EE 321 Analog Electronics, Fall 2009
Homework #4 solution

3.4. In each of the ideal-diode circuits shown in Fig. P3.4, v_I is a 1 − kHz, 10 − V peak sine wave. Sketch the waveform resulting at v_O. What are its positive and negative peak values?

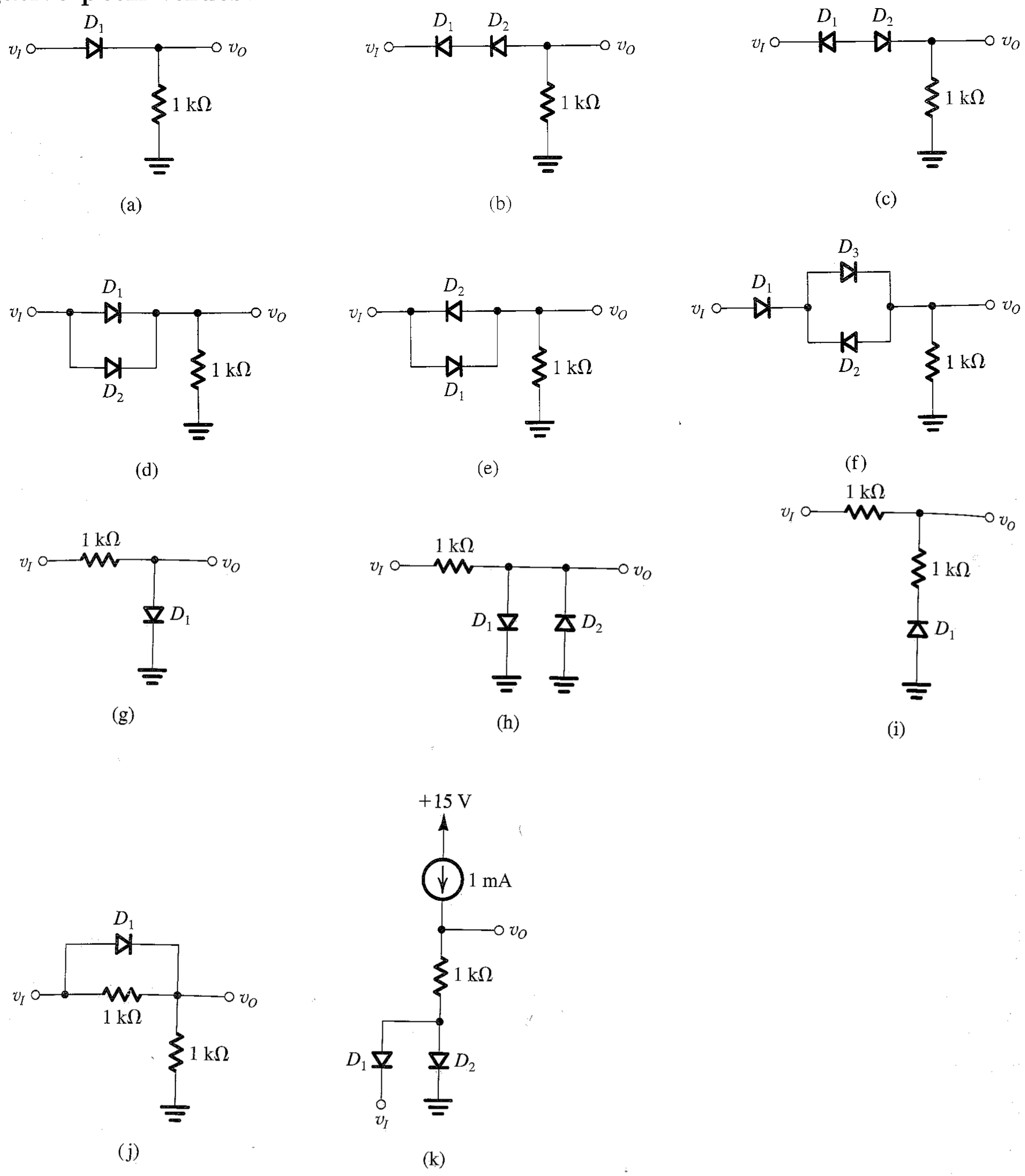
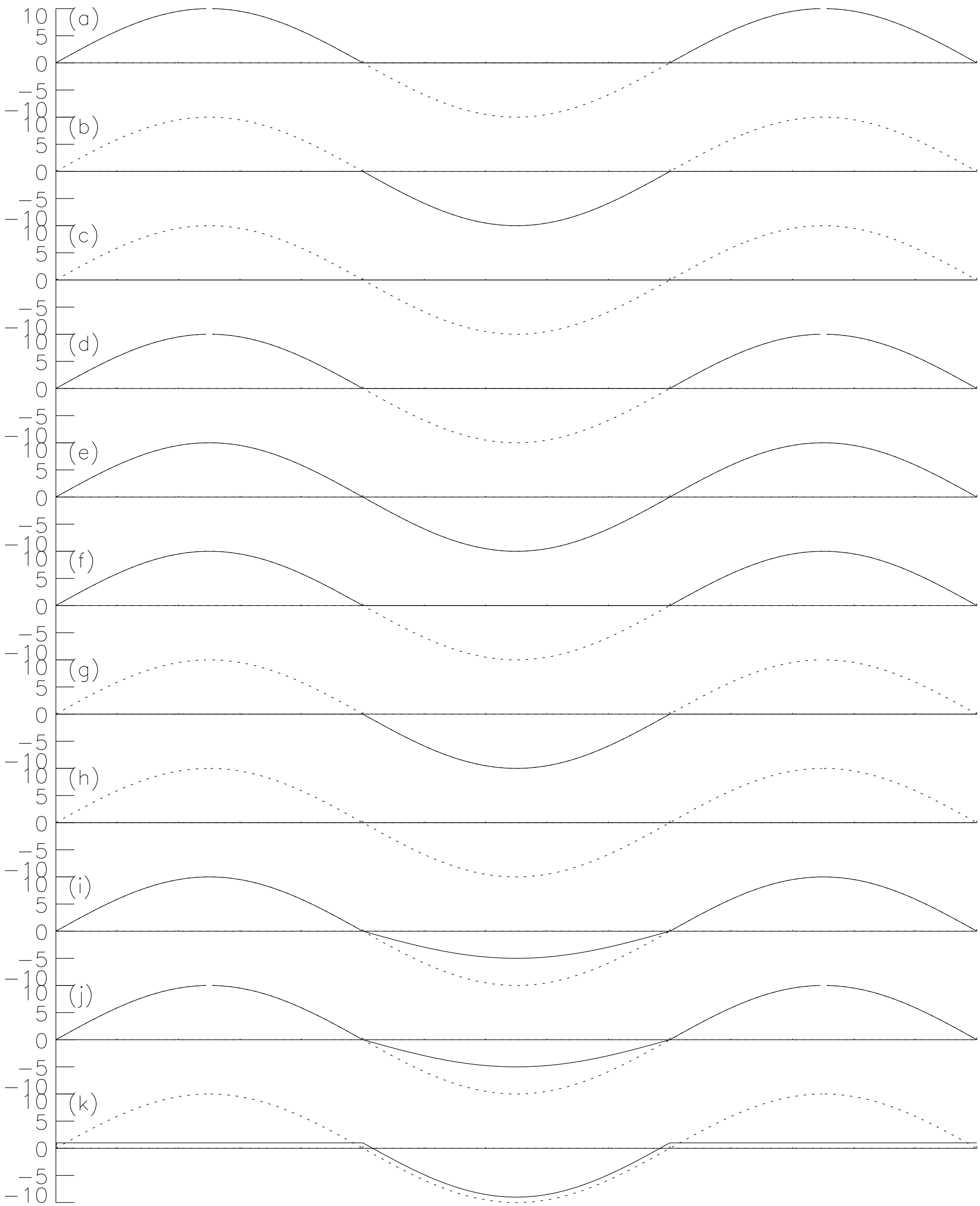


FIGURE P3.4 *(Continued)*


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
10
5
0
−5
−10

Circuit	Min	Max
a	0	10
b	-10	0
c	0	0
d	0	10
e	-10	10
f	0	10
g	-10	0
h	0	0
i	-5	10
j	-5	10
k	-9	1

3.26. For the circuit shown in Fig. P3.26, both diodes are identical, conducting 10 mA at 0.7 V, and 100 mA at 0.8 V. Find the value of R for which $V = 80\,\text{mV}$.

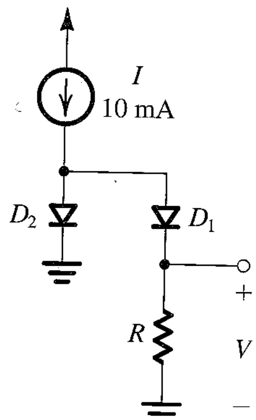


FIGURE P3.26

V_1 is the voltage across diode 1, V_2 across diode 2, I_1 is current through diode 1, and I_2 through diode 2. Then we have (using exponential model)

$$I = I_1 + I_2 \quad I_2 = I_s \exp\left(\frac{V_2}{nV_T}\right) \quad I_1 = I_s \exp\left(\frac{V_2 - V}{nV_T}\right) = I_s \exp\left(\frac{V_2}{nV_T}\right) \exp\left(-\frac{V}{nV_T}\right)$$

or alternatively,

$$I_2 = I_1 \exp\left(\frac{V}{nV_T}\right)$$

and thus

$$I = I_1 + I_2 = I_1 \left[1 + \exp\left(\frac{V}{nV_T}\right)\right]$$

and

$$I_1 = \frac{I}{1 + \exp\left(\frac{V}{nV_T}\right)}$$

and

$$R = \frac{V}{I_1} = \frac{V}{I}\left[1 + \exp\left(\frac{V}{nV_T}\right)\right]$$

Now we only need to determine nV_T. Note that

$$\log\left(\frac{I_D}{I_S}\right) = \frac{V}{nV_T}$$

and

$$\log\left(\frac{I_{Da}}{I_S}\right) - \log\left(\frac{I_{Db}}{I_S}\right) = \frac{V_a - V_b}{nV_T}$$

$$\log\left(\frac{I_{Da}}{I_{Db}}\right) = \frac{V_a - V_b}{nV_T}$$

$$nV_T = \frac{V_a - V_b}{\log\left(\frac{I_{Da}}{I_{Db}}\right)}$$

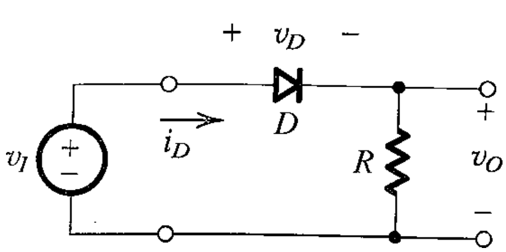
Now inserting $V_a = 0.8\,\mathrm{V}$, $I_a = 100\,\mathrm{mA}$, $V_b = 0.7\,\mathrm{V}$, and $I_b = 10\,\mathrm{mA}$, we get

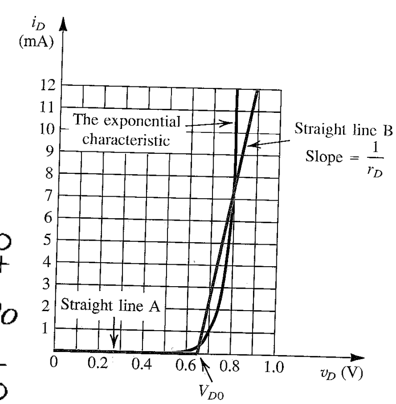
$$nV_T = \frac{0.1}{\log 10} = 43.4\,\mathrm{mV}$$

Inserting that in the expression for R, we get

$$R = \frac{80}{10}\left[1 + \exp\left(\frac{80}{43.4}\right)\right] = 58.5\,\Omega$$

3.44. Consider the half-wave rectifier of Fig. 3.3(a) with $R = 1\,\mathrm{k}\Omega$ and the diode having the characteristics and the piecewise-linear model shown in Fig 3.12 ($V_{D0} = 0.65\,\mathrm{V}$), $r_D = 20\,\Omega$). Analyze the rectifier circuit using the piecewise-linear model for the diode, and thus find the output voltage v_O as a function of v_I. Sketch the transfer characteristic v_O versus v_I for $0 \le v_I \le 10\,\mathrm{V}$. For v_I being a sinusoid with 10 V peak amplitude, sketch and clearly label the waveform of v_O.

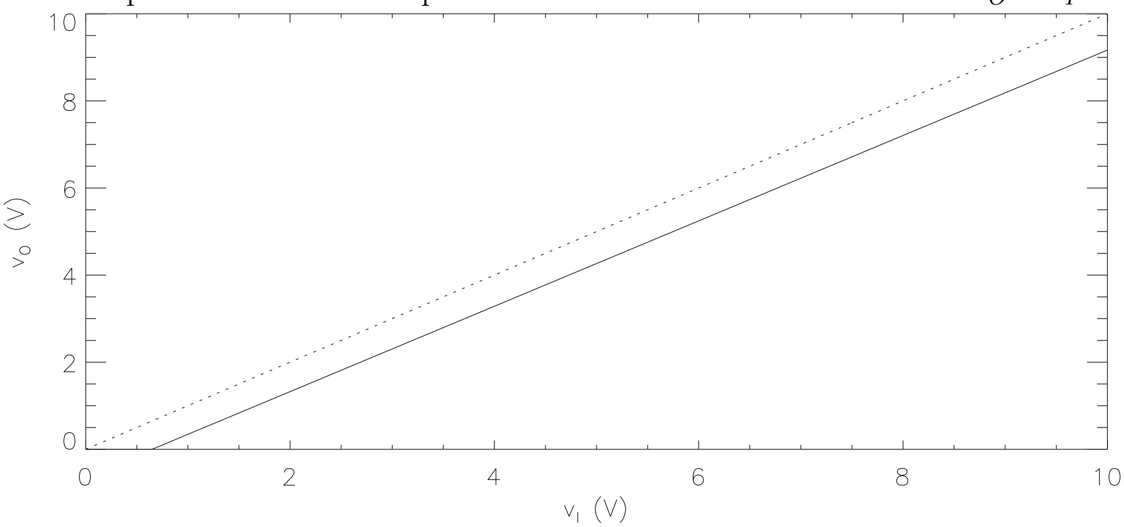




When the diode is conducting, that is when $v_I > V_{D0}$, the output is a voltage division between the load resistance and the diode resistance of the input voltage less the diode drop. When the input voltage is less than the diode voltage the output is zero. Thus,

$$v_O = \begin{cases} 0 & v_I < V_{D0} \\ (v_I - V_{D0})\,\frac{R}{R+r_D} & \end{cases}$$

That curve is plotted here. For comparison the dotted curve is the function $v_O = v_I$.



3.54. In the circuit shown in Fig. P3.54, I is a dc current and v_s is a sinusoidal signal. Capacitors C_1 and C_2 are very large; their function is to couple the signal to and from the diode but block the dc current from flowing into the signal source or the load (not shown). Use the diode small-signal model to show that the signal component of the output voltage is

$$v_o = v_s \frac{nV_T}{nV_T + IR_s}$$

If $v_s = 10\,\text{mV}$ find v_o for $I = 1\,\text{mA}$, 0.1 mA, and $1\,\mu\text{A}$. Let $R_s = 1\,\text{k}\Omega$ and $n = 2$. At what value of I does v_o become one-half of v_s? Note that this circuit functions as a singla attentuator with the attenuation factor controlled by the value of the dc current I.

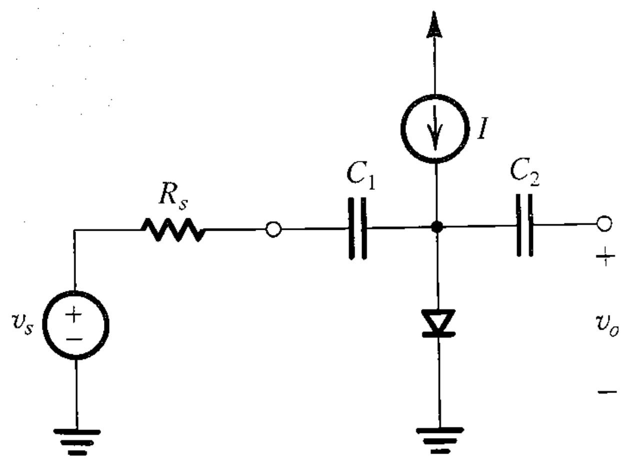


FIGURE P3.54

In the small-signal approximation we assume a small-signal current-voltage relationship between resistance and current through the diode,

$$\frac{di_D}{dv_D} = I_D \frac{1}{nV_T}$$

This is a resistance, $r_d = \left(\frac{di_D}{dv_D}\right)^{-1} = \frac{nV_T}{I_D}$. Now then, for small signals we can transform the circuit in the following way. Eliminate all DC sources, replace the diode with the resistor, r_d, and replace the capacitors with shorts. In that case the circuit is a simple voltage divider between r_d and R_s,

$$v_o = v_s \frac{r_D}{r_D + R_s} = v_s \frac{\frac{nV_T}{I_D}}{\frac{nV_T}{I_D} + R_s} = v_s \frac{nV_T}{nV_T + R_s I_D}$$

Now, $R_s = 1\,\text{k}\Omega$, $n = 2$, $V_T = 25.2\,\text{mV}$. Then for $I = 1\,\text{mA}$,

$$\frac{v_o}{v_I} = \frac{2 \times 25.2 \times 10^{-3}}{2 \times 25.2 \times 10^{-3} + 10^3 \times 10^{-3}} = 0.048,$$

for $I = 0.1\,\text{mA}$,

$$\frac{v_o}{v_s} = \frac{2 \times 25.2 \times 10^{-3}}{2 \times 25.2 \times 10^{-3} + 10^3 \times 10^{-4}} = 0.34,$$

and for $I = 1\,\mu\text{A}$,

$$\frac{v_o}{v_s} = \frac{2 \times 25.2 \times 10^{-3}}{2 \times 25.2 \times 10^{-3} + 10^3 \times 10^{-6}} = 0.98$$

The value of I for which the output is half the input is found from

$$nV_T = I_D R_s$$

$$I_D = \frac{nV_T}{R_s} = \frac{2 \times 25.2 \times 10^{-3}}{10^3} = 50\,\mu\text{A}$$