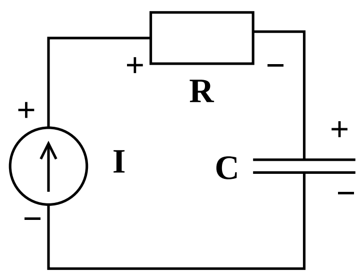
EE 211 Circuit and Signals I, Fall 2012
Exam 2 October 17, 2012
Solution

Rules: This is a closed-book exam. You may use only your brain, a calculator and pen/paper. Each numbered question counts equally toward your grade.

Grading policy: Only boxed answers count. If you have boxed the correct answer and show enough relevant math you get 10. If only one of them you get 5. Otherwise you get 0. Always include the unit on every answer and reduce to simple form!



1. **If the current is $I(t) = I_0 u(t)$ write the expression for the voltage across the capacitor.**

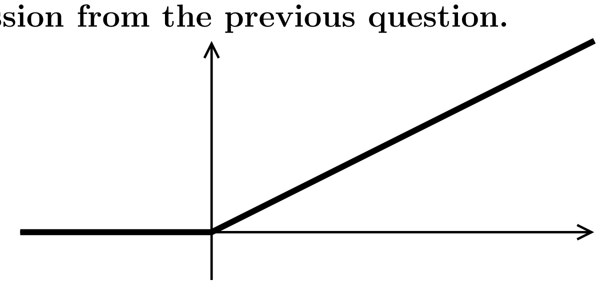
   We know that

$$i = C\frac{dV}{dt}$$

   and we know that $v(t) = 0$ for $t < 0$. We can integrate from $t = 0$ to get

$$V(t) = \int_0^V dV = \int_0^t \frac{I_0}{C}\,dt$$
$$= \frac{I_0}{C} t u(t) = \frac{I_0}{C} r(t)$$

2. **Plot the expression from the previous question.**

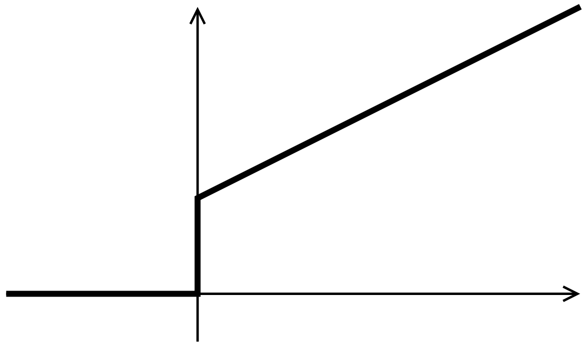


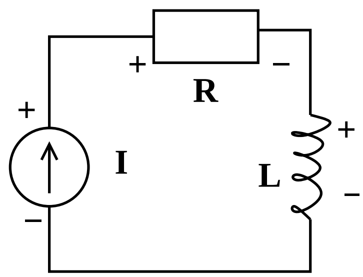
3. **Write the expression for voltage across the current source.**

   It is the sum of voltage across the resistor and the capacitor,

$$V_s = V_R + V_C = RI_0u(t) + \frac{I_0}{C}r(t) \tag{1}$$

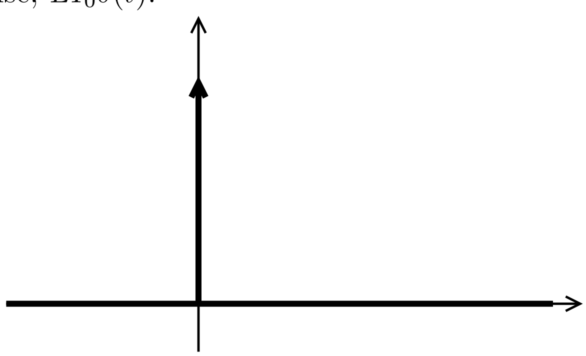
4. Plot it.





5. If $I(t) = I_0u(t)$ plot the voltage across the inductor.

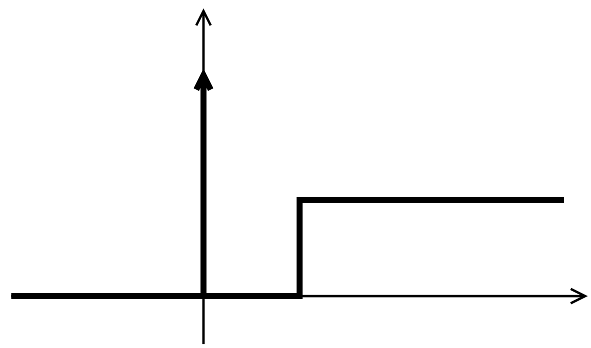
It will be an impulse, $LI_0\delta(t)$.



6. If $I(t) = I_0[u(t) + r(t-1)]$ derive an expression for the voltage across the inductor.

$$V_L = L\frac{dI}{dt} = I_0L[\delta(t) + u(t-1)]$$

7. Plot the expression from the previous question.

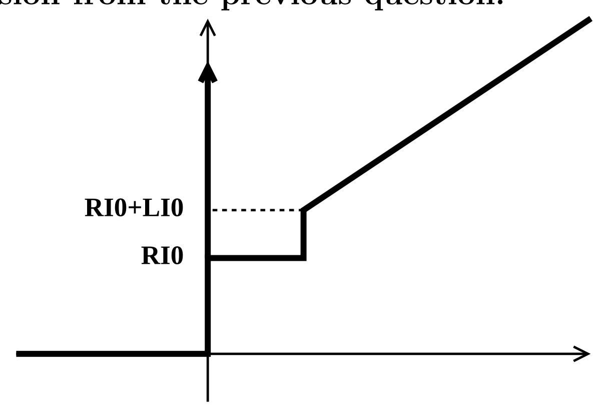


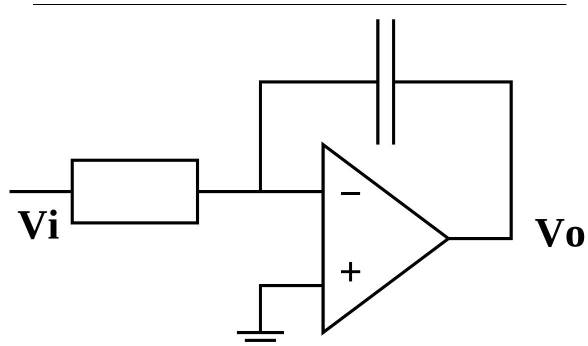
8. **Derive an expression for the voltage across the current source.**

   This is simply the voltage across the inductor plus the voltage across the resistor

$$V_s = V_R + V_L = RI_0\left[u(t) + r(t-1)\right] + LI_0\left[\delta(t) + u(t-1)\right]$$

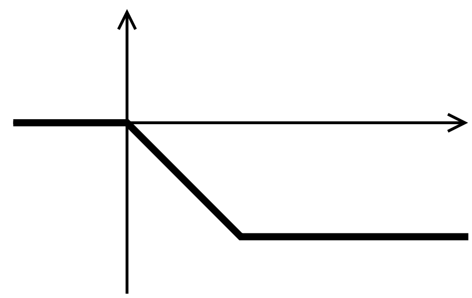
9. **Plot the expression from the previous question.**





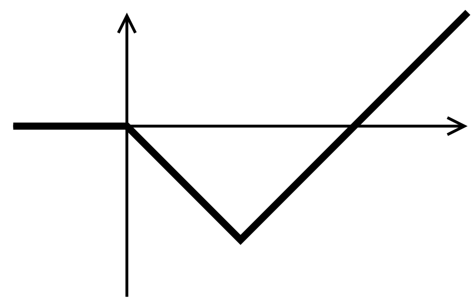
10. **Carefully plot V_o when V_I is**

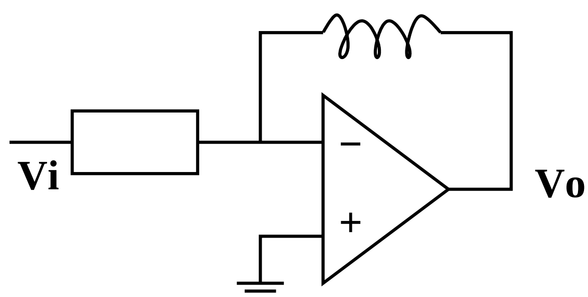
$$\boldsymbol{V_I = V_1\left[u(t) - u(t-1)\right]}$$



11. Carefully plot V_o when V_I is

$$\boldsymbol{V_I = V_1\left[u(t) - 2u(t-1)\right]}$$





12. Assume $R = 10\,\Omega$ and $L = 2\,\mathrm{H}$. If $V_I(t) = \alpha r(t)$ what should α be to make $V_o = -10\,\mathrm{V}$ for $t > 0$?

Begin with

$$V_L = L\frac{dI}{dt}$$

and realize that $V_o = -V_L$, and that $I = \frac{V_i}{R}$. Insert all this and we get

$$V_O = -L\frac{dI}{dt} = -\frac{L}{R}\frac{dV_i}{dt} = -\frac{L\alpha}{R}\frac{dr}{dt} = -\frac{L\alpha}{R}u(t)$$

For any value of $t > 0$ we thus have

$$V_O = -\frac{L\alpha}{R}$$

or

$$\alpha = -\frac{V_O R}{L} = -\frac{-10 \times 10}{2} = 50\,\text{V/s}$$