(**20 points**) **5.4** Consider the system in 5.3(ii) with a required steady-state error of 20%, and an adjustable PI controller zero location. Show that the corresponding closed-loop characteristic equation is given by

$$1 + K\left(\frac{s+a}{s}\right)\frac{1}{(s+3)(s+5)} = 0$$

Next, rewrite the equation as

 $1 + K_f G_f(s) = 0$

where $K_f = K$ is a constant and $G_f(s)$ is a function of *s*, and examine the effect of shifting the zero on the closed-loop poles.

a) Design the system for a dominant second order pair with a damping ratio of 0.5. What is ω_n for this design?

b) Obtain the time response using a CAD program. How does the time response compare with that of a second order system with the same ω_n and ζ as the dominant pair? Give reasons for the differences.

c) Discuss briefly the tradeoff between error, speed of response and relative stability in this problem.

(**20 points**) **5.8** Repeat Problem 5.7 with a required settling time less than 0.5 s and an undamped natural frequency of 10 rad/s.

Design a controller for the plant transfer function

$$G(s) = \frac{1}{(s+1)(s+5)}$$

(**20 points**) **6.5** Design a proportional controller for the system in Problem 6.1(a) to meet the following specifications here possible. If the design specification cannot be met, explain why and suggest a more appropriate controller.

(a) A damping ratio of 0.7.

- (b) A steady-state error of 10 % due to a unit step.
- (c) A steady-state error of 10 % due to a unit ramp.

(20 points) 6.11 Consider the DC motor position control system of Example 3.6, where the (type 1) analog plant has the transfer function

$$G(s) = \frac{1}{s(s+1)(s+10)}$$

and design a digital controller by using frequency response methods to obtain a settling time of about one second and an overshoot of less than 5%.

(20 points) 7.14 Autonomous underwater vehicle (AUV) are robotic submarines that can be used for a variety of studies of the underwater environment. The vertical and horizontal dynamics of the vehicle must be controlled to remotely operate the AUV. The INFANTE is a research AUV operated by the Instituto Superior Tecnico of Lisbon, Portugal. The variables of interest in horizontal motion are the sway speed and the yaw angle. A linearized model of the horizontal plane motion of the vehicle is given by

$$\begin{bmatrix} \dot{x}1\\ \dot{x}2\\ \dot{x}3 \end{bmatrix} = \begin{bmatrix} -0.14 & -0.69 & 0.0\\ -0.19 & -0.048 & 0.0\\ 0.0 & 1.0 & 0.0 \end{bmatrix} \begin{bmatrix} x1\\ x2\\ x3 \end{bmatrix} + \begin{bmatrix} 0.056\\ -0.23\\ 0.0 \end{bmatrix} u$$
$$\begin{bmatrix} y1\\ y2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x1\\ x2\\ x3 \end{bmatrix}$$

where x_1 is the sway speed, x_2 is the yaw angle, x_3 is the yaw rate and u is the rudder deflection. Obtain the discrete state-space model for the system with a sampling period of 50 ms.



The INFANTE vehicle (Source: from paper by Silvester and Pascoal, "Control of the INFANTE UAV using gain scheduled static output feedback", *Control Engineering Practice*, 12 (2004) 1501-1509).