EE 321 Analog Electronics, Fall 2009
Homework #2 solution

1.77. For the circuit shown in Fig. P1.77 first, evaluate $T_i(s) = V_i(s)/V_s(s)$ and the corresponding cutoff (corner) frequency. Second, evaluate $T_o(s) = V_o(s)/V_i(s)$ and the corresponding cutoff frequency. Put each of the transfer functions in the standard form (see Table 1.2), and combine them to form the overall transfer function, $T(s) = T_i(s) \times T_o(s)$. Provide a Bode magnitude plot for $|T(j\omega)|$. What is the bandwidth between 3-dB cutoff points?

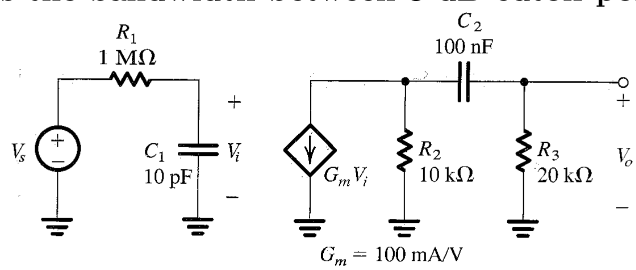


FIGURE P1.77

First the input transfer function

$$T_i(s) = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{1}{1 + sC_1R_1}$$

The cutoff frequency is

$$f_{ci} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2 \times \pi \times 10 \times 10^{-12} \times 10^6} = 15.9\,\text{kHz}$$

On the output side we have

$$V_x = -G_m V_i \left(R_2 || \frac{1}{sC_2} + R_3 \right)$$

And

$$V_o = V_x \frac{R_3}{R_3 + \frac{1}{sC_2}}$$

or

$$V_x = V_o \frac{1}{R_3} \left(R_3 + \frac{1}{sC_2} \right)$$

Eliminating V_x between the two we get

$$V_o \frac{1}{R_3} \left(R_3 + \frac{1}{sC_2} \right) = -G_m V_i \left(R_2 || \frac{1}{sC_2} + R_3 \right)$$

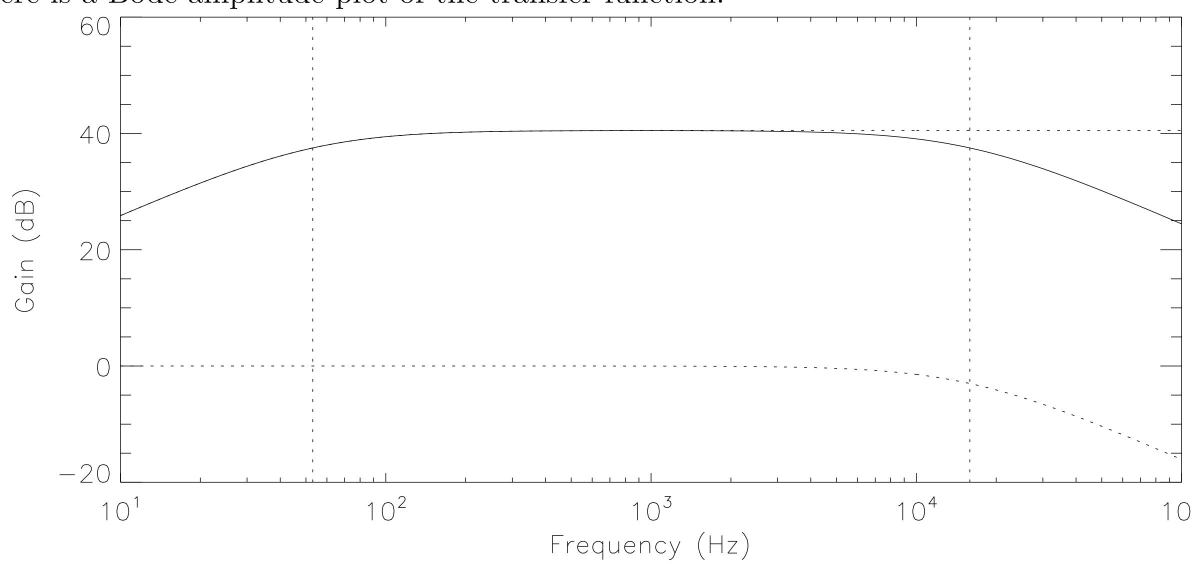
$$\frac{V_o}{R_3} = -G_m V_i \frac{1}{\frac{1}{R_2}\left(R_3 + \frac{1}{sC_2}\right) + 1}$$

$$\begin{aligned}\frac{V_o}{V_i} &= -\,G_m \frac{R_3}{\frac{R_3}{R_2} + \frac{1}{sC_2R_2} + 1} \\ &= -\,\frac{sG_mR_3C_2R_2}{sC_2R_3 + 1 + sC_2R_2} \\ &= -\,\frac{sG_MR_3R_2C_2}{1 + s\left(R_2 + R_3\right)C_2}\end{aligned}$$

The cutoff frequency is

$$f_{co} = \frac{1}{2\pi C_2\left(R_2 + R_3\right)} = \frac{1}{2 \times \pi \times 100 \times 10^{-9} \times \left(10 \times 10^3 + 20 \times 10^3\right)} = 53.1\,\mathrm{Hz}$$

Here is a Bode amplitude plot of the transfer function.



and the bandwidth between 3-dB frequencies is $\mathrm{BW} = 15.9 \times 10^3 - 53.1 = 15.9\,\mathrm{kHz}$.

2.4. A set of experiments are run on an op amp that is ideal except for having a finite gain A. The results are tabulated below. Are the results consistent? If not, are they reasonable, in view of the possibility of experimental error? What do they show the gain to be? Using this value, predict values of the measurements that were accidentally omitted (the blank entries).

Experiment #	V_1	V_2	V_O
1	0.00	0.00	0.00
2	1.00	1.00	0.00
3		1.00	1.00
4	1.00	1.10	10.1
5	2.01	2.00	–0.99
6	1.99	2.00	1.00
7	5.10		–5.10

Experiments 1 and 2 show that there is no common-mode gain. Experiment 4 is a differential gain of $10.1/0.1 = 101$. Experiment 5 is a differential gain of $-0.99/(-0.01) = 99$. Experiment 6 is a differential gain of $1/0.01 = 100$. The mean measured gain is thus $A = 100$.
In experiment 3 the missing value is $V_1 = 0.99$.
In experiment 7 the missing value is found from

$$V_o = A\,(V_2 - V_1) \qquad V_2 = \frac{V_o}{A} + V_1 = 5.1 - \frac{5.1}{100} = 5.05$$

2.31. The circuit in Fig. P2.31 can be considered an extension of the circuit in Fig. 2.8.
(a) Find the resistances looking into node 1, R_1; node 2, R_2; node 3, R_3; and node 4, R_4.
(b) Find the currents, I_1, I_2, I_3, and I_4 in terms of the input current I.
(c) Find the voltages at nodes 1, 2, 3, and 4. That is, V_1, V_2, V_3, and V_4, in terms of (IR).

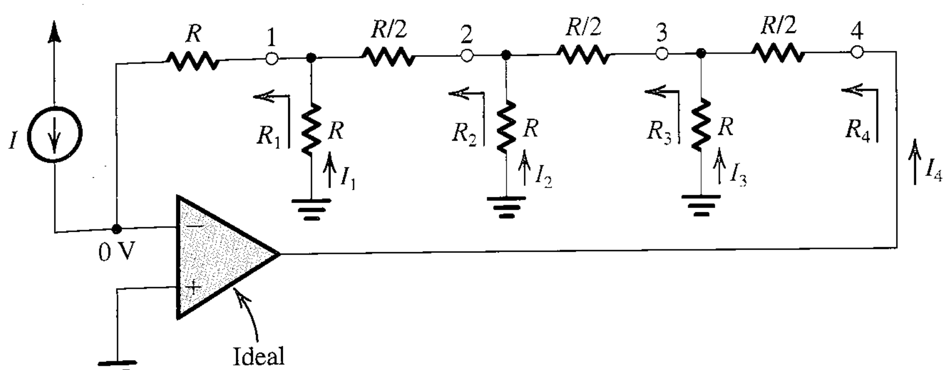


FIGURE P2.31

(a) The resistance looking into node 1 is R. R carrying I_1, and R left of 1 combine to $R/2$, added with $R/2$ left of 2, we get a resistance looking into 2 of R. By the same reasoning the resistance looking into 3 and 4 are also R.
(b) At each of the 4 points the input resistance is R. This means that the current must equal the right-ward current in order that the voltage drops match. At node 1 we have I going right, so $I_1 = I$. At node 2 we have $2I$ going right, so $I_2 = 2I$. At node 3 we have $4I$ going right, so $I_3 = 4I$, and at node 4 we have $8I$ going right, so $I_4 = 8I$.
(c) Since the input resistances are R at each node and the input currents are $-I$, $-2I$, $-4I$, and $-8I$, the voltages must be $V_1 = -RI$, $V_2 = -2RI$, $V_3 = -4I$, and $V_4 = -8I$.

2.49. Derive an expression for the voltage gain v_o/v_i of the circuit in Fig. P2.49.

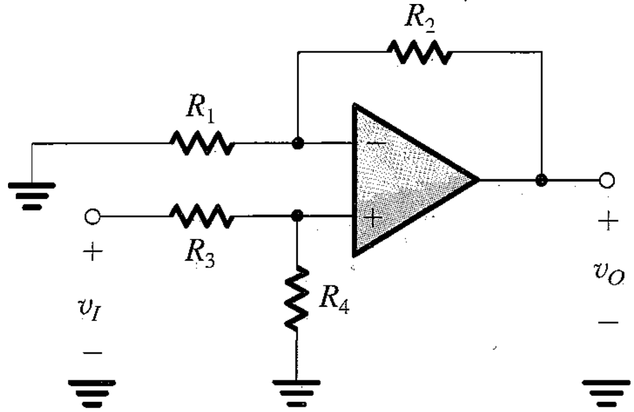


FIGURE P2.49

$$v_+ = \frac{R_4}{R_3 + R_4} v_I$$

$$v_- = v_o \frac{R_1}{R_1 + R_2}$$

We assume the op-amp is ideal, so $v_+ = v_-$,

$$\frac{R_4}{R_3 + R_4} v_I = v_o \frac{R_1}{R_1 + R_2}$$

$$\frac{v_O}{v_I} = \frac{R_1 + R_2}{R_1} \frac{R_4}{R_3 + R_4}$$

It is simply a ordinary non-inverting amplifier with gain $1 + \frac{R_2}{R_1}$ preceded by a voltage divider with gain $\frac{R_4}{R_3+R_4}$.