EE 321 Analog Electronics, Fall 2009
Homework #3 solution

2.108. The circuit in Fig. P2.108 uses an op-amp having a ±4 mV offset. What is its output offset voltage? What does the output offset become with the input AC coupled through a capacitor C? If instead, the 1 kΩ resistor is capacitively coupled to ground, what does the output offset becomes?

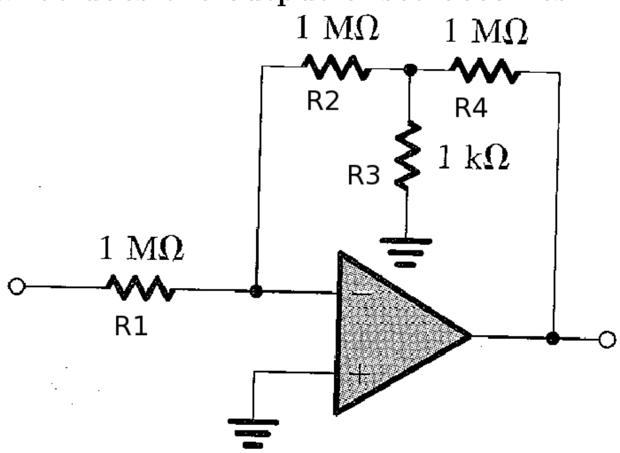


FIGURE P2.108

I have labeled the resistors on the figure. First, what is the output offset voltage for the circuit as shown. Begin with $V_- = V_{OS}$ and input current $I_I = -\frac{V_{OS}}{R_1}$. If the voltage at the 3-resistor node is V_x, we can write the following relationships:

$$V_X = V_{OS} - R_2 I_I = V_{OS} - \frac{R_2}{R_1} V_{OS} = V_{OS}\left(1 - \frac{R_2}{R_1}\right)$$

$$I_4 = I_3 - I_I = \frac{V_X}{R_3} + \frac{V_{OS}}{R_1}$$

where I_3 flows through R_3 to ground, I_4 flows through R_4 towards the node.

$$V_O = V_x + I_4 R_4 = V_X + V_X \frac{R_4}{R_3} + V_{OS}\frac{R_4}{R_1} = V_X\left(1 + \frac{R_4}{R_3}\right) + V_{OS}\frac{R_4}{R_1}$$

Now insert the expression for V_X from above

$$\begin{aligned} V_O =& V_{OS}\left(1 - \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right) + V_{OS}\frac{R_4}{R_1} \\ =& V_{OS}\left[\left(1 - \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right) + \frac{R_4}{R_1}\right] \\ =& V_{OS}\left[1 + \frac{R_4}{R_3} - \frac{R_2}{R_3} - \frac{R_2}{R_1}\frac{R_4}{R_3} + \frac{R_4}{R_1}\right] \end{aligned}$$

Inserting values we get

$$\begin{aligned} V_O =& V_{OS}\left[1+\frac{10^6}{10^3}-\frac{10^6}{10^3}-\frac{10^6}{10^6}\frac{10^6}{10^3}+\frac{10^6}{10^3}\right] \\ =& V_{OS}\left[1+10^3-10^3-10^3+10^3\right] = V_{OS} = \pm 4\,\mathrm{mV} \end{aligned}$$

(but the signal gain is 1 also).
If we capacitively couple the input then $I_I = 0$ (at DC), and, and we have (still at DC) $V_X = V_{OS}$, and then, since $I_4 = I_3$,

$$V_O = \frac{R_4}{R_3}V_X = V_{OS}\frac{R_4}{R_3} = 10^3 V_{OS} = \pm 4\,\mathrm{V}$$

If instead we capacitively couple R_3 to ground then we have $I_3 = 0$ and thus $I_4 = -I_I$ so that

$$V_O - V_- = I_4\left(R_2 + R_3\right)$$

or

$$\begin{aligned} V_O =& V_- - I_I\left(R_2+R_3\right) \\ =& V_{OS} + \frac{V_{OS}}{R_1}\left(R_2+R_3\right) \\ =& V_{OS}\left(1+\frac{R_2+R_3}{R_1}\right) \\ =& V_{OS}\left(1+\frac{2\times 10^6}{10^6}\right) \\ =& 3V_{OS} \\ =& \pm\, 12\,\mathrm{mV} \end{aligned}$$

2.115. Design a Miller integrator that has a unity-gain frequency of 1 krad/s and an input resistance of 100 kΩ. Sketch the output you would expect for the situation in which the output initially at 0 V, a 2 V, 2 ms pulse is applied to the input. Characterize the output that results when a sine wave $2\sin(1000t)$ is applied to the input?
A miller integrator has an input resistor, R, and a negative feedback capacitor, C. It's transfer function is like the transfer function of a inverting feedback amplifier,

$$\frac{V_o}{V_i} = -\frac{\frac{1}{sC}}{R} = -\frac{1}{sCR}$$

R is the input resistance, so $R = 100\,\mathrm{k\Omega}$. We want unity gain at $\omega = \omega_0$, so

$$1 = \frac{1}{\omega_0 RC}$$

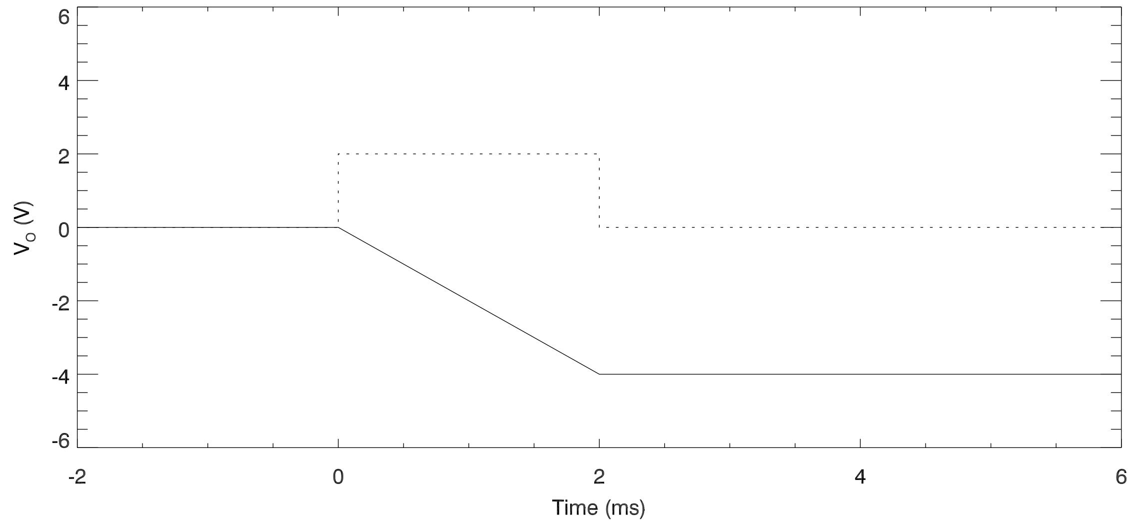
or

$$C = \frac{1}{\omega_0 R} = \frac{1}{10^3 \times 10^5} = 10^{-8}\,\mathrm{F} = 10\,\mathrm{nF}$$

Next, the input goes from 0 V to 2 V for 2 ms and then back to 0 V again. While in the pulse the output rises linearly according to the expression

$$V_O = -\frac{V_I t}{RC}$$

Because the time-constant is $RC = 1\,\mathrm{ms}$, the output increases to $-2 \times V_I = 4\,\mathrm{V}$ during the pulse and then remains constant after the pulse. That looks like this:



If we apply the sine wave to the input (with frequency $\omega = 1000$) the output will be shifted by $-90°$ and have same amplitude, because we designed the integrator to have unity gain at $\omega_0 = 10^3$.

$$V_o = -2\sin(1000t - 90°) = 2\cos(1000t)$$

2.122. An op-amp differentiator with 1 ms time constant is drive by the rate-controlled step down in Fig P02.122. Assuming v_o to be zero initially, sketch and label its waveform.

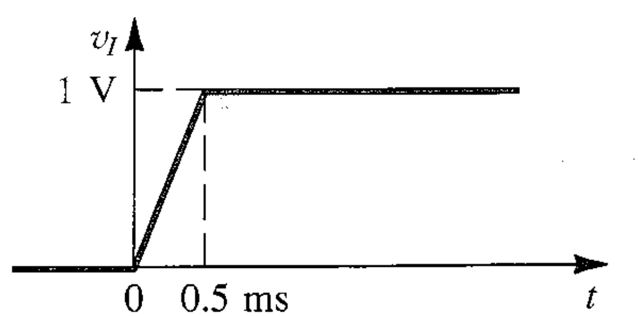


FIGURE P2.122

Before the non-zero input signal begins, the output is zero. During the linear ramp in the input, the output is constant. During the constant value of the input after the linear ramp, the output is zero.

$$V_i = \begin{cases} 0 & t < 0 \\ \frac{V_I t}{t_0} & t \leq 0 < t_0 \\ V_I & 0 \leq t_0 \end{cases}$$

Next, the output voltage

$$V_o = -\tau \frac{dV_I}{dt}$$

First note that $V_o = 0$ for $t < 0$.
Next, for $0 \leq t < t_0$,

$$\begin{aligned} V_o &= -\tau \frac{dV_I}{dt} \\ &= -\tau \frac{V_I}{t_0} \end{aligned}$$

$$V_o = \begin{cases} 0 & t < 0 \\ -\frac{\tau}{t_0} V_I & 0 \leq t < t_0 \\ 0 & t_0 \leq t \end{cases}$$

Inserting $V_I = 1\,\mathrm{V}$, $\tau = 1\,\mathrm{ms}$, and $t_0 = 0.5\,\mathrm{ms}$ we get

$$V_o = \begin{cases} 0 & t < 0 \\ -2\,\mathrm{V} & 0 \leq t < t_0 \\ 0 & t_0 \leq t \end{cases}$$

Here is what it looks like plotted. Note that it is discontinuous at $t = 0$ and $t = t_0$.