EE 321 Analog Electronics, Fall 2009
Homework #5 solution

3.74. Consider a half-wave rectifier circuit with a triangular wave input of $5-$V peak-to-peak amplitude and zero average and with $R = 1\,\mathrm{k}\Omega$. Assume that the diode can be represented by the piecewise-linear model $V_{D0} = 0.65\,\mathrm{V}$ and $r_D = 20\,\Omega$. Find the aveage value of v_o.

The relationship between the input and the output is

$$v_o = \begin{cases} (v_I - V_{D0})\frac{R}{R+r_D} & v_I \geq v_{D0} \\ 0 & v_I < v_{D0} \end{cases}$$

If the period of the signal is T, and the input voltage is $v_I = V\sin\left(\frac{2\pi t}{T}\right)$, then the diode is turned on between times t_1 and t_2, where

$$\sin\left(\frac{2\pi t_1}{T}\right) = \frac{V_{D0}}{V} \quad 0 \leq t_1 \leq \frac{T}{4}$$

$$t_1 = \frac{T}{2\pi}\sin^{-1}\frac{V_{D0}}{V} = \sin^{-1}\frac{0.65}{5}\frac{T}{2\pi} = \frac{0.130369}{2\pi}T$$

and

$$t_2 = \frac{T}{2} - t_1 = \frac{T}{2} - \frac{0.130369}{2\pi}T = \left(\frac{1}{2} - \frac{0.130369}{2\pi}\right)T = \frac{\pi - 0.130369}{2\pi}T$$

The average value of v_O is found by integrating it over the period and dividing by the period,

$$\begin{aligned}
\langle v_o\rangle =& \frac{1}{T}\int_0^T v_o\,dt \\
=& \frac{1}{T}\int_{t_1}^{t_2} v_o\,dt \\
=& \frac{1}{T}\int_{t_1}^{t_2}\left[V\sin\left(\frac{2\pi t}{T}\right) - V_{D0}\right]\frac{R}{R+r_D}\,dt \\
=& \frac{1}{T}\frac{R}{R+r_D}\left[V\int_{t_1}^{t_2}\sin\left(\frac{2\pi t}{T}\right)\,dt - (t_2 - t_1)\,V_{D0}\right]
\end{aligned}$$

Change integration variable, $x = 2\pi\frac{t}{T}$, $dx = \frac{2\pi}{T}\,dt$, $x_1 = 2\pi\frac{t_1}{T} = 0.130369$, $x_2 = 2\pi\frac{t_2}{T} = \pi - 0.130369$, we get

$$
\begin{aligned}
\langle v_o \rangle =& \frac{1}{T} \frac{R}{R + r_D} \left[V \frac{T}{2\pi} \int_{x_1}^{x_2} \sin x \, dx - (t_2 - t_1) V_{D0} \right] \\
=& \frac{1}{T} \frac{R}{R + r_D} \left[V \frac{T}{2\pi} \int_{x_1}^{x_2} \sin x \, dx - \frac{T}{2\pi} (x_2 - x_1) V_{D0} \right] \\
=& \frac{1}{2\pi} \frac{R}{R + r_D} \left[V \int_{x_1}^{x_2} \sin x \, dx - (x_2 - x_1) V_{D0} \right] \\
=& \frac{1}{2\pi} \frac{R}{R + r_D} \left[V \left[-\cos x \right]_{x_1}^{x_2} - (x_2 - x_1) V_{D0} \right]
\end{aligned}
$$

Now inserting all the numbers

$$
\begin{aligned}
\langle v_o \rangle =& \frac{1}{2\pi} \frac{10^3}{20 + 10^3} \left[5 \times (\cos 0.130369 - \cos (\pi - 0.130369)) - 0.65 \times (\pi - 2 \times 0.130369) \right] \\
=& 1.2549 \, \mathrm{V}
\end{aligned}
$$

3.78. A full-wave bridge rectifier circuit with a 1− kΩ load operates from a 120−V (rms) 60− Hz household supply through a 10-to-1 step-down transformer having a single secondary winding. It uses four diodes, each of which can be modeled to have a 0.7 − V drop for any current. What is the peak value of the rectified voltage across the load? For what fraction of the cycle does each diode conduct? What is the average voltage across the load? What is the average current through the load?

The peak value of the rectified voltage across the load is

$$
V_O = V_I - 2v_D
$$

where $V_I = 120 \, \mathrm{V} \frac{\sqrt{2}}{10} = 16.97 \, \mathrm{V}$, so

$$
V_O = 16.97 - 2 \times 0.7 = 15.57 \, \mathrm{V}
$$

Each dioded conducts for a fraction, f, of time which is equal to the time the input voltage is greater than $2v_D$, or the fraction of the time that a sinusoid exceeds the value $2v_D / V_I$.

$$
\begin{aligned}
f =& \frac{1}{2\pi} \left(\pi - 2 \sin^{-1} \left(\frac{2v_D}{V_I} \right) \right) \\
=& \frac{1}{2\pi} (\pi - 2 \times 0.0825924) \\
=& 0.4737 \\
=& 47.37\%
\end{aligned}
$$

(the answer in the book is incorrect). The average voltage across the load is found by integrating the output voltage over a period and dividing by the period,

$$\langle v_o \rangle = \frac{1}{T}\int_0^T v_o \, dt$$

The relationship between the input and the output voltage is

$$v_o = \begin{cases} |v_I| - 2V_D & |v_I| > 2V_D \\ 0 & |v_I| \leq 2V_D \end{cases}$$

where $v_I = V_I \sin \frac{2\pi t}{T}$. Because of the symmetry we can just integrate over the half period corresponding to the positive peak in v_I,

$$\begin{aligned} \langle v_o \rangle =& \frac{2}{T}\int_0^{\frac{T}{2}} v_o \, dt \\ =& \frac{2}{T}\int_{t_1}^{t_2} V_I \sin\left(\frac{2\pi t}{T}\right) - 2V_D \, dt \end{aligned}$$

Now change variables, $x = \frac{2\pi t}{T}$, and thus $dx = \frac{2\pi}{T} dt$, $x_1 = \frac{2\pi t_1}{T}$, and $x_2 = \frac{2\pi t_2}{T}$,

$$\begin{aligned} \langle v_o \rangle =& \frac{2}{T}\frac{T}{2\pi}\int_{x_1}^{x_2} V_I \sin\, x - 2V_D \, dx \\ =& \frac{1}{\pi}\left[V_I \left[-\cos x\right]_{x_1}^{x_2} - 2\left(x_2 - x_1\right) V_D\right] \end{aligned}$$

Now,

$$x_1 = \sin^{-1}\left(\frac{2V_D}{V_I}\right) = \sin^{-1}\left(\frac{2 \times 0.7}{16.97}\right) = 0.0825924$$

and $x_2 = \pi - x_1 = \pi = -0.0825924$, and we can insert

$$\langle v_o \rangle = \frac{1}{\pi}\left[16.97 \times 2 \times \cos(0.0825924) - 2 \times 0.7 \times (\pi - 2 \times 0.0825924)\right] = 9.44\,\mathrm{V}$$

The average current is simply the average voltage divided by the load resistance,

$$\langle i_o \rangle = \frac{1}{R}\langle v_o \rangle = \frac{9.44}{10^3} = 9.44\,\mathrm{mA}$$

3.91. The op amp in the precision rectifier circuit of Fig P3.91 is ideal with output saturation levels of $\pm 12\,\mathrm{V}$. Assume that when conducting the diode exhibits a constant voltage drop of 0.7 V. Find v_-, v_a, and v_A for:

(a) $v_I = +1\,\mathrm{V}$

(b) $v_I = +2\,\mathrm{V}$

(c) $v_I = -1\,\mathrm{V}$

(d) $\boldsymbol{v_I = -2\,\mathrm{V}}$

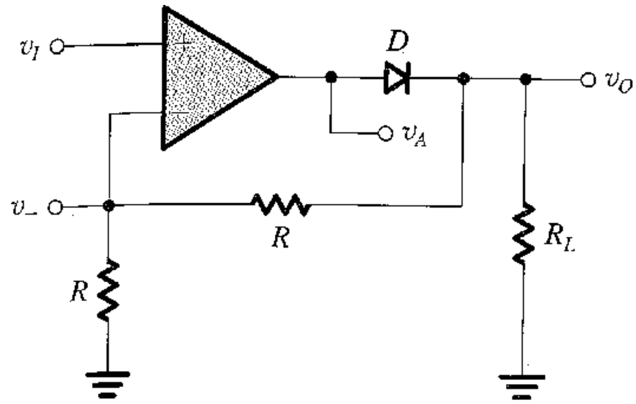

FIGURE P3.91

(a) When $v_I > v_D$, the op-amp will attempt to output current to raise v_- to v_I by raising its output voltage. Therefore I expect the diode to be conducting. In that case we have,

$$i_- = \frac{v_-}{R} = \frac{v_D}{R}$$

and thus

$$v_o = 2Ri_- = 2R\frac{v_I}{R} = 2v_I$$

and

$$v_A = v_O + V_D$$

Inserting values we get

$$v_- = v_I = 1\,\mathrm{V}$$

$$v_o = 2v_I = 2 \times 1 = 2\,\mathrm{V}$$

$$v_A = v_o + V_D = 2 + 0.7 = 2.7\,\mathrm{V}$$

(b) In this case the derivation is exactly the same as for case (a), so

$$v_- = v_I = 2\,\mathrm{V}$$

$$v_o = 2v_I = 4\,\mathrm{V}$$

$$v_A = v_o + V_D = 4 + 0.7 = 4.7\,\mathrm{V}$$

(c) In this case, the op-amp output will attempt to draw current by lowering its voltage. It cannot draw current so the op-amp output will go to negative rail. There is no current anywhere else in the circuit so $v_- = v_o = 0$. Thus,

$$v_- = 0\,\text{V}$$

$$v_o = 0\,\text{V}$$

$$v_A = -12\,\text{V}$$

(d) In this case the situation is identical to case (c), with the same voltages.

3.92. The op-amp in the circuit of Fig P3.92 is ideal with saturation levels of $\pm 12\,\text{V}$. The diodes exhibit a constant $0.7\,\text{V}$ drop when conducting. Find v_-, v_A, and v_o for:

- **(a)** $v_I = +1\,\text{V}$
- **(b)** $v_I = +2\,\text{V}$
- **(c)** $v_I = -1\,\text{V}$
- **(d)** $v_I = -2\,\text{V}$

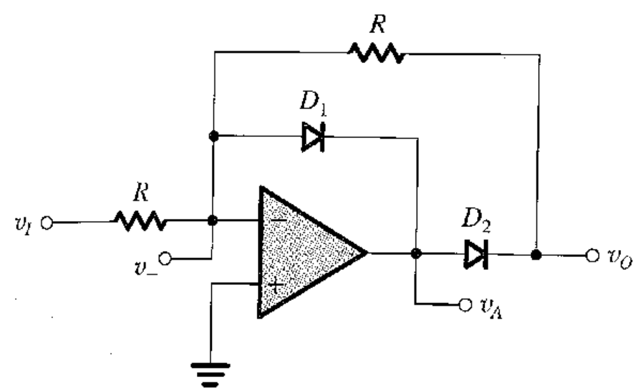

FIGURE P3.92

(a) In this case the input voltage is above ground, and the op-amp will attempt to adjust by drawing current in. It can draw current through D_1, and then D_2 will not be conducting. Thus, $v_A = -V_D = -0.7\,\text{V}$, $v_- = 0\,\text{V}$. For the ground we realize that no current flows through the loop containing ground, and thus $v_o = v_- = 0\,\text{V}$.
(b) This case is the same as case (a). The op-amp is simply drawing twice as much current through D_1. The voltages are the same as case (a).
(c) In this case the input is below ground and the op-amp will attempt to compensate by supplying current, raising its output voltage. In this case diode D_2 is conducting, and the op-amp current will rise until $v_- = 0$. At that point, $v_o = -v_I = 1\,\text{V}$, and $v_A = v_o + V_D = 1.7\,\text{V}$.
(d) This case is similar to case (c). The op-amp will output current through D_2 to make $v_- = 0$, and then $v_o = -v_I = 2\,\text{V}$, and $v_A = v_o + V_D = 2.7\,\text{V}$.