EE 321 Analog Electronics, Fall 2009
Homework #6 solution

3.108. Holes are being steadily injected into a region of n-type silicon (connected to other devices, the detail of which are not important for this question.). In the steady state, the excess-hole concentration profile shown in Fig. P3.108 is established in the n-type silicon region. Here, "excess" means over and above the concentration p_{n0}. If $N_D = 10^{16}\,\mathrm{cm}^{-3}$, $n_i = 1.5 \times 10^{10}\,\mathrm{cm}^{-3}$, and $W = 5\,\mu\mathrm{m}$, find the density of the current that will flow in the x direction.

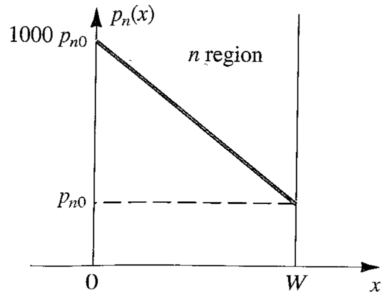

FIGURE P3.108

The hole diffusion current density is related to the gradient of the hole concentration,

$$J_p = -qD_p \frac{dp}{dx}$$

and

$$\frac{dp}{dx} = \frac{p(W) - p(0)}{W} = -\frac{999\,p_{n0}}{W}$$

p_{n0} is found from

$$p_{n0} N_D = n_i^2$$

$$p_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.3 \times 10^4\,\mathrm{cm}^{-3}$$

so that

$$\frac{dp}{dx} = -\frac{2.3 \times 10^4}{5 \times 10^{-6} \times 10^2} = -4.6 \times 10^7\,\mathrm{cm}^{-4}$$

and finally

$$J_p = -qD_p \frac{dp}{dx} = 1.602 \times 10^{19} \times 12 \times 4.6 \times 10^7 = 8.84 \times 10^{-11}\,\mathrm{A\,cm}^{-2} = 88.4\,\mathrm{pA\,cm}^{-2}$$

3.111. In a $10\,\mu$m long bar of donor-doped silicon, what donor concentration is needed to realize a current density of $1\,\mathrm{mA}/\mu\mathrm{m}^2$ in response to an applied voltage of 1 V. (Note: although the carrier mobilities change with doping concentration

[see table associated with Problem 3.113], as a first approximation you may assume μ_n to be constant and use the value for intrinsic silicon, 1350 cm^2 V^{-1} s^{-1}.
The relationship between the electron concentration and electron current is

$$J = qn\mu_n E$$

(where I ignore p because I assume $N_D \gg n_i$. I can check that at the end).

$$n = \frac{J}{q\mu_n E} = \frac{JL}{q\mu_n V}$$

where L is the bar length. Inserting (and being careful of the units, converting to cm),

$$n = \frac{1 \times 10 \times 10^{-4}}{1.602 \times 10^{-19} \times 1350 \times 1} = 4.6 \times 10^{12}$$

This is two orders of magnitude above the intrinsic density, so we can safely ignore p, and set

$$N_D = n$$

3.116. Estimate the total charge stored in a 0.1 μm depletion layer on one side of a 10 μm $\times$ 10 μm junction. The doping concentration on that side of the junction is 10^{16} cm^{-3}.
Assuming that all the charge carriers diffuse across the junction, and an equal number of the opposite charge carrier diffuse across the junction also, in the opposite direction, the total amount of charge is

$$Q = 2qNV = 2 \times 1.602 \times 10^{-19} \times 10^{16} \times \left(10 \times 10^{-4}\right) \times 0.1 \times 10^{-4} = 1.602 \times 10^{-14}\,\mathrm{C}$$

3.117. Combine Eqs. (3.51) and (3.52) to find q_J in terms of V_R. Differentiate this expression to find an expression for the junction cpacitanace C_J. Show that the expression you find is the same as the result obtained using Eq. (3.54) in conjunction with Eq (3.52).
Equation 3.51:

$$q_J = q\frac{N_A N_D}{N_A + N_D} A W_{\mathrm{dep}}$$

Equation 3.52:

$$W_{\mathrm{dep}} = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o + V_R)}$$

Combining them we get

$$q_J = q\frac{N_A N_D}{N_A + N_D} A \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$$

The junction capacitance is then

$$
\begin{aligned}
C_J =& \frac{dq_J}{dV_R} \\
=& q\frac{N_A N_D}{N_A+N_D} A\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}\frac{1}{2\sqrt{V_o+V_R}} \\
=& q\frac{N_A N_D}{N_A+N_D} A\sqrt{\frac{2\epsilon_s}{q}\frac{N_A+N_D}{N_A N_D}}\frac{1}{2\sqrt{V_0+V_R}} \\
=& A\sqrt{2q\epsilon_s\frac{N_A N_D}{N_A+N_D}}\frac{1}{2\sqrt{V_0+V_R}} \\
=& A\sqrt{\frac{q\epsilon_s}{2}\frac{N_A N_D}{N_A+N_D}}\frac{1}{\sqrt{V_0+V_R}}
\end{aligned}
$$

Next compare with the combination of Equation 3.54 and 3.52,

$$
\begin{aligned}
C_J =& \frac{\epsilon_s A}{W_{\text{dep}}} = \frac{\epsilon_s A}{\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0+V_R)}} \\
=& \frac{\epsilon_s A}{\sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)(V_0+V_R)}} \\
=& A\sqrt{\frac{q\epsilon_s}{2}\frac{N_A N_D}{N_A+N_D}\frac{1}{V_0+V_R}}
\end{aligned}
$$

which is the same expression as above.