EE 321 Analog Electronics, Fall 2009
Homework #1 solution

1.42. Symmetrically saturating amplifiers, operating in the so-called clipping mode, can be used to convert sine waves to pseudo-square waves. For an amplifier with a small-signal gain of 1000 and clipping levels of ±9 V, what peak value of input sinusoid is needed to produce an output whose extremes are just at the edge of clipping? Clipped 90% of the time? Clipped 99% of the time?
Ignoring clipping, the relationship between the input and the output is

$$v_o = Av_i = AV_i \sin \omega t$$

To be at the edge of clipping we require that $AV_i = V_C$ V, or $V_i = V_C/1000 = 9\,\text{mV}$.
To be clipped 90% of the time, the output sine wave must reach the clipping level after 2.5% of the period, so

$$V_C = AV_i \sin (2.5\% \times 2 \times \pi)$$

or

$$V_i = \frac{V_c}{A \times 0.1564} = 57.5\,\text{mV}$$

To be clipped 99% of the time, the output sine wave must reach the clipping level after 0.25% of the period, so

$$V_i = \frac{V_c}{A \times 0.01570} = 0.573\,\text{V}$$

1.50. You are given two amplifiers, A and B, to connect in cascade between a 10-mV, 100-kΩ source and a 100-Ω load. The amplifiers have voltage gain, input resistance, and output resistance as follows: for A, 100 V/V, 10 kΩ, 10 kΩ, respectively; for B, 1 V/V, 100 kΩ, 100 kΩ, respectively. Your problems is to decide how the amplifiers should be connected. To proceed, evaluate the two possible connections between sources S and load L, namely, SABL, and SBAL. Find the voltage gain for each both as a ratio and in dB. Which amplifier arrangement is best?
We have $R_s = 100\,\text{k}\Omega$, $R_{ia} = 10\,\text{k}\Omega$, $R_{oa} = 10\,\text{k}\Omega$, $A_{voa} = 100$, $R_{ib} = 100\,\text{k}\Omega$, $R_{oa} = 100\,\text{k}\Omega$, $A_{vob} = 1$, $R_L = 100\,\Omega$.
First consider SABL:

$$\begin{aligned}\frac{v_o}{v_s} =& \frac{R_{ia}}{R_{ia} + R_s} A_{voa} \frac{R_{ib}}{R_{ib} + R_{oa}} A_{vob} \frac{R_L}{R_L + R_{ob}}\\ =& \frac{10}{10+100} \times 100 \times \frac{100}{10+100} \times 1 \times \frac{0.1}{0.1+100}\\ =& 0.00826\\ =& 20 \log_{10} 0.00826\,\text{dB}\\ =& -41.7\,\text{dB}\end{aligned}$$

Next consider SBAL:

$$
\begin{aligned}
\frac{v_o}{v_s} =& \frac{R_{ib}}{R_{ib}+R_s} A_{vob} \frac{R_{ia}}{R_{ia}+R_{ob}} A_{voa} \frac{R_L}{R_L+R_{oa}} \\
=& \frac{100}{100+100} \times 1 \times \frac{10}{10+100} \times 100 \times \frac{0.1}{0.1+10} \\
=& 0.0450 \\
=& 20\log_{10} 0.045\,\text{dB}
\end{aligned}
$$

$$= -26.94\,\text{dB}$$

Which one is better? The problem does not define what it means by "better," so I will assume it is the larges voltage drop across the load resistor. In that case SBAL is better.

1.63. For the circuit in Fig. P1.63, show that

$$\boldsymbol{\frac{v_c}{v_b} = \frac{-\beta R_L}{r_\pi + (\beta+1)\, R_E}}$$

and

$$\boldsymbol{\frac{v_e}{v_b} = \frac{R_E}{R_E + \frac{r_\pi}{\beta+1}}}$$

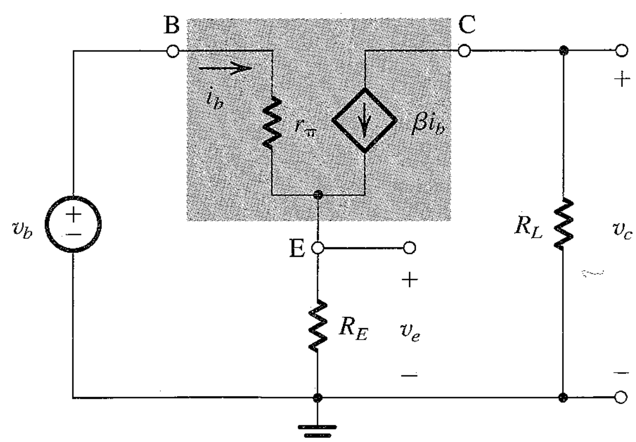

FIGURE P1.63

$$v_e = R_E\,(i_b + \beta i_b) = R_E i_b\,(\beta+1)$$

$$v_c = -\beta i_b R_L$$

$$v_e = v_b - i_b r_\pi$$

Rewriting the first and third equation to have i_b on the left-hand-side and elimintaing i_b we get.

$$\frac{v_e}{R_E(\beta+1)} = \frac{v_b - v_e}{r_\pi}$$

$$v_e r_\pi = v_b R_E(\beta+1) - v_e R_E(\beta+1)$$

$$v_e \left[r_\pi + R_E(\beta+1)\right] = v_b R_E(\beta+1)$$

$$\begin{aligned}\frac{v_e}{v_b} =& \frac{R_E(\beta+1)}{r_\pi + R_E(\beta+1)}\\ =& \frac{R_E}{R_E + \frac{r_\pi}{\beta+1}}\end{aligned}$$

Which is the second expression we were asked to show. Next, eliminate v_e between the first and third equation.

$$R_E i_b(\beta+1) = v_b - i_b r_\pi$$

Next isolate i_b in the second equation and insert

$$i_b = -\frac{v_c}{\beta R_L}$$

$$-R_E\frac{v_c}{\beta R_L}(\beta+1) = v_b + \frac{v_c}{\beta R_L}r_\pi$$

$$\begin{aligned}v_b =& - v_c\left[\frac{R_E(\beta+1)}{\beta R_L} + \frac{r_\pi}{\beta R_L}\right]\\ =& - v_c\frac{R_E(\beta+1) + r_\pi}{\beta R_L}\end{aligned}$$

and thus

$$\frac{v_c}{v_b} = -\frac{\beta R_L}{r_\pi + R_E(\beta+1)}$$

which is the first equation that we were asked to show.

1.68. For the circuit in Fig. P1.68, find the transfer function $T(s) = V_o(s)/V_i(s)$, and arrange it in the appropriate standard form from Table 1.2. Is this a high-pass or a low-pass network? What is its transmission at very high frequencies? [Estimate this directly, as well as by letting $s \to \infty$ in your expression for $T(s)$.] What is the corner frequency ω_0? For $R_1 = 10\,\mathrm{k\Omega}$, $R_2 = 40\,\mathrm{k\Omega}$, and $C = 0.1\,\mu\mathrm{F}$, find f_0. What is the value of $|T(j\omega_0)|$?

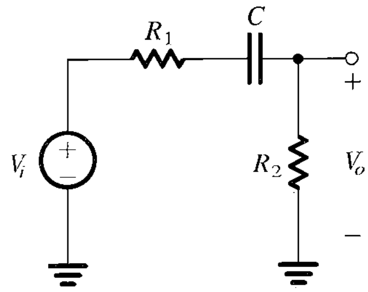

FIGURE P1.68

The transfer function is a simple voltage divider,

$$\frac{V_o}{V_i}(s) = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}}$$
$$= \frac{sR_2C}{1 + sC\left(R_1 + R_2\right)}$$

This is a high-pass network. The transmission at high frequency is

$$\frac{V_o}{V_i}\left(s \to \infty\right) = \frac{sR_2C}{sC\left(R_1 + R_2\right)} = \frac{R_2}{R_1 + R_2}$$

This makes sense because at very high frequency a capactior is essentially a short. I have no idea what is meant by "Estimate this directly." The corner frequency is

$$\omega_0 = \frac{1}{C\left(R_1 + R_2\right)}$$

For the given values of R_1, R_2, and C, we have

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi C\left(R_1 + R_2\right)}$$
$$= \frac{1}{2 \times \pi \times 0.1 \times 10^{-6} \times \left(10 \times 10^3 + 40 \times 10^3\right)}$$
$$= 31.8\,\text{Hz}$$