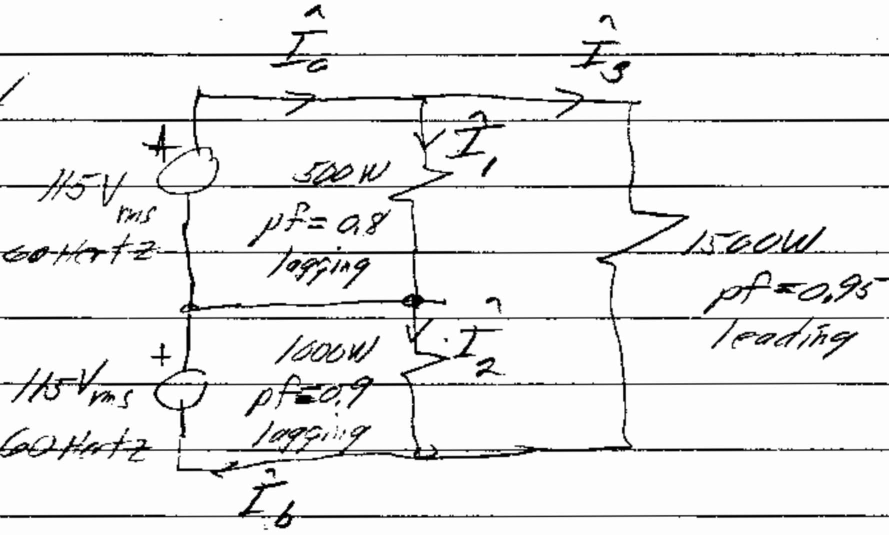
EE 212 Homework 1

9.34



$P = V_{rms} I_{rms} (pf)$

$\hat{I}_1 = \frac{500}{115 \times 0.8} e^{-j\cos^{-1}0.8} = 5.435 e^{-j36.87^\circ} = 4.348 - j3.26$

$\hat{I}_2 = \frac{1000}{115 \times 0.9} e^{-j\cos^{-1}0.9} = 9.66\, e^{-j25.842^\circ} = 8.69 - j4.21$

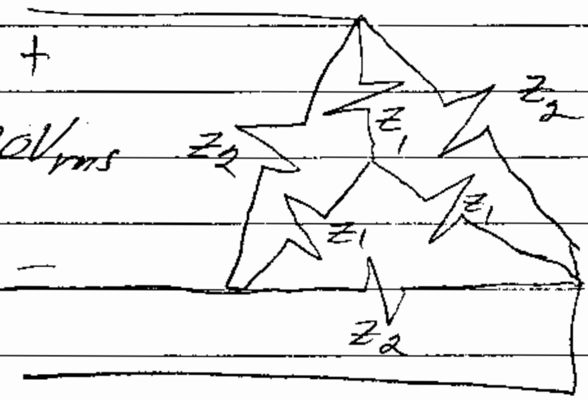
$\hat{I}_3 = \frac{1500}{230 \times 0.95} e^{j\cos^{-1}0.95} = 6.865 e^{j18.19^\circ} = 6.52 + j2.14$

$$\left\{ \begin{array}{l} \text{so } \hat{I}_a = \hat{I}_1 + \hat{I}_3 = 10.87 - j1.12 = 10.93 e^{-j5.88^\circ} \leftarrow \\ \hat{I}_b = \hat{I}_2 + \hat{I}_3 = 15.21 - j2.07 = 15.35 e^{-j7.75^\circ} \leftarrow \end{array} \right\}$$

EE 212 Homework 2

9.44

{balanced 3φ source L-L voltages are 120 V_{rms}} 120 V_{rms}



$Z_1 = 3+j4$

$Z_2 = 5-j12$

{for balanced loa[d] $Z_Y = \frac{1}{3} Z_\Delta$

converting Y to Δ gives $Z_{\Delta_{equiv.}} = 3Z_Y = 9+j12$

this gives a balanced Δ load with impedances of $(5-j12)$ in parallel with $9+j12$

$$Z_{\Delta_{total}} = \frac{(5-j12)(9+j12)}{14} = \frac{13 e^{-j67.38°}\, 15 e^{j53.13}}{14} = 13.93 e^{-j14.25}$$

$$I_\Delta = \frac{120}{Z_{\Delta_{total}}} = \frac{120}{13.93} e^{j14.25} = 8.615 e^{j14.25}$$

$$P_{ave_{total}} = 3\times \mathrm{Re}\{120 \times 8.615 e^{-j14.25°}\} = 3005.97\,W$$

$$pf = \cos(14.25°) = 0.97 \text{ leading}$$

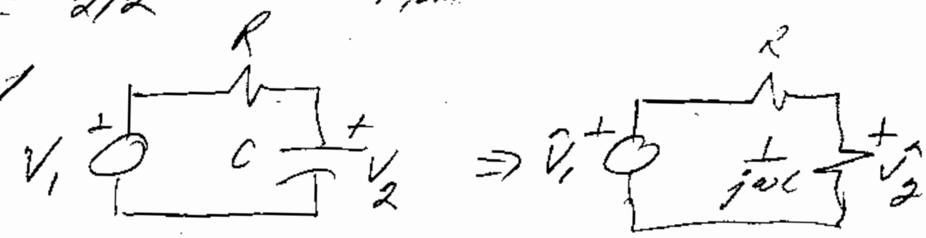
Δ

If you assume the 120 V_{rms} is line to neutral then

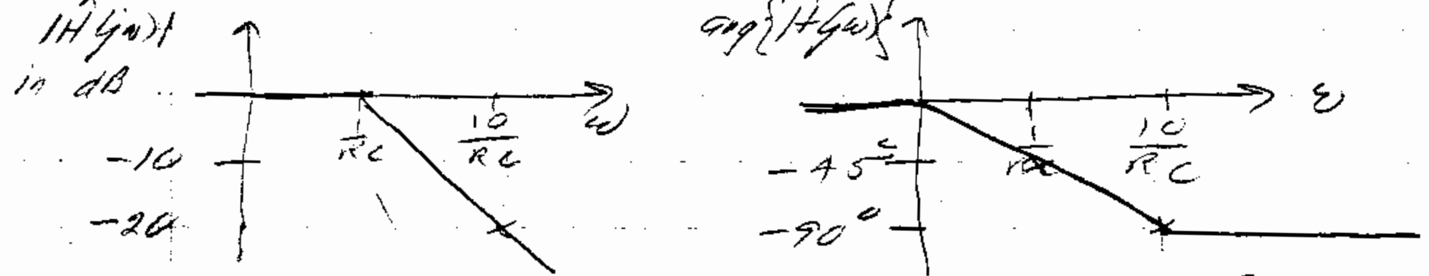
$$P_{ave_{total}} = 3 \times 3005.97 = 9{,}017.9 \text{ Watts}$$

EE 212 Homework 3

10.1

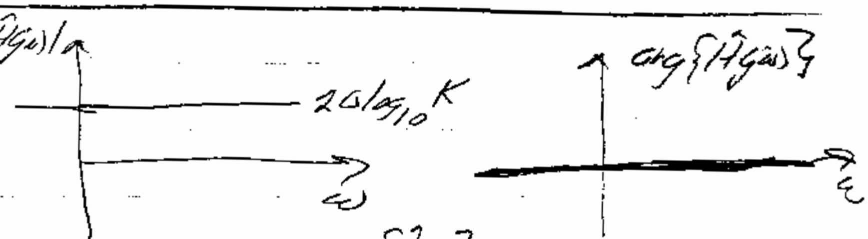


$$\hat{H}(j\omega) = \frac{\hat{V}_2}{\hat{V}_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} \quad (\text{low pass})$$

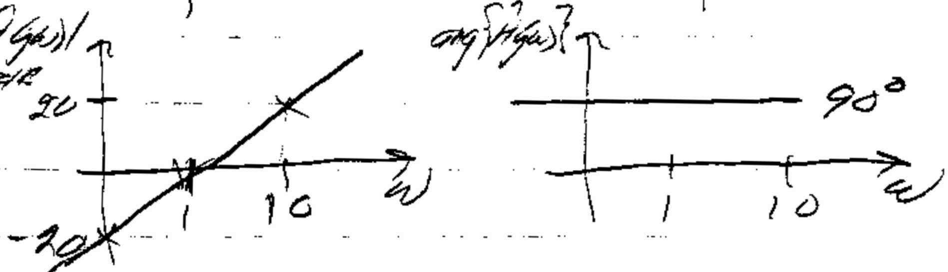


half power when $\omega RC = 1$ or $\boxed{\omega = \frac{1}{RC}}$

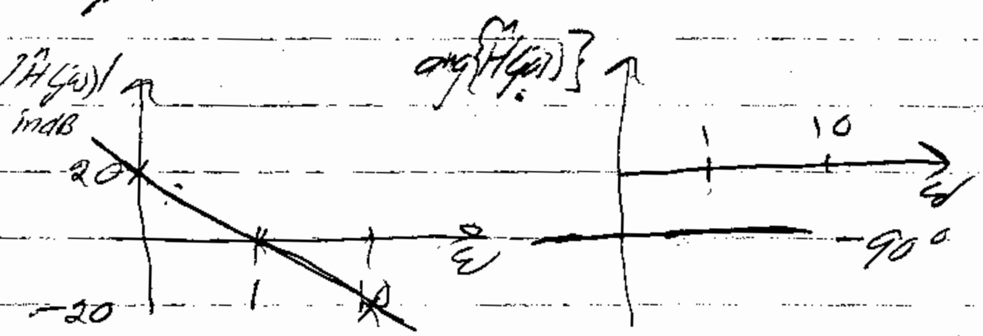
10.11 a) $\hat{H}(j\omega) = K$



b) $\hat{H}(j\omega) = j\omega$



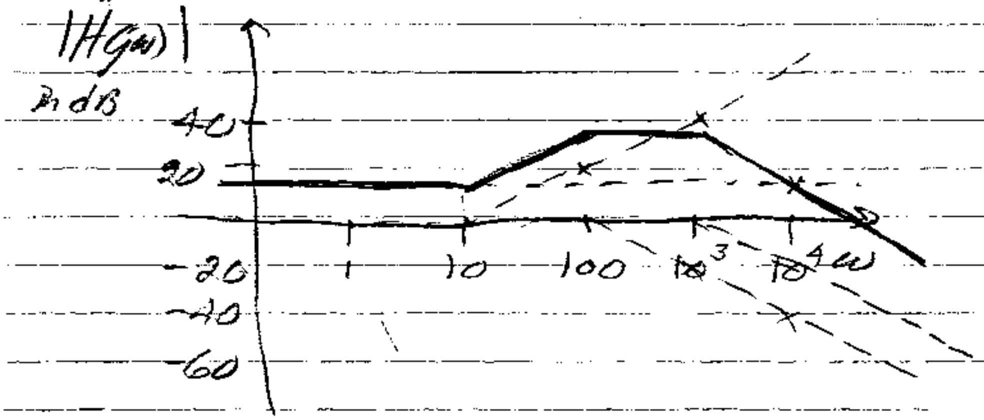
c) $H(j\omega) = \frac{1}{j\omega}$

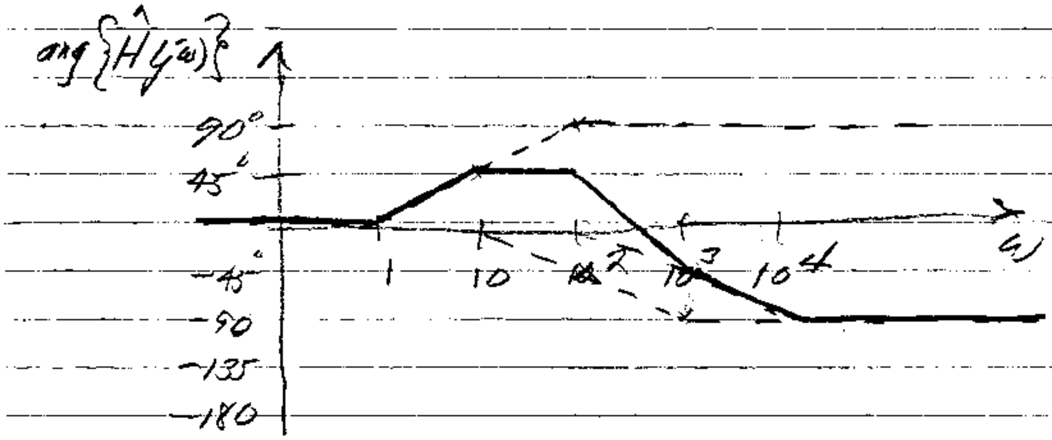


EE 211 Homework 4

Special! Bode plot for $H(j\omega) = \frac{5(1+j\frac{\omega}{10})}{(1+j\frac{\omega}{100})(1+j\frac{\omega}{1000})}$

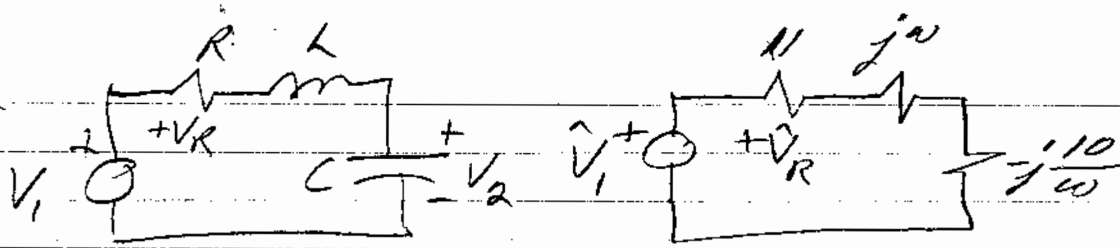
$20\log_{10} 5 \cong 14\,dB$





EE 212 Homework 5

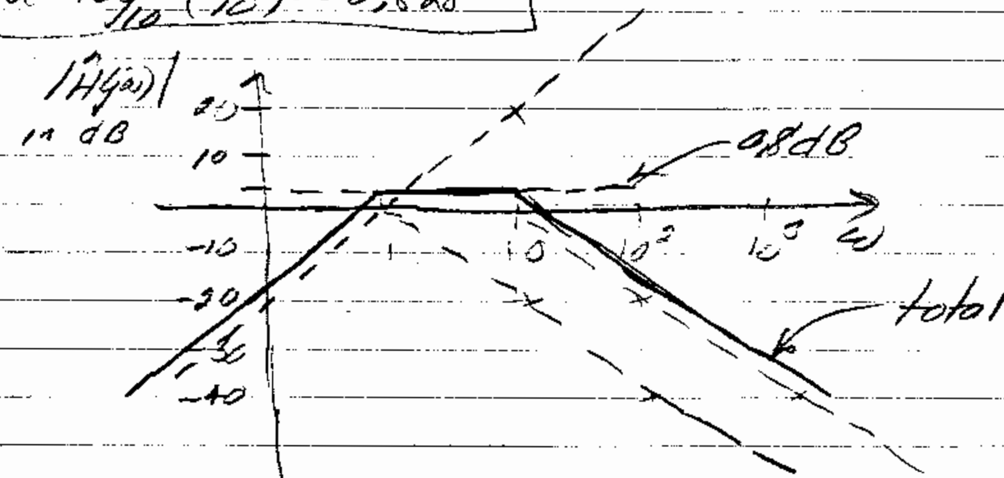
10.12

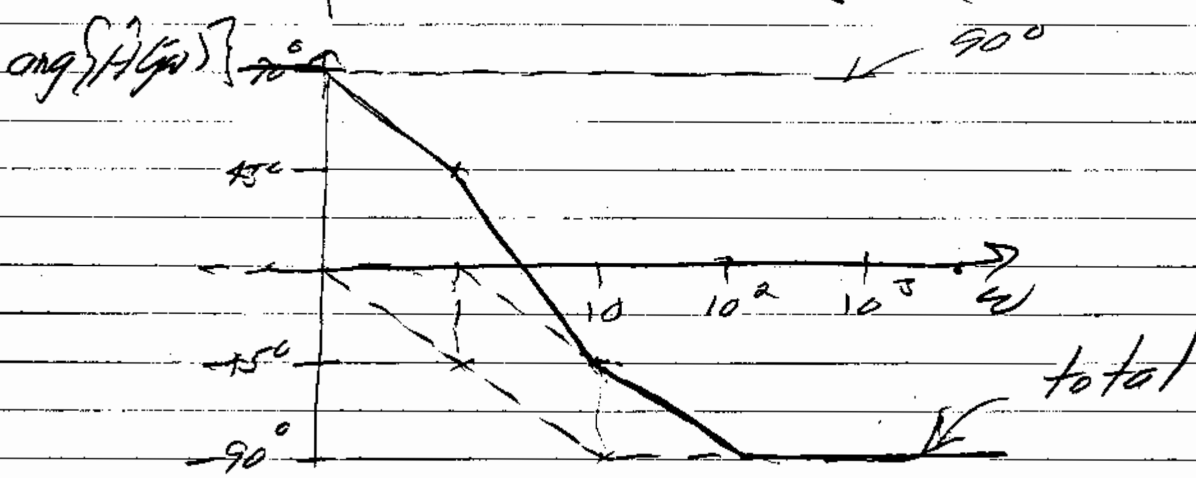


$$\hat{H}(j\omega) = \frac{\hat{V}_R}{\hat{V}_1} = \frac{11}{11 + j\omega + \frac{10}{j\omega}} = \frac{11 j\omega}{(j\omega)^2 + 11 j\omega + 10}$$

$$\hat{H}(j\omega) = \frac{11 j\omega}{(j\omega+1)(j\omega+10)} = \frac{11}{10} \times \frac{j\omega}{(1+j\omega)(1+j\frac{\omega}{10})}$$

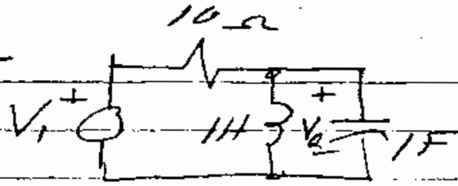
$$\boxed{20\log_{10}\left(\frac{11}{10}\right) = 0.828}$$



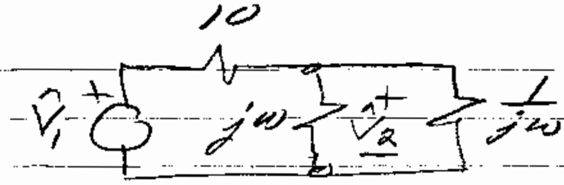


EE 212 Homework 6

10.15



$\Rightarrow$



$$\hat{H}(j\omega) = \frac{\hat{V}_2}{\hat{V}_1} = \frac{\frac{1}{j\omega + \frac{1}{j\omega}}}{10 + \frac{1}{j\omega + \frac{1}{j\omega}}} = \frac{\frac{j\omega}{(j\omega)^2+1}}{10 + \frac{j\omega}{(j\omega)^2+1}}$$

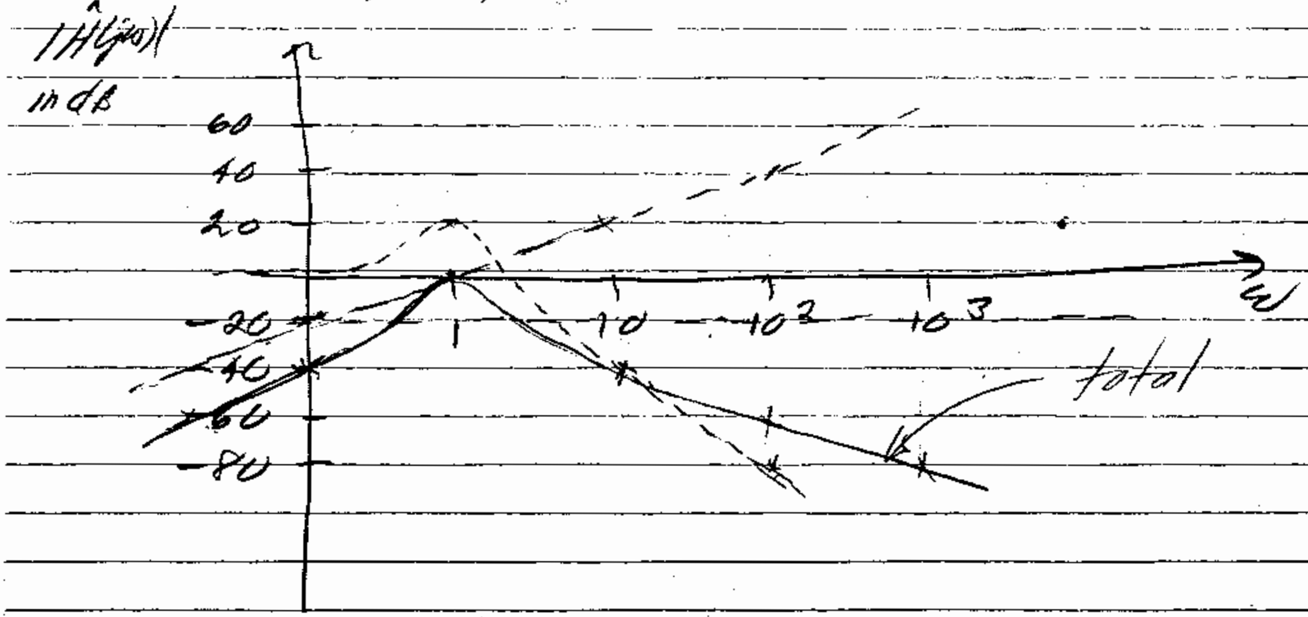
$$\hat{H}(j\omega) = \frac{j\omega}{10(j\omega)^2 + j\omega + 10} = \frac{\left(\frac{1}{10}\right) j\omega}{(j\omega)^2 + 1 + j\frac{\omega}{10}}$$

$$20\log_{10}|\hat{H}(j\omega)| = 20\log_{10}\omega + \underbrace{20\log_{10}(10^{-1})}_{= -20\,dB} - \underbrace{20\log_{10}\sqrt{[1-(\omega)^2]^2 + \left[\frac{\omega}{10}\right]^2}}_{\text{for large } \omega \text{ this becomes } -20\log_{10}\omega^2}$$

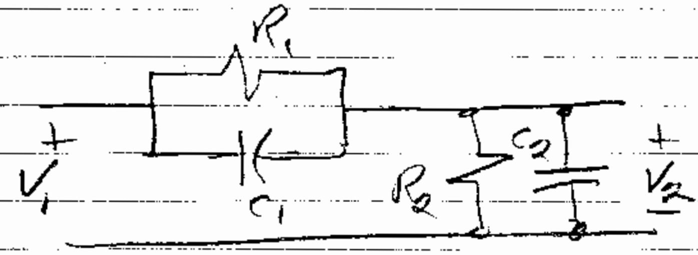
→ This is a curve of slope -40 dB/decade and a corner frequency of $\omega = 1$

At $\omega=1$ the magnitude of this last term is:

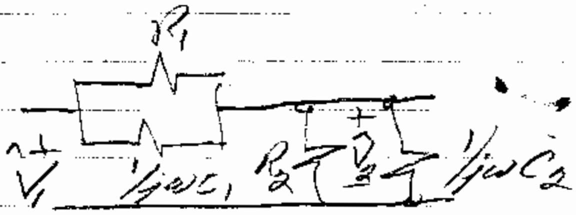
$$-20\log_{10}\sqrt{\frac{1}{10^2}} = 20\,dB$$



Special Problem $\times 10$ scope probe



$\Rightarrow$

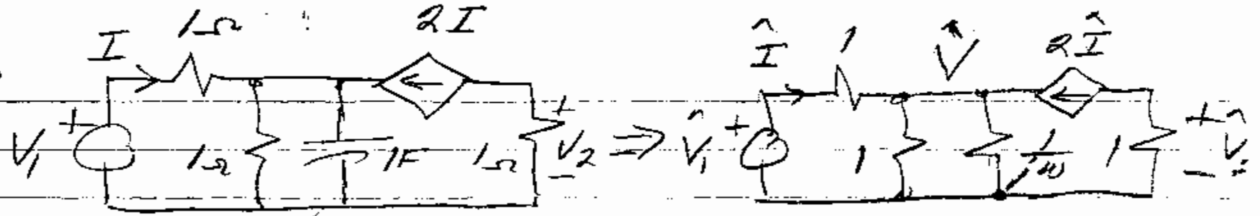


$$\frac{\hat{V}_2}{\hat{V}_1} = \frac{\dfrac{R_2/j\omega C_2}{R_2 + 1/j\omega C_2}}{\dfrac{R_1\left(\frac{1}{j\omega C_1}\right)}{R_1 + \frac{1}{j\omega C_1}} + \dfrac{R_2/j\omega C_2}{R_2 + 1/j\omega C_2}} = \frac{\dfrac{R_2}{1 + j\omega R_2 C_2}}{\dfrac{R_1}{1 + j\omega R_1 C_1} + \dfrac{R_2}{1 + j\omega R_2 C_2}}$$

if $R_1 C_1 = R_2 C_2$ then $\dfrac{\hat{V}_2}{\hat{V}_1} = \dfrac{R_2}{R_1 + R_2}$

EE 212 Homework 7

10.9



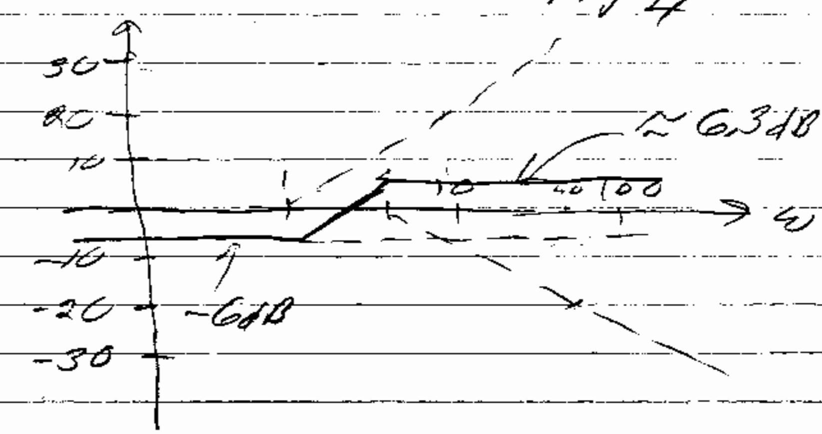
$$\boxed{\hat H(j\omega) = \frac{\hat V_2}{\hat V_1}} \qquad \frac{\hat V - \hat V_1}{1} + \frac{\hat V}{1} + j\omega \hat V - 2\hat I = 0 \quad \text{but } \hat I = \frac{\hat V_1 - \hat V}{1}$$

$$\therefore \hat V(1+1+j\omega+2) = \hat V_1 + 2\hat V_1$$

$$\hat V = \frac{3\hat V_1}{4+j\omega} \quad \text{and } \hat V_2 = -2\hat I = -2\left(\frac{\hat V_1 - \hat V}{1}\right)$$

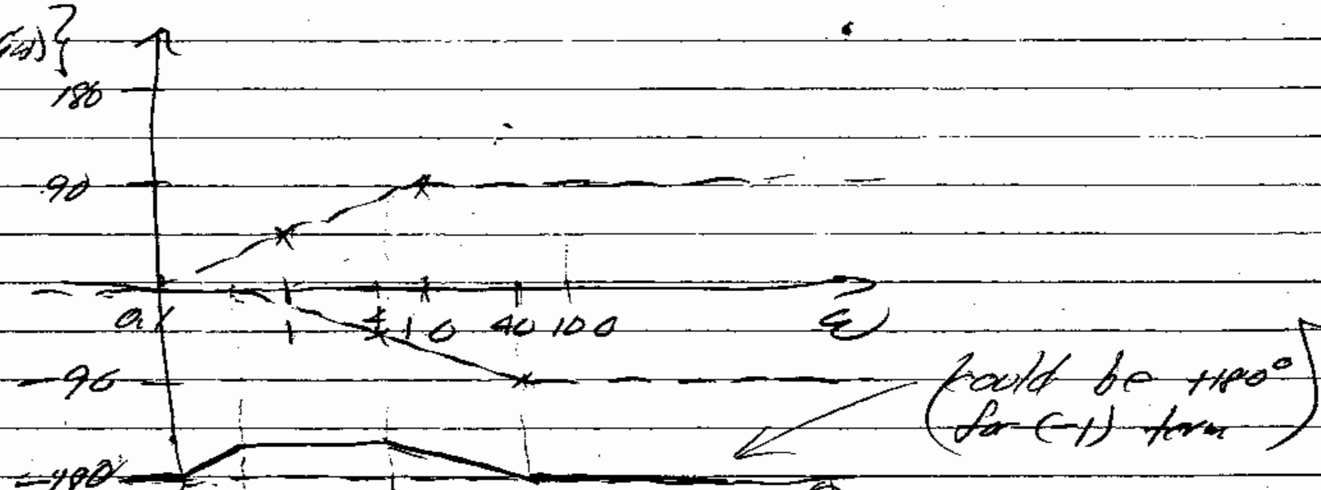
$$\therefore \hat V_2 = -2\left(\hat V_1 - \frac{3\hat V_1}{4+j\omega}\right) = -2\hat V_1\left(\frac{4+j\omega-3}{4+j\omega}\right) = \hat V_1 \frac{-2-j2\omega}{4+j\omega}$$

$$\text{or } |\hat H(j\omega)| = \frac{\hat V_2}{\hat V_1} = -\frac{1}{2}\,\frac{1+j\omega}{1+j\frac{\omega}{4}} \quad ; \quad 20\log_{10}\left(\tfrac{1}{2}\right) = -6\,dB$$



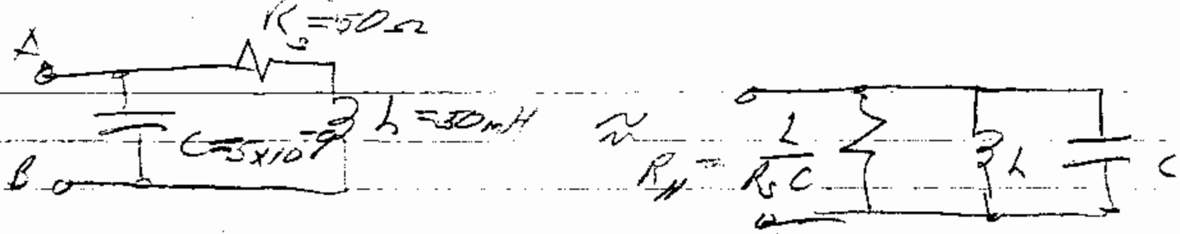
@ω=4 $20\log_{10}\sqrt{17} = 12.3$

$12.3 - 6 = 6.3\,dB$



EE 212 Homework 8

10-33

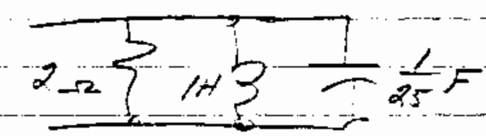


$$R_H = \frac{L}{R_s C} = \frac{50\times10^{-3}}{50\times5\times10^{-9}} = 0.2\times10^{6} = \boxed{200\,k\Omega}$$

$$Q = \frac{\omega_r L}{R_s} = \frac{L}{R_s}\cdot\frac{1}{\sqrt{LC}} = \frac{1}{R_s}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{50}\sqrt{\frac{50\times10^{-3}}{5\times10^{-9}}} = 0.02\sqrt{\frac{10^{-2}}{10^{-9}}} = \boxed{63.24}$$

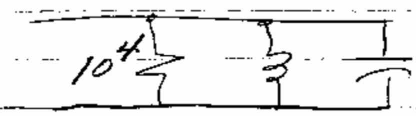
10-34



$K_m = 5\times10^{3}$

$K_f = 10^{5}$

scaled circuit

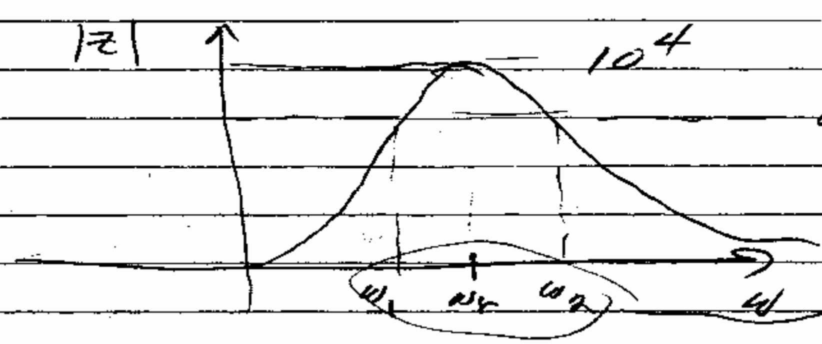


$$\begin{cases} R_{new} = 5\times10^{3}\times2 = 10^{4} \\ L_{new} = \frac{5\times10^{3}}{10^{5}}L = 5\times10^{-2} \\ C_{new} = \frac{C}{5\times10^{3}\times10^{5}} = 8\times10^{-11} \end{cases}$$

$$\omega_r = \frac{1}{\sqrt{L_{new}C_{new}}} = \frac{1}{\sqrt{40\times10^{-13}}} = \boxed{5\times10^{5}}$$

$$Q = R\sqrt{\frac{C}{L}} = 10^{4}\sqrt{\frac{8\times10^{-11}}{5\times10^{-2}}} = \boxed{0.4}$$

$$\omega_{1,2} = \mp\frac{\omega_r}{2Q} + \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2+1} = \pm 6.25\times10^{5} + 8\times10^{5} = \boxed{\begin{Bmatrix} 14.25\times10^{5} \\ 1.75\times10^{5} \end{Bmatrix}}$$



see above for values

EE 212 Homework 9

11.1 a) $\mathcal{L}\left[(2e^{-8t} - e^{-2t})u(t)\right] = \dfrac{2}{s+8} - \dfrac{1}{s+2}$ ←

b) $\mathcal{L}\left[(6 + 2e^{-6t} - 12e^{-t})u(t)\right] = \dfrac{6}{s} + \dfrac{2}{s+6} - \dfrac{12}{s+1}$ ←

c) $\mathcal{L}\left[(2+3t)e^{-2t}u(t)\right] = \dfrac{2}{s+2} + \dfrac{3}{(s+2)^2}$ ←

d) $\mathcal{L}\left[(\cos 4t - \sin 4t)e^{-3t}u(t)\right] = \dfrac{s+3}{(s+3)^2+16} - \dfrac{4}{(s+3)^2+16}$ ←

11.3 a) $\mathcal{L}\left[\sin(\beta t - \phi)u(t)\right] = \cos\phi \dfrac{\beta}{s^2+\beta^2} - \sin\phi \dfrac{s}{s^2+\beta^2}$ ←

b) $\mathcal{L}\left[\cos(\beta t - \phi)u(t)\right] = \cos\phi \dfrac{s}{s^2+\beta^2} + \sin\phi \dfrac{\beta}{s^2+\beta^2}$ ←

c) $\mathcal{L}\left[e^{-\alpha t}\sin(\beta t - \phi)u(t)\right] = \cos\phi \dfrac{\beta}{(s+\alpha)^2+\beta^2} - \sin\phi \dfrac{(s+\alpha)}{(s+\alpha)^2+\beta^2}$ ←

d) $\mathcal{L}\left[e^{-\alpha t}\cos(\beta t - \phi)u(t)\right] = \cos\phi \dfrac{s+\alpha}{(s+\alpha)^2+\beta^2} + \sin\phi \dfrac{\beta}{(s+\alpha)^2+\beta^2}$ ←

EE 212 Homework 10

11.11 a) $F(s) = \frac{6}{s(s+1)(s+3)} = \frac{2}{s} + \frac{-3}{s+1} + \frac{1}{s+3}$

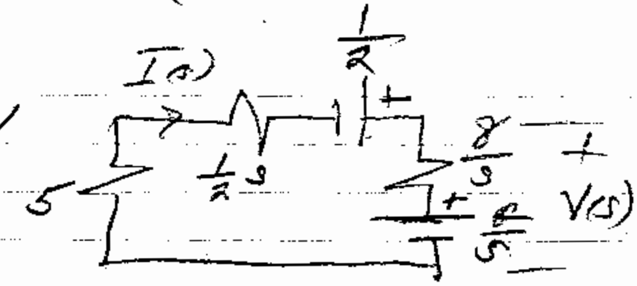
$\therefore f(t) = \{2 - 3e^{-t} + e^{-3t}\} u(t)$ ←

b) $F(s) = \frac{60(s+4)}{s(s+2)(s+12)} = \frac{10}{s} + \frac{-6}{s+2} + \frac{-4}{s+12}$

$f(t) = \{10 - 6e^{-2t} - 4e^{-12t}\} u(t)$ ←

EE 212 Homework 11

11.21



$i(0) = 1$

$v(0) = 2$

$$I(s) = \frac{\frac{1}{2} - \frac{2}{s}}{5 + \frac{s}{2} + \frac{8}{s}} = \frac{s-4}{s^2+10s+16} = \frac{s-4}{(s+2)(s+8)}$$

or $I(s) = \frac{-1}{s+2} + \frac{2}{s+8}$

$$\boxed{i(t) = \{2e^{-8t} - e^{-2t}\}u(t)} \leftarrow$$

$$V(s) = I(s)\frac{8}{s} + \frac{2}{s} = \frac{8(s-4)}{s(s+2)(s+8)} + \frac{2}{s}$$

$$V(s) = \frac{0}{s} + \frac{\frac{8(-6)}{-12}}{s+2} + \frac{\frac{\cancel{8}(-12)}{-\cancel{8}(-6)}}{s+8} = \frac{4}{s+2} - \frac{2}{s+8}$$

$$v(t) = \{4e^{-2t} - 2e^{-8t}\}u(t) \leftarrow$$

EE 212 Homework 12

11.14 b) $F(s) = \dfrac{(s+2)(s+3)}{s(s+1)^2} = \dfrac{1}{s+1}\left\{\dfrac{(s+2)(s+3)}{s(s+1)}\right\}$

$$\begin{array}{r} 1 \\ s^2+s\,\overline{)\,s^2+5s+6} \\ \underline{s^2+s} \\ 4s+6 \end{array} \quad \therefore \; F(s) = \frac{1}{s+1}\Big\{\underbrace{\frac{4s+6}{s(s+1)}}_{\frac{6}{s}+\frac{-2}{s+1}} + 1\Big\}$$

$$F(s) = \frac{1}{s+1}\left\{1 + \frac{6}{s} - \frac{2}{s+1}\right\} = \frac{1}{s+1} + \frac{6}{s(s+1)} - \frac{2}{(s+1)^2}\}$$

$$F(s) = \frac{1}{s+1} + \frac{6}{s} + \frac{-6}{s+1} - \frac{2}{(s+1)^2} = \frac{6}{s} - \frac{5}{s+1} - \frac{2}{(s+1)^2}$$

$$\therefore f(t) = \{6 - 5e^{-t} - 2te^{-t}\}u(t) \leftarrow$$

11.14 c) $F(s) = \dfrac{4(1-e^{-2s})}{s^2(s+2)} = \left\{\dfrac{K_1}{s} + \dfrac{2}{s^2} + \dfrac{1}{s+2}\right\}(1-e^{-2s})$

$F' \rightarrow$

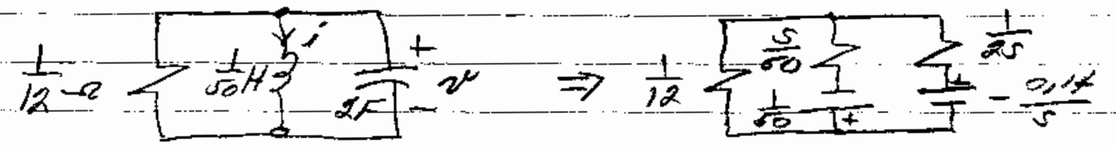
$$\frac{d}{ds}\{F'(s)s^2\}\Big|_{s=0} = K_1 = 4(-1)(s+2)^{-2}\Big|_{s=0} = -1$$

$$\therefore F(s) = \left\{-\frac{1}{s} + \frac{2}{s^2} + \frac{1}{s+2}\right\}(1-e^{-2s})$$

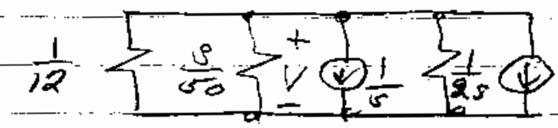
$$f(t) = \{-1 + 2t + e^{-2t}\}u(t) - \{-1 + 2(t-2) + e^{-2(t-2)}\}u(t-$$

EE 212 Homework 13

11.22 ($v(t)$ only) $v(0) = -0.14\,V$; $i(0) = 1A$



converting to current sources



$\frac{0.14/s}{1/2s} = 0.28$

KCL $12V + \frac{50V}{s} + \frac{1}{s} + V(2s) + 0.28 = 0$

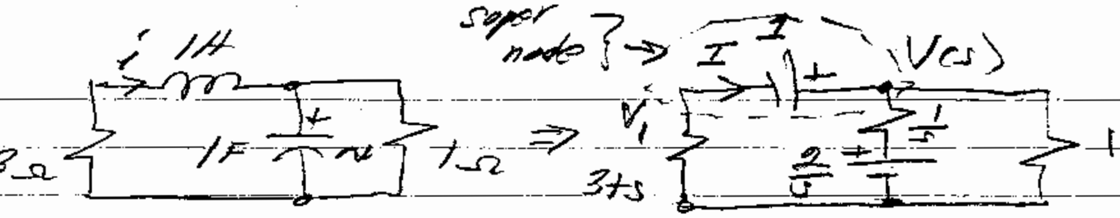
$$V = -\frac{\frac{1}{s} + 0.28}{12 + \frac{50}{s} + 2s} = -\frac{1 + 0.28s}{2s^2 + 12s + 50} = -\frac{0.5 + 0.14s}{s^2 + 6s + 25}$$

$$V(s) = -\frac{0.14(s+3) + 0.08\left(\frac{4}{4}\right)}{(s+3)^2 + 16}$$

$\therefore v(t) = -\left\{0.14e^{-3t}\cos 4t + 0.02e^{-3t}\sin 4t\right\} u(t)$

EE 212 Homework 14

11.23



$v(0)=2,\ i(0)=1$

$$\frac{V_1}{3+s}+\frac{V-\frac{2}{s}}{\frac{1}{s}}+\frac{V}{1}=0 \quad ; \quad V-V_1=1$$

so $\frac{V-1}{s+3}+\frac{V-\frac{2}{s}}{\frac{1}{s}}+V=0=\frac{V-1}{s+3}+Vs-2+V=0$

or $V-1+(Vs-2+V)(s+3)=0=V-1+Vs^2+3Vs-2s-6+Vs+3V$

$V(s^2+4s+4)=1+6+2s=2s+7$

$$V=\frac{2s+7}{s^2+4s+4}=\frac{2s+7}{(s+2)^2}=\frac{2}{(s+2)}+\frac{3}{(s+2)^2}$$

so $\boxed{v(t)=[2e^{-2t}+3te^{-2t}]u(t)}$

$$I(s)=-\frac{V_1}{3+s}=\frac{1-V}{3+s}=\frac{1-\frac{2s+7}{(s+2)^2}}{s+3}=\frac{1}{s+3}-\frac{2s+7}{(s+3)(s+2)^2}$$

$$\frac{2s+7}{(s+3)(s+2)^2}=\frac{1}{s+3}+\frac{K}{s+2}+\frac{3}{(s+2)^2}$$

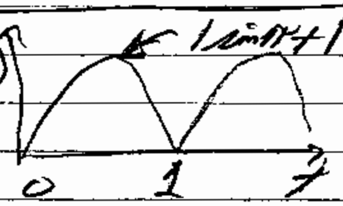
$2s+7=(s+2)^2+K(s+2)(s+3)+3(s+3)$

s^2 terms: $0=1+K$; $K=-1$

so $I(s)=\cancel{\frac{1}{s+3}}-\cancel{\frac{1}{s+3}}+\frac{1}{s+2}-\frac{3}{(s+2)^2}$

$\boxed{\text{or } i(t)=[e^{-2t}-3te^{-2t}]u(t)}$

11.5 f(t)



Periodic function!

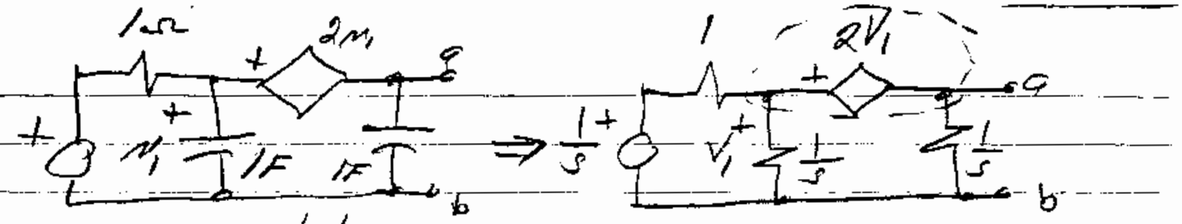
$f_1(t)=\sin\pi t\,[u(t)-u(t-1)]$

$F_1(s)=\mathcal{L}\{\sin\pi t\,[u(t)-u(t-1)]\}$

but $\sin\pi(t-1)=\sin\pi t\cos\pi-\cos\pi t\sin\pi=-\sin\pi t$

$\therefore F_1(s)=\mathcal{L}\{\sin\pi t\,u(t)-\sin\pi(t-1)\,u(t-1)\}$; $\boxed{\mathcal{L}[f(t)]=\left\{\frac{\pi}{s^2+\pi^2}-\frac{\pi e^{-s}}{s^2+\pi^2}\right\}\frac{1}{1-e^{-s}}}$

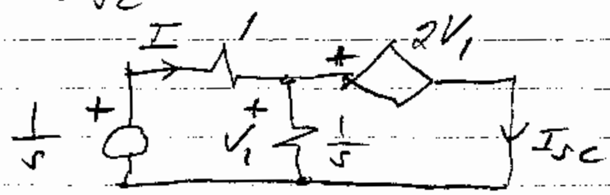
EE 212 Homework 13 soper noa

11.49 

$$\frac{V_1 - \frac{1}{s}}{1} + \frac{V_1}{1/s} + \frac{V_{Th}}{1/s} = 0$$ but $V_1 - V_{Th} = 2V_1$; $V_{Th} = -V_1$

∴ $V_1 = \frac{1}{s}$ or $\boxed{V_{Th} = -\frac{1}{s}}$

For I_{sc} we have:

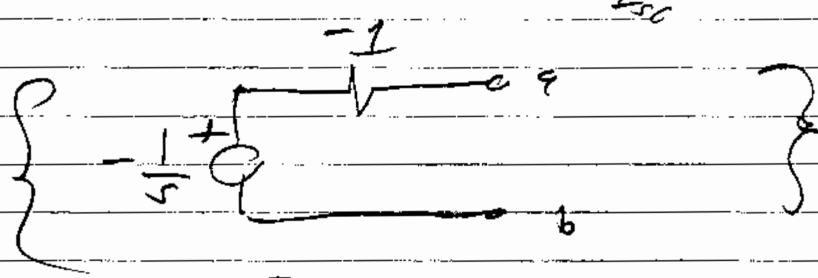


We see from circuit $V_1 = 2V_1$

∴ $V_1 = 0$

∴ $I = I_{sc} = \frac{1}{s}/1 = \frac{1}{s}$

so $Z_{Th} = \frac{V_{Th}}{I_{sc}} = -1$



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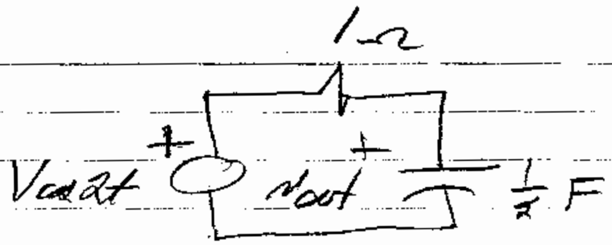
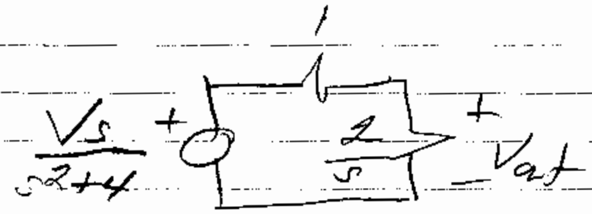
Special

zero state





$$V_{out} = \frac{\frac{2}{s}}{1+\frac{2}{s}} \cdot \frac{Vs}{s^2+4}$$

$$V_{out} = V\frac{2s}{(s+2)(s^2+4)} = V\left\{\frac{K_1}{s+2} + \frac{K_2 s + K_3}{s^2+4}\right\}$$

$$\boxed{K_1 = -\frac{1}{2}} \;;\; 2s = -\frac{1}{2}(s^2+4) + (K_2 s + K_3)(s+2)$$

s^2 terms; $0 = -\frac{1}{2} + K_2$; $\boxed{K_2 = \frac{1}{2}}$

s terms: $2 = 1 + K_3$; $\boxed{K_3 = 1}$

$$V_{out} = V\left\{\frac{-\frac{1}{2}}{s+2} + \frac{\frac{1}{2}s + 1}{s^2+4}\right\}$$

$$\boxed{v_{out}(t) = V\left\{-\frac{1}{2}e^{-2t} + \frac{1}{2}\cos 2t + \frac{1}{2}\sin 2t\right\}u(t)}$$

EE 212 Homework 17

11.65 $h(t)=e^{-2t}u(t)$; $x(t)=2u(t)-2u(t-1)$

$$y(t)=\int_0^\infty e^{-2\tau}u(\tau)\left[2u(t-\tau)-2u(t-\tau-1)\right]d\tau$$

$$y(t)=\int_0^t 2e^{-2\tau}d\tau\, u(t)-2\int_0^{t-1} e^{-2\tau}d\tau\, u(t-1)$$

$$y(t)=-e^{-2\tau}\Big|_0^t u(t)+e^{-2\tau}\Big|_0^{t-1}u(t-1)$$

$$y(t)=\left[1-e^{-2t}\right]u(t)-\left[1-e^{-2(t-1)}\right]u(t-1) \leftarrow$$

11.51 $H(s)=\frac{1}{s+2}$

a) $x(t)=u(t)$; $Y(s)=\frac{1}{s(s+2)}=\frac{\frac{1}{2}}{s}+\frac{-\frac{1}{2}}{s+2}$

so $y(t)=\left[\frac{1}{2}-\frac{1}{2}e^{-2t}\right]u(t) \leftarrow$

b) $x(t)=e^{-t}u(t)$; $Y(s)=\frac{1}{(s+1)(s+2)}=\frac{1}{s+1}+\frac{-1}{s+2}$

$y(t)=\left[e^{-t}-e^{-2t}\right]u(t) \leftarrow$

c) $x(t)=(1-e^{-t})u(t)$; $X(s)=\frac{1}{s}-\frac{1}{s+1}$

$Y(s)=\frac{1}{s(s+2)}-\frac{1}{(s+1)(s+2)}$

$\therefore$ from a) + b) $y(t)=\left[\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}\right]u(t) \leftarrow$

d) $x(t)=e^{-2t}u(t)$; $X(s)=\frac{1}{s+2}$ so $Y(s)=\frac{1}{(s+2)^2}$

$y(t)=te^{-2t}u(t) \leftarrow$

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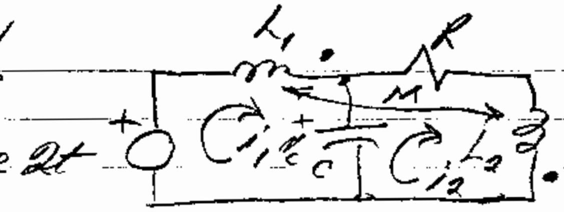
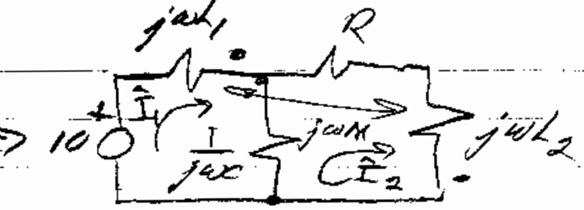
$v_c(0)=0$
$i_1(0)=1$
$i_2(0)=2$

Special



$\Rightarrow$



$\boxed{\text{time domain}}$

a)
$$\left.\begin{cases} 10\cos 2t = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} + \frac{1}{C}\int_0^t (i_1 - i_2)\,dt + v_c(0) \\ 0 = \frac{1}{C}\int_0^t (i_2 - i_1)\,dt - v_c(0) + i_2 R + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{cases}\right\} \leftarrow$$

$\boxed{\text{sinusoidal steady state}}$

b)
$$\left.\begin{cases} 10 = j\omega L_1 \hat{I}_1 + j\omega M \hat{I}_2 + \frac{1}{j\omega C}(\hat{I}_1 - \hat{I}_2) \\ 0 = \frac{1}{j\omega C}(\hat{I}_2 - \hat{I}_1) + R\hat{I}_2 + j\omega L_2 \hat{I}_2 + j\omega M \hat{I}_1 \end{cases}\right\} \leftarrow$$

$\boxed{\text{s-domain}}$

c)
$$\begin{cases} \frac{10s}{s^2+4} = sL_1 I_1 - L_1 i_1(0) + sM I_2 - M i_2(0) + \frac{1}{sC}(I_1 - I_2) + \frac{v_c(0)}{s} \\ 0 = \frac{1}{sC}(I_2 - I_1) - \frac{v_c(0)}{s} + RI_2 + sL_2 I_2 - L_2 i_2(0) + Ms I_1 - M i_1(0) \end{cases}$$

$$\left.\begin{cases} \frac{10s}{s^2+4} = sL_1 I_1 - L_1 + sMI_2 - 2M + \frac{1}{sC}(I_1 - I_2) \\ 0 = \frac{1}{sC}(I_2 - I_1) + RI_2 + sL_2 I_2 - 2L_2 + Ms I_1 - M \end{cases}\right\} \leftarrow$$

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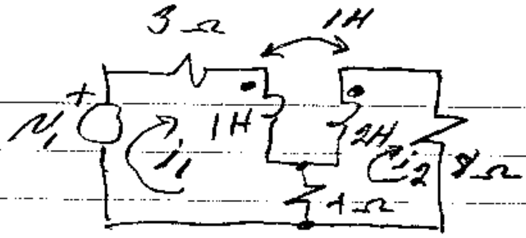
12.11



$H(s) = \frac{V_2(s)}{V_1(s)}$

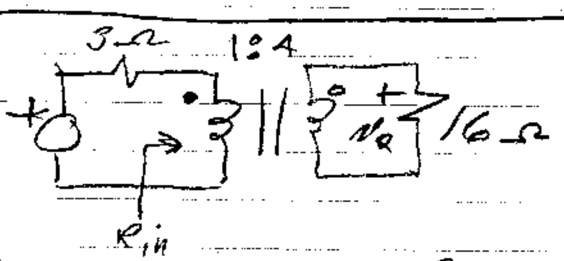
$V_2(s) = 8I_2(s)$

$$\left.\begin{aligned} V_1 &= 3I_1 + sI_1 - sI_2 + 4(I_1 - I_2) \\ 0 &= 4(I_2 - I_1) + 2sI_2 - sI_1 + 8I_2 \end{aligned}\right\} \quad \begin{aligned} V_1 &= I_1(7+s) - I_2(s+4) \\ 0 &= -I_1(s+4) + I_2(12+2s) \end{aligned}$$

$$\text{so } I_2 = \frac{\begin{vmatrix} 7+s & V_1 \\ -(s+4) & 0 \end{vmatrix}}{\begin{vmatrix} 7+s & -(s+4) \\ -(s+4) & 12+2s \end{vmatrix}} = \frac{V_1(s+4)}{(7+s)(12+2s) - (s+4)^2} = \frac{V_1(s+4)}{84 + 26s + 2s^2 - s^2 - 8s - 16}$$

$$I_2 = \frac{V_1(s+4)}{s^2 + 18s + 68} \; ; \; \boxed{H(s) = \frac{8(s+4)}{s^2+18s+68}} \longleftarrow$$

12.17

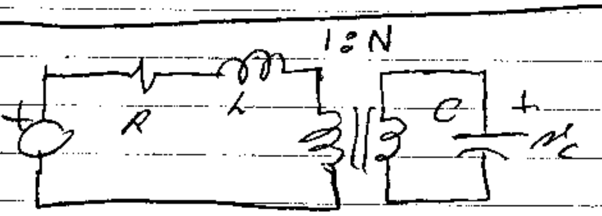


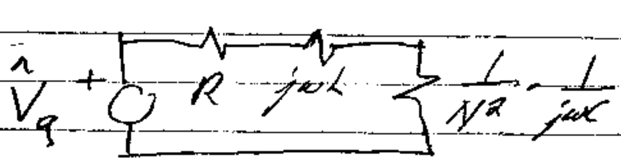
a) $\boxed{Z = 3 + 1 = 4\,\Omega} \longleftarrow$

b) $v_2 = \frac{1}{4} v_g \times 4 \; ; \; \boxed{\frac{v_2}{v_g} = 1} \longleftarrow$

c) for maximum power $R_{in} = 3 = \frac{1}{16} R_x \; \therefore \; \boxed{R_x = 48\,\Omega} \longleftarrow$

12.22





RLC series circuit with equivalent capacitance equal N^2C !

a) $\omega_0 = \frac{1}{\sqrt{LC_{eq}}} = \boxed{\frac{1}{N\sqrt{LC}}} \longleftarrow$

b) $Q = \omega_0 \frac{L}{R} = \frac{L}{RN\sqrt{LC}} = \boxed{\sqrt{\frac{L}{C}} \cdot \frac{1}{NR}} \longleftarrow$

c) $BW = \frac{\omega_0}{Q} = \frac{\frac{1}{N\sqrt{LC}}}{\sqrt{\frac{L}{C}} \frac{1}{NR}} = \boxed{\frac{R}{L}} \longleftarrow$

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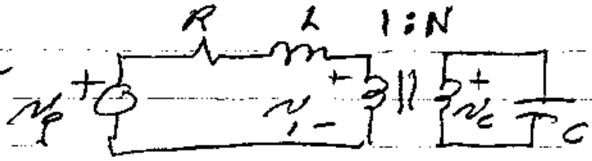
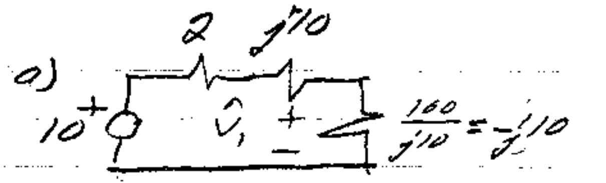
12.25



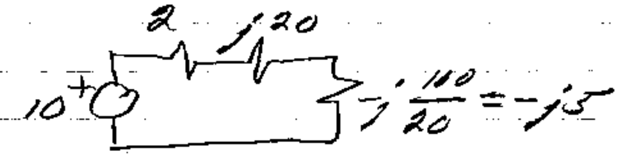
a)



$\omega = 5,\ R = 2,\ L = 2,\ C = 2,\ N = 1/10$

$\hat{V}_1 = 10\ \frac{-j10}{2} = -j50 \quad \therefore \hat{V}_c = -j5,\quad \boxed{v_c(t) = 5\cos(5t - 90^\circ)}$

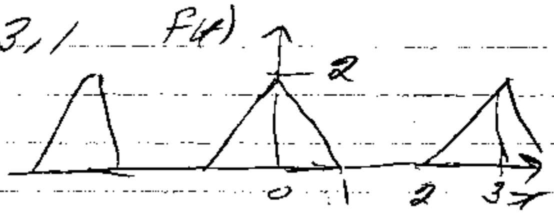
b) $v_g(t) = 10\cos 10t$



$\hat{V}_1 = 10\ \frac{-j5}{2+j15} = \frac{5e^{-j90^\circ}}{15.13e^{j82.4^\circ}} = 0.33e^{-j172.4^\circ}$

$\boxed{v_c(t) = 0.33\cos(10t - 172.4^\circ)} \leftarrow$

13.1



even function $\therefore\ b_n = 0$

$\boxed{a_0 = \frac{2}{3}} \leftarrow \qquad T = 3$

$a_n = \frac{4}{T}\int_0^1 (-2t+2)\cos\frac{n2\pi t}{3}\,dt$

$a_n = -\frac{8}{3}\int_0^1 t\cos\frac{2\pi n t}{3}\,dt + \frac{8}{3}\int_0^1 \cos\frac{2\pi n t}{3}\,dt$

$u = t \quad dv = \cos\frac{2\pi n t}{3}\,dt$

$du = dt \quad v = \frac{3}{2\pi n}\sin\frac{2\pi n t}{3}$

$a_n = -\frac{8}{3}t\frac{3}{2\pi n}\sin\frac{2\pi n t}{3}\Big|_0^1 + \frac{8}{3}\int_0^1 \frac{3}{2\pi n}\sin\frac{2\pi n t}{3}\,dt + \frac{8}{3}\frac{3}{2\pi n}\sin\frac{2\pi n t}{3}\Big|_0^1$

$a_n = \cancel{\frac{-8}{2\pi n}\sin\frac{2\pi n}{3}} - \frac{8}{2\pi n}\cdot\frac{3}{2\pi n}\cos\frac{2\pi n t}{3}\Big|_0^1 + \cancel{\frac{8}{2\pi n}\sin\frac{2\pi n}{3}}$

$a_n = \frac{6}{(\pi n)^2}\left[1 - \cos\left(\frac{2\pi n}{3}\right)\right] \leftarrow$

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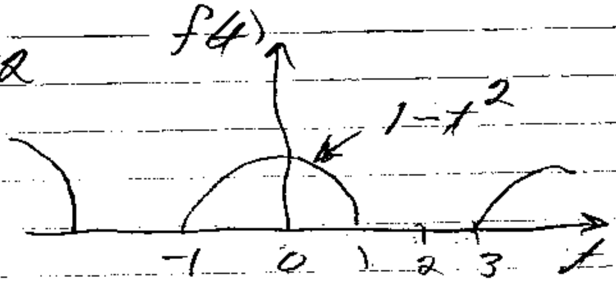
13.2



$T = 4$; $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

even $f(t)$ so $b_n = 0$

$$a_0 = \frac{1}{4}\int_{-1}^{1}(1-t^2)\,dt = \frac{1}{4}\left[t - \frac{t^3}{3}\right]_{-1}^{1} = \frac{1}{4}\left[\frac{2}{3} + 1 - \frac{1}{3}\right] = \boxed{\frac{1}{3}}$$

$$a_n = \frac{2}{T}\int_{-1}^{1}(1-t^2)\cos n\frac{\pi}{2}t\,dt = \int_0^1 (1-t^2)\cos\frac{n\pi}{2}t\,dt \quad \left\{ u = \frac{n\pi}{2}t \quad du = \frac{n\pi}{2}dt \right\}$$

$\boxed{\text{from table}}$

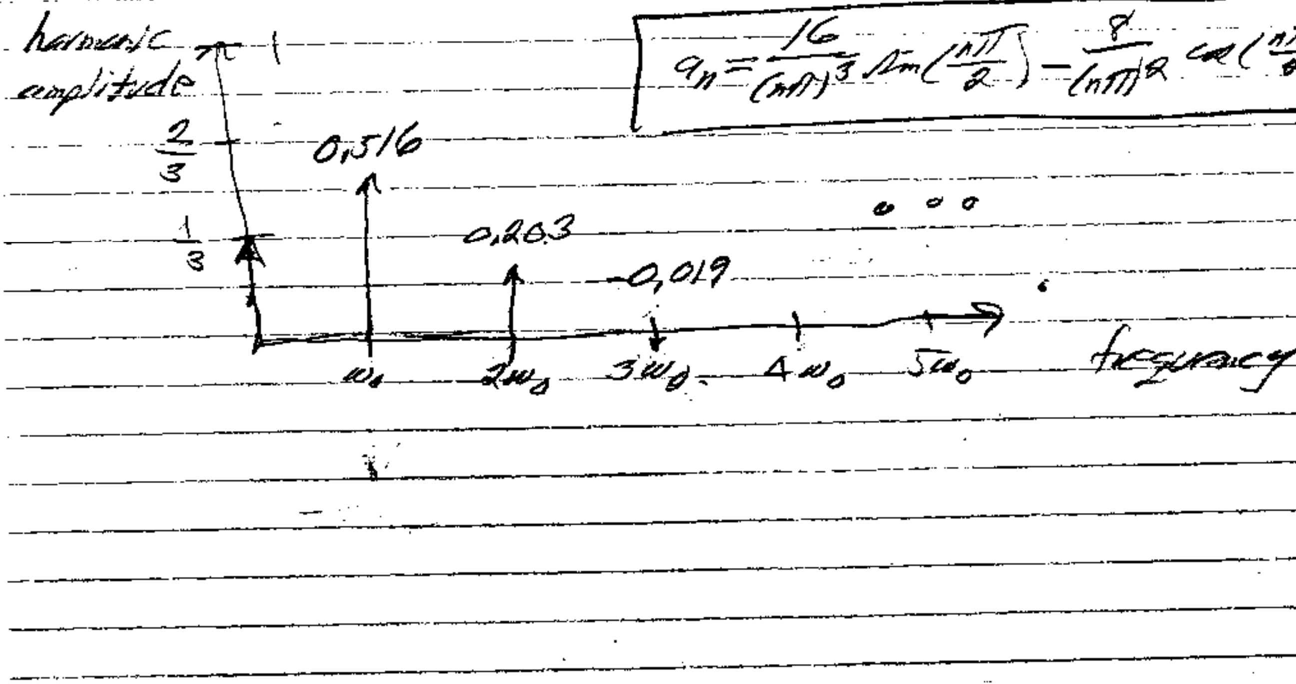
$$a_n = \frac{2}{n\pi}\sin\frac{n\pi t}{2}\Big|_0^1 - \int_0^{\frac{n\pi}{2}} \frac{4\times 2}{(n\pi)^3}u^2\cos u\,du$$

$$a_n = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{8}{(n\pi)^3}\left\{2u\cos u + (u^2-2)\sin u\right\}_0^{\frac{n\pi}{2}}$$

$$a_n = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{8}{(n\pi)^3}\left\{n\pi\cos\left(\frac{n\pi}{2}\right) + \left(\frac{n^2\pi^2}{4} - 2\right)\sin\left(\frac{n\pi}{2}\right)\right\}$$

$$a_n = \cancel{\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)} - \frac{8}{(n\pi)^2}\cos\left(\frac{n\pi}{2}\right) - \cancel{\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)} + \frac{16}{(n\pi)^3}\sin\left(\frac{n\pi}{2}\right)$$

$$\boxed{a_n = \frac{16}{(n\pi)^3}\sin\left(\frac{n\pi}{2}\right) - \frac{8}{(n\pi)^2}\cos\left(\frac{n\pi}{2}\right)}$$



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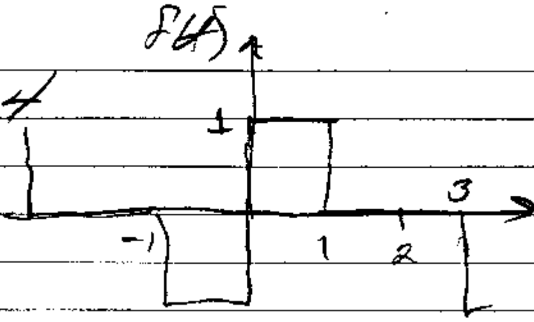
13.4



$T = 4 \therefore \omega_0 = \frac{\pi}{2}$

odd function $\therefore a_n = 0$

$\boxed{a_0 = 0}$

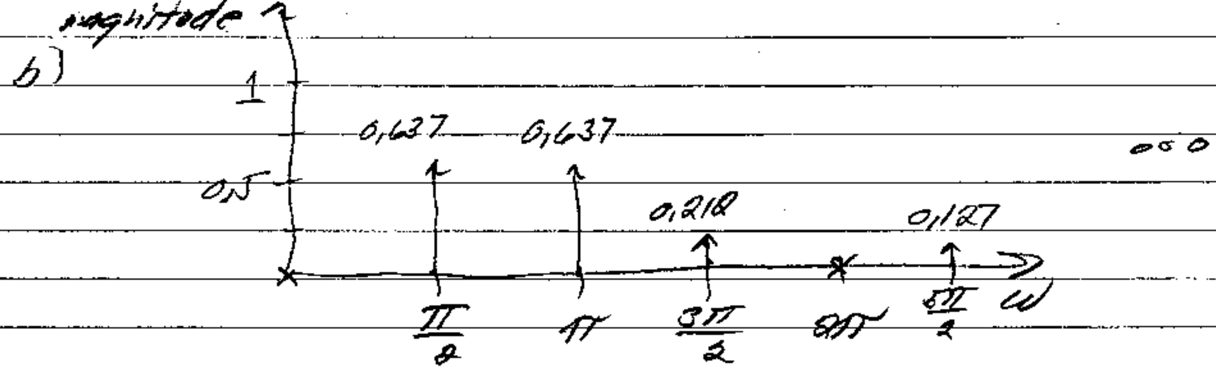
a)

$$b_n = \frac{4}{T}\int_0^1 1 \sin\left(\frac{n\pi}{2}t\right) dt = \frac{-2}{n\pi}\cos\left(\frac{n\pi t}{2}\right)\Big|_0^1$$

$$\boxed{b_n = \frac{2}{n\pi}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]}$$

$$\therefore f(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]\sin\left(\frac{n\pi}{2}t\right)$$

b)



13.22 f(t) is odd + periodic.

in general $\hat{c}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t} dt$

$\hat{c}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\left[\cos n\omega_0 t - j\sin n\omega_0 t\right] dt$ but f(t) is odd

$$\therefore \hat{c}_n = -\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \underbrace{f(t)\, j\sin n\omega_0 t}_{\text{even function}}\, dt = \boxed{-\frac{2j}{T}\int_0^{\frac{T}{2}} f(t)\sin(n\omega_0 t)\, dt}$$

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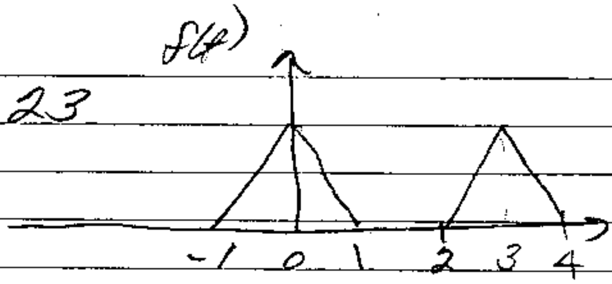
13.23



$T = 3$

$$\hat{C}_n = \frac{1}{3}\int_{-1}^{1} f(t)\, e^{-jn\frac{2\pi}{3}t}\,dt = \frac{2}{3}\int_0^1 (2-2t)\cos\left(n\frac{2\pi}{3}t\right)dt$$

let $u = \frac{2\pi n t}{3}$; $du = \frac{2\pi n}{3}dt$

$$\hat{C}_n = \frac{4}{3}\times\frac{3}{n2\pi}\sin\left(\frac{2\pi n t}{3}\right)\Big|_0^1 - \frac{4}{3}\int_0^{\frac{2\pi n}{3}}\left(\frac{3}{2\pi n}\right)^2 u\cos u\,du$$

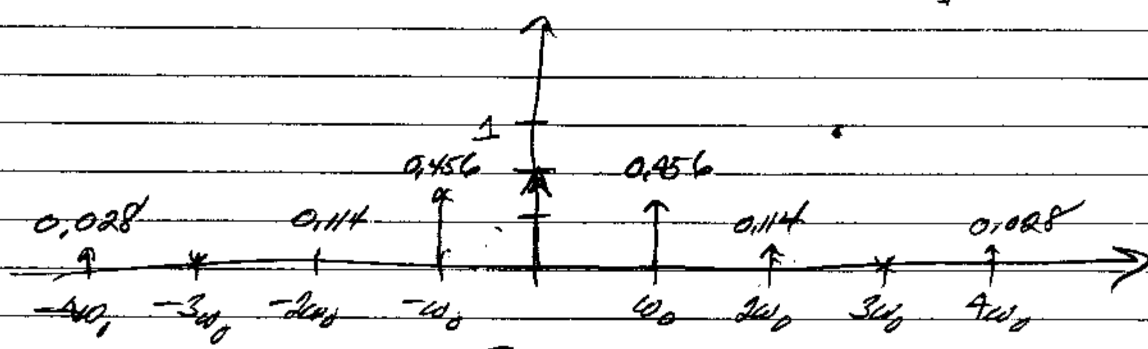
$$\hat{C}_n = \frac{2}{n\pi}\sin\frac{2\pi n}{3} - \frac{3}{(n\pi)^2}\left\{\cos u + u\sin u\right\}_0^{\frac{2\pi n}{3}}$$ ← from tables

$$\hat{C}_n = \cancel{\frac{2}{n\pi}\sin\frac{2\pi n}{3}} - \frac{3}{(n\pi)^2}\cos\frac{2\pi n}{3} + \frac{3}{(n\pi)^2} - \cancel{\frac{3}{(n\pi)^2}\cdot\frac{2\pi n}{3}\sin\frac{2\pi n}{3}}$$

$$\hat{C}_n = \frac{3}{(n\pi)^2}\left(1-\cos\frac{2\pi n}{3}\right)$$ ←

$$\hat{C}_0 = \lim_{n\to 0}\hat{C}_n = \lim_{n\to 0}\left\{\frac{2\pi\sin\frac{2\pi n}{3}}{\pi^2 2n}\right\} = \lim_{n\to 0}\frac{\frac{(2\pi)^2}{3}\cos\frac{2\pi n}{3}}{2\pi^2} = \boxed{\frac{2}{3}}$$

harmonic magnitude of frequency spectrum



not asked for in problem

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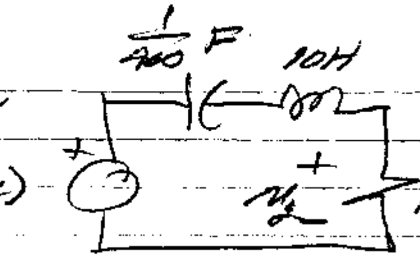
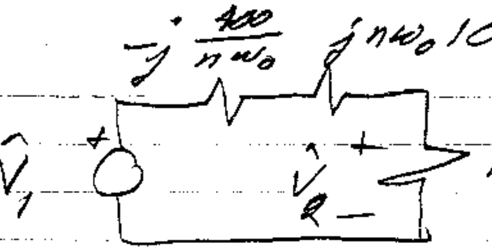
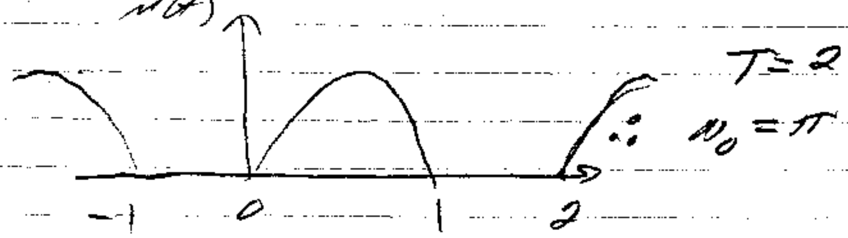
13.15







$T = 2$

$\therefore\ \omega_0 = \pi$

from page 602 $v(t) = \frac{1}{\pi} + \frac{1}{2}\sin \pi t + \sum_{n=2}^{\infty} \frac{1-\cos(1+n)\pi}{\pi(1-n^2)} \cos n\pi t$

$[\,v(t) = 0.32 + \frac{1}{2}\sin\pi t - 0.21\cos 2\pi t - 0.04\cos 4\pi t \cdots$

$\therefore\ \hat{V}_{2_n} = \hat{V}_{1_n} \frac{1}{1+j\left(n\pi 10 - \frac{400}{n\pi}\right)}$ ← for cosine terms

(fundamental ($n=1$) will have 90° phase shift)

$\hat{V}_{2_n} = \hat{V}_{1n} \frac{n\pi}{n\pi + j(10n^2\pi^2 - 400)}$

$\therefore\ |\hat{V}_{2n}| = |\hat{V}_{1n}| \frac{1}{\sqrt{1+\left(10n\pi - \frac{400}{n\pi}\right)^2}}$

n	$\lvert\hat{V}_{2n}\rvert$
0	0
1	5.213×10^{-3}
2	-0.162
3	0
4	-4.5×10^{-4}
5	0
6	-1.1×10^{-4}

We see that second harmonic is dominant

$\therefore\ \hat{V}_2 \simeq -0.162\, e^{-j\tan^{-1}\left(\frac{40\pi^2 - 400}{2\pi}\right)}$

or $\hat{V}_2 \cong -0.162\, e^{+j39.7^\circ}$

$v_2(t) \cong -0.162\cos(2\pi t + 39.7^\circ)$ ←

or $v_2(t) \cong 0.162\cos(2\pi t - 140.3^\circ)$ ←

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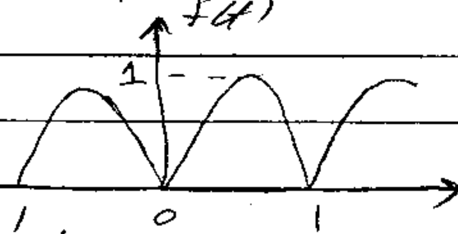
13.25



$T' = 1$

$\omega_0 = \frac{2\pi}{T'} = 2\pi$

even function

$$\hat{C}_n = \frac{1}{T'}\int_{-\frac{1}{2}}^{\frac{1}{2}} |\sin \pi t|\, e^{-jn2\pi t}\, dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} |\sin \pi t| \left[\cos(2\pi n t) - j\sin(2\pi n t)\right] dt$$

$$\hat{C}_n = 2\int_0^{\frac{1}{2}} \sin \pi t \cos(2\pi n t)\, dt = \int_0^{\frac{1}{2}} \left[\sin(\pi + 2\pi n)t + \sin(\pi - 2\pi n)t\right] dt$$

$$\hat{C}_n = \left\{ -\frac{\cos(\pi+2\pi n)t}{\pi + 2\pi n} - \frac{\cos(\pi - 2\pi n)t}{\pi - 2\pi n} \right\}_0^{\frac{1}{2}}$$

$$\hat{C}_n = \left\{ \frac{-\cos(\pi+2\pi n)\frac{1}{2} + 1}{\pi + 2\pi n} + \frac{-\cos(\pi - 2\pi n)\frac{1}{2} + 1}{\pi - 2\pi n} \right\}$$

$$\hat{C}_n = \frac{(2\pi n - \pi)\cos(\pi + 2\pi n)\frac{1}{2} - (\pi + 2\pi n)\cos(\pi - 2\pi n)\frac{1}{2} + \pi + 2\pi n + \pi - 2\pi n}{\pi^2 - 4\pi^2 n^2}$$

$$\hat{C}_n = \frac{(2\pi n - \pi)\left[\cos\frac{\pi}{2}\cos n\pi - \sin\frac{\pi}{2}\sin n\pi\right] - (\pi + 2\pi n)\left[\cos\frac{\pi}{2}\cos n\pi + \sin\frac{\pi}{2}\sin n\pi\right] + 2\pi}{\pi^2 - 4\pi^2 n^2}$$

$$\hat{C}_n = \frac{-4\pi n \sin n\pi + 2\pi}{\pi^2 - 4\pi^2 n^2} = \frac{2}{\pi - 4\pi n^2} = \frac{2}{\pi(1 - 4n^2)}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} e^{j2\pi n t}}$$

$$f(t) = 0.637 - 0.212\, e^{j2\pi t} - 0.212\, e^{-j2\pi t} - 0.0424\, e^{j4\pi t} - 0.0424\, e^{-j4\pi t} \ldots$$

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6/9/13

14.6 $\mathcal{F}[f(t-a)] = \int_{-\infty}^{\infty} f(t-a) e^{-j\omega t} dt$ (let $x = t-a$)

$\mathcal{F}[f(t-a)] = \int_{-\infty}^{\infty} f(x) e^{-j\omega(x+a)} dx = e^{-j\omega a} F(j\omega)$

14.9 $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$ ($f(t)$ is even)

$\therefore F(j\omega) = \int_{-\infty}^{\infty} f(t) \cos\omega t \, dt = 2\int_{0}^{\infty} f(t) \cos\omega t \, dt$

14.13 $\mathcal{F}[e^{-\alpha t} f(t)] = \int_{-\infty}^{\infty} e^{-\alpha t} f(t) e^{-j\omega t} dt$

$\mathcal{F}[e^{-\alpha t} f(t)] = \int_{-\infty}^{\infty} f(t) e^{-(\alpha+j\omega)t} dt = F(j\omega + \alpha)$

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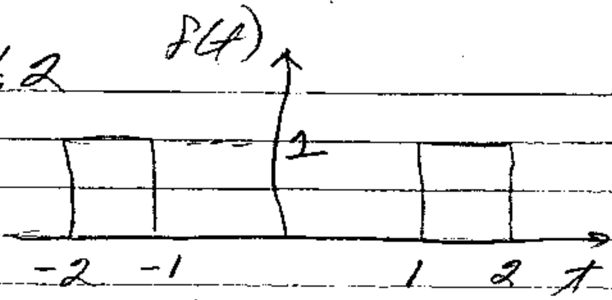
14.2

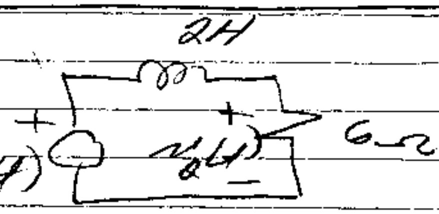


even function

$$F(j\omega) = 2\int_0^{\infty} \cos \omega t \, dt$$

$$F(j\omega) = 2\int_{+1}^{+2} \cos \omega t \, dt = 2\left(\frac{1}{\omega}\right) \sin \omega t \Big|_1^2 = \frac{2}{\omega}\left[\sin 2\omega - \sin \omega\right]$$

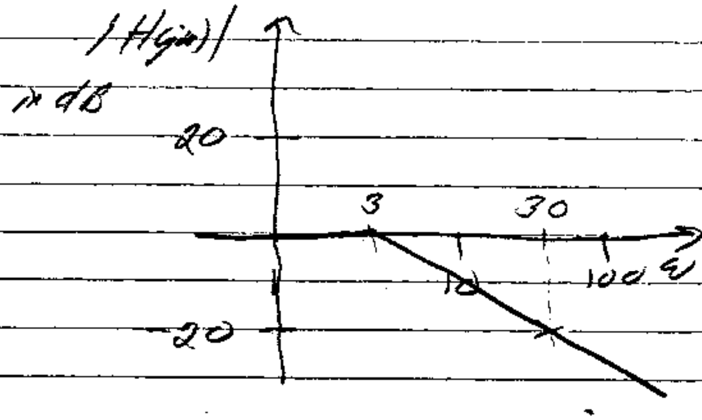
14.19

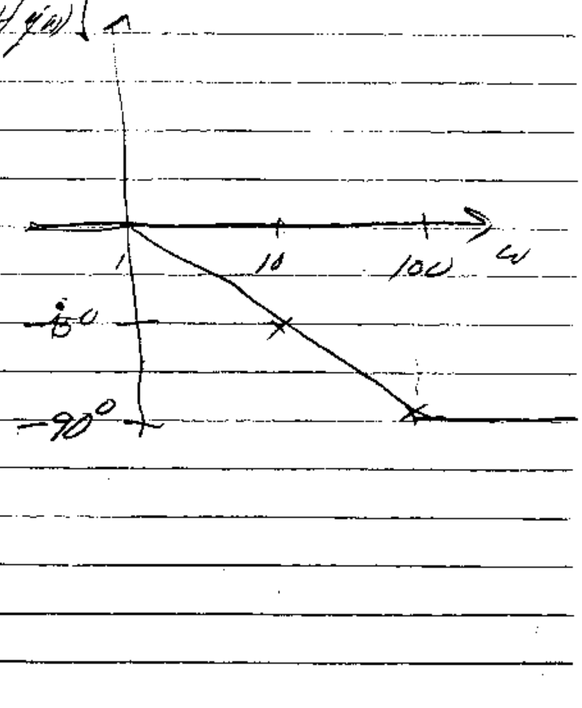


a) $H(j\omega) = \boxed{\dfrac{6}{6+j2\omega}}$

b) $h(t) = \mathcal{F}^{-1}\left[\dfrac{3}{3+j\omega}\right] = \boxed{3e^{-3t}u(t)}$

c) $H(j\omega) = \dfrac{1}{1+\frac{j\omega}{3}}$





d) $\boxed{\text{low pass}}$

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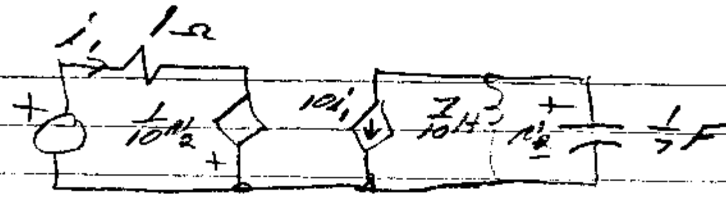
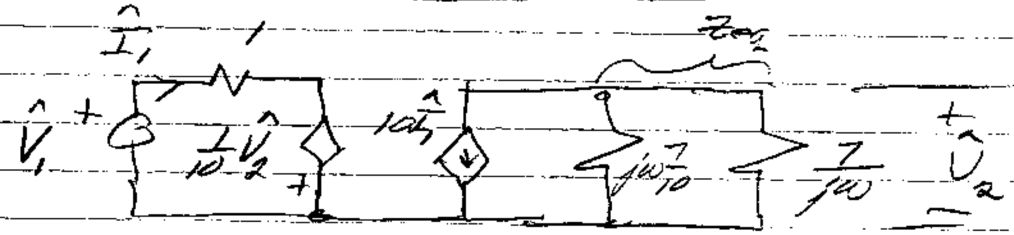
14.23





a) $-\hat{V}_2 = 10\hat{I}_1 Z_{eq} = 10\hat{I}_1 \frac{\frac{49}{10}}{j\omega\frac{7}{10}+\frac{7}{j\omega}} = \hat{I}_1 \frac{49 j\omega}{7+\frac{7}{10}(j\omega)^2} = \hat{I}_1 \frac{70 j\omega}{10+(j\omega)^2}$

but $\hat{I}_1 = \left(\hat{V}_1 + \frac{1}{10}\hat{V}_2\right)/1$

so $\hat{V}_2 = -\left(\hat{V}_1 + \frac{1}{10}\hat{V}_2\right)\frac{70 j\omega}{10+(j\omega)^2}$

or $\hat{V}_2\left[1+\frac{7j\omega}{10+(j\omega)^2}\right] = -\hat{V}_1 \frac{70j\omega}{10+(j\omega)^2} = \hat{V}_2\left[\frac{(j\omega)^2+7j\omega+10}{10+(j\omega)^2}\right]$

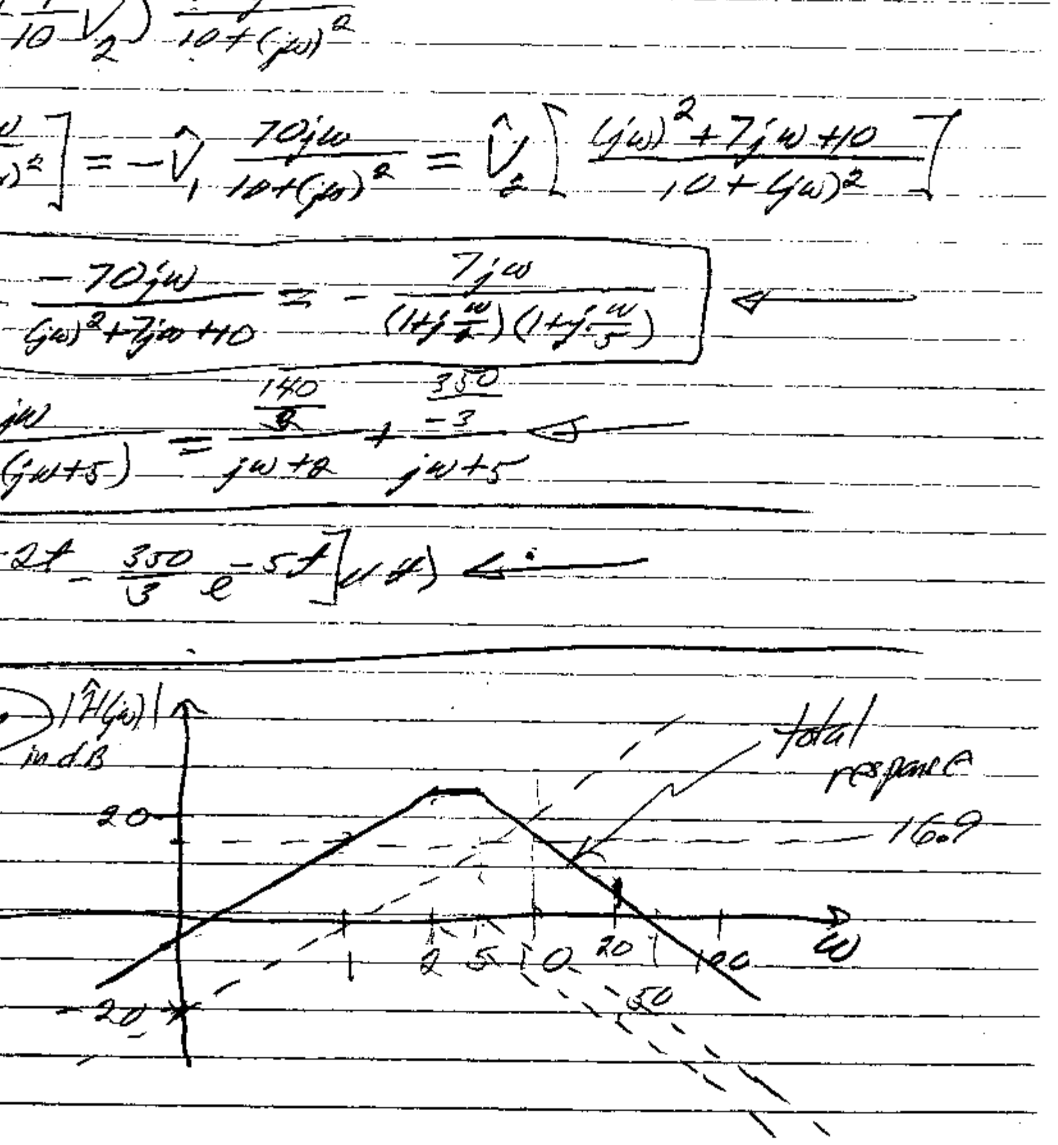
$\therefore \boxed{\frac{\hat{V}_2}{\hat{V}_1} = H(j\omega) = \frac{-70j\omega}{(j\omega)^2+7j\omega+10} = -\frac{7j\omega}{(1+j\frac{\omega}{2})(1+j\frac{\omega}{5})}}$

$H(j\omega) = \frac{-70j\omega}{(j\omega+2)(j\omega+5)} = \frac{\frac{140}{3}}{j\omega+2} + \frac{\frac{-350}{3}}{j\omega+5}$

b) $h(t) = \left[\frac{140}{3}e^{-2t} - \frac{350}{3}e^{-5t}\right]u(t)$

c) $20\log_{10} 7 = 16.978$

(band pass)



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14.17 $v(t) = t e^{-at} u(t)$ (applied to $1\,\Omega$)

a) $W_{1\Omega} = \int_0^\infty t^2 e^{-2at}\,dt = \left[e^{-2at}\left(\frac{t^2}{-2a} - \frac{2t}{4a^2} - \frac{2}{8a^3}\right)\right]_0^\infty$ (from tables)

$$\boxed{W_{1\Omega} = \frac{1}{4a^3}}$$

2nd method $W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{(j\omega+a)^2}\right|^2 d\omega$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{a^2-\omega^2+j2\omega a}\right|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(a^2-\omega^2)^2+4\omega^2a^2}\,d\omega$$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{a^4+\omega^4+2\omega^2a^2}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{(a^2+\omega^2)^2}$$

$$W_{1\Omega} = \frac{1}{2\pi}\left\{\frac{\omega}{2a^2(a^2+\omega^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{\omega}{a}\right)\right\}_{-\infty}^{\infty} = \frac{1}{2\pi}\cdot\frac{1}{2a^3}\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$$

$$\boxed{W_{1\Omega} = \frac{1}{4a^3}}$$

b) $W = \frac{1}{2\pi}\int_{-a}^{a}\frac{d\omega}{(a^2+\omega^2)^2} = \frac{1}{2\pi}\left\{\frac{\omega}{2a^2(a^2+\omega^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{\omega}{a}\right)\right\}_{-a}^{a}$

from $\omega = -a$ to $+a$

$$= \frac{1}{2\pi}\left\{\frac{a}{4a^4} + \frac{1}{2a^3}\cdot\frac{\pi}{4} + \frac{a}{4a^4} + \frac{1}{2a^3}\frac{\pi}{4}\right\}$$

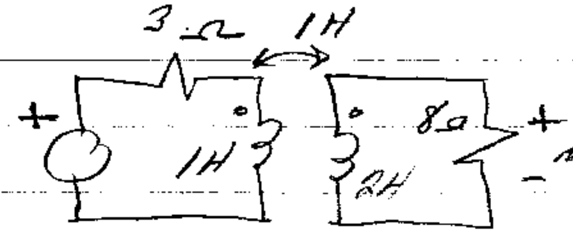
$$= \frac{1}{2\pi}\left\{\frac{1}{2a^3} + \frac{\pi}{4a^3}\right\} = \frac{1}{4a^3}\left\{\frac{1}{\pi}+\frac{1}{2}\right\}$$

so the % of the energy from $\omega = 0$ to a

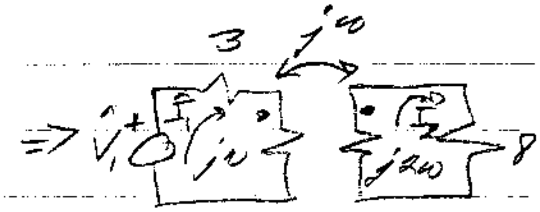
is $\left\{\frac{1}{\pi}+\frac{1}{2}\right\}\times 100 = 81.83\%$

EE 212 Homework 30

14.28



$\Rightarrow$



$$\left.\begin{aligned} \hat{V}_1 &= 3\hat{I}_1 + j\omega \hat{I}_1 - j\omega \hat{I}_2 \\ 0 &= j2\omega \hat{I}_2 - j\omega \hat{I}_1 + 8\hat{I}_2 \end{aligned}\right\} \quad \begin{aligned} \hat{V}_1 &= \hat{I}_1 (3+j\omega) - j\omega \hat{I}_2 \\ 0 &= -j\omega \hat{I}_1 + (8+j2\omega)\hat{I}_2 \end{aligned}$$

$$\hat{I}_2 = \frac{\begin{vmatrix} 3+j\omega & \hat{V}_1 \\ -j\omega & 0 \end{vmatrix}}{\begin{vmatrix} 3+j\omega & -j\omega \\ -j\omega & 8+j2\omega \end{vmatrix}} = \frac{j\omega \hat{V}_1}{(3+j\omega)(8+j2\omega)+\omega^2} = \frac{j\omega \hat{V}_1}{24+2(j\omega)^2+14j\omega-(j\omega)^2}$$

$$\hat{H}(j\omega) = \frac{\hat{V}_2}{\hat{V}_1} = \frac{8j\omega}{(j\omega)^2+14j\omega+24} = \frac{8j\omega}{(j\omega+2)(j\omega+12)}$$

$$H(j\omega) = \frac{\frac{-16}{10}}{j\omega+2} + \frac{\frac{-8\times 12}{-10}}{j\omega+12} = \frac{-\frac{8}{5}}{j\omega+2} + \frac{\frac{48}{5}}{j\omega+12}$$

$$\therefore\ h(t) = \left[\frac{48}{5} e^{-j12t} - \frac{8}{5} e^{-2t}\right] u(t)$$