k=0}^{\infty} \alpha^k u[n-k]$$

$= \{ \sum^{n} \alpha^k$ $n \geq 0$ From tables $1 - \alpha^{k+1}$

This is O.K.

(b) $n\alpha^n \mu[n] \circledast \mu[n]$

$$n\alpha^n \mu[n] \circledast \mu[n] = \sum_{k=-\infty}^{\infty} k\alpha^k \mu[n]\mu[n-k] = \sum_{k=0}^{\infty} k\alpha^k \mu[n-k]$$

$$= \begin{cases} \sum_{k=0}^{n} k\alpha^k, & n \geq 0 \\ 0 & n \leq 0 \end{cases}$$

M2.6 Using program 2-4, investigate the effect of smoothing a signal by a moving-average filter of lengths 5, 7, and 9. Does the smoothing improve with increase in length? Effect of length on the delay between smoothed vs. nonsm

3.5 The finite-energy function $x_a(t) = \sin(t)/\pi t$ is not absolutely summable. Show that CTFT is

$$X_a(j\Omega) = \begin{cases} 1, & |\Omega| \leq 1 \\ 0 & |\Omega| > 1 \end{cases}$$

$$x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}\,d\Omega = \frac{1}{2\pi}\int_{-1}^{1} e^{j\Omega t}$$

$$= \frac{1}{2\pi j t}\left(e^{j\Omega} - e^{j\Omega}\right) = \frac{\sin(t)}{\pi t}$$

3.13 Determine the DTFT of the two-sided sequence $y[n] = \alpha^{|n|}$, $|\alpha| < 1$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \alpha^{|n|} e^{-j\omega n}$$

$$T(e^{jw}) = \sum_{n=1}^{\infty} (\alpha e^{jw})^n + \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$

$$= \sum_{n=0}^{\infty} (\alpha e^{jw})^n - 1 + \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$

$$= \frac{\alpha e^{jw}}{1 - \alpha e^{jw}} + \frac{1}{1 - \alpha e^{-jw}}$$

$$= \frac{\alpha e^{jw}(1 - \alpha e^{-jw}) + 1 - \alpha e^{jw}}{1 - \alpha e^{jw} - \alpha e^{-jw} + \alpha^2}$$

$$= \frac{\cancel{\alpha e^{jw}} - \alpha^2 + 1 - \cancel{\alpha e^{jw}}}{1 - \alpha(e^{jw} + e^{-jw}) + \alpha^2}$$

$$= \frac{1 - \alpha^2}{1 - 2\alpha \cos(jw) + \alpha^2}$$