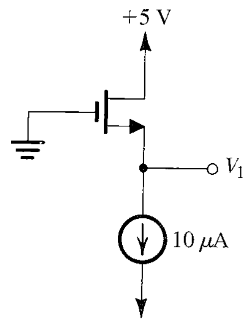
EE 321 Analog Electronics, Fall 2009
Homework #7 solution

4.43. For each of the circuits in Fig. P4.43, find the labeled node voltages. For all transistors, $k_n' \frac{W}{L} = 0.4\,\text{mA/V}^2$, $V_t = 1\,\text{V}$, $\lambda = 0$.

(a) Figure P3.43a



(a)

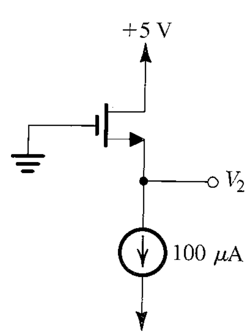
Since $v_{DS} = v_{GS} + V$, $(V = 5\,\text{V})$ we are in the triode region. Insert expression for v_{DS} to get

$$i_D = k_n' \frac{W}{L} \left[(v_{GS} - V_t)(v_{GS} - V_t) - \frac{(v_{GS} - V_t)^2}{2} \right] = \frac{k_n'}{2} \frac{W}{L} (v_{GS} - V_t)^2$$

which gives

$$\begin{aligned} V_1 &= -\,v_{GS} = -\sqrt{2 i_D \left(k_n' \frac{W}{L} \right)^{-1}} - V_t \\ &= -\,\sqrt{2 \times \frac{10 \times 10^{-6}}{0.4 \times 10^{-3}}} - 1 \\ &= -\,1.22\,\text{V} \end{aligned}$$

(b) Figure P3.43b

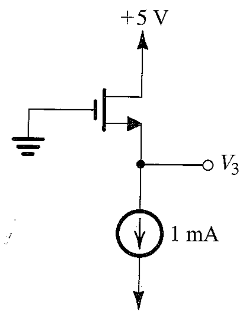


(b)

Same as (a), except larger current, so we get

$$V_2 = -\sqrt{\frac{2 \times 100 \times 10^{-6}}{0.4 \times 10^{-3}}} - 1$$
$$= -1.71\,\text{V}$$

(c) Figure P3.43c

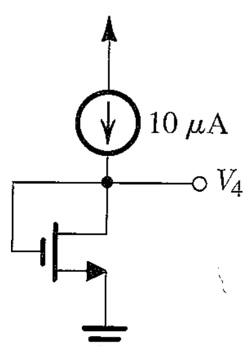


(c)

Same as (a), except larger current still, so we get

$$V_3 = -\sqrt{\frac{2 \times 1 \times 10^{-3}}{0.4 \times 10^{-3}}} - 1$$
$$= -3.24\,\text{V}$$

(d) Figure P3.43d



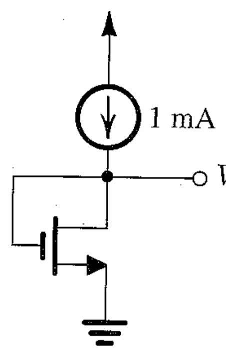
(d)

In this case we are in the saturation region because $v_{DS} > v_{GS} - V_t$,

$$i_D = \frac{k_n'}{2}\frac{W}{L}\left(v_{GS} - V_t\right)^2$$

$$
\begin{aligned}
V_4 = & v_{GS} = \sqrt{2 i_D \left(k_n' \frac{W}{L}\right)^{-1}} + V_t \\
= & \sqrt{\frac{2 \times 10 \times 10^{-6}}{0.4 \times 10^{-3}}} + 1 \\
= & 1.22\,\text{V}
\end{aligned}
$$

(e) Figure P3.43e

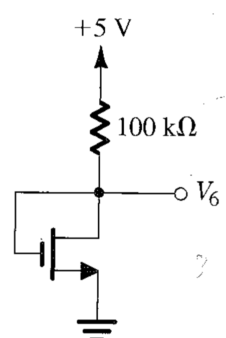


(e)

Same as previous case except larger current.

$$
\begin{aligned}
V_5 = & v_{GS} = \sqrt{\frac{2 \times 10^{-3}}{0.4 \times 10^{-3}}} + 1 \\
= & 3.24\,\text{V}
\end{aligned}
$$

(f) Figure P3.43f



(f)

Saturation mode, so

$$
i_D = \frac{k_n'}{2} \frac{W}{L} \left(v_{GS} - V_t\right)^2
$$

and

$$i_D = \frac{V - v_{GS}}{R}$$

$$V - v_{GS} = R\frac{k_n'}{2}\frac{W}{L}\left[v_{GS}^2 + V_t^2 - 2v_{GS}V_t\right]$$

$$V - v_{GS} = Av_{GS}^2 + AV_t^2 - 2Av_{GS}V_t$$

$$Av_{GS}^2 + v_{GS}\left(1 - 2AV_t\right) + AV_t^2 - V = 0$$

$$Av_{GS}^2 + Bv_{GS} + C = 0$$

$$v_{GS} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = \frac{R}{2}k_n'\frac{W}{L} = 20\,\mathrm{V}^{-1} \qquad B = 1 - 2AV_t = 1 - 2 \times 20 \times 1 = -39$$

$$C = AV_t^2 - V = 20 \times 1^2 - 5 = 15\,\mathrm{V}$$

so that

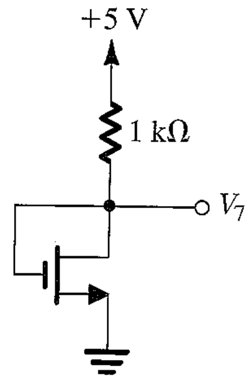
$$v_{GS} = \frac{39 \pm \sqrt{39^2 - 4 \times 20 \times 15}}{2 \times 20}$$

$$v_{GS} = 1.43\,\mathrm{V} \;\text{ or }\; v_{GS} = 0.53\,\mathrm{V}$$

Only one of these solutions is correct. We assume non-zero current and thus conducting mode, which is only true for $v_{GS} > V_t$. Therefore the correct solution is

$$v_{GS} = 1.43\,\mathrm{V}$$

(g) Figure P3.43g



(g)

This is the same problem except a different resistance. In this case we have

$$A = \frac{R}{2} k_n' \frac{W}{L} = 0.2\,\mathrm{V}^{-1} \qquad B = 1 - 2AV_t = 1 - 2 \times 0.2 \times 1 = 0.6$$

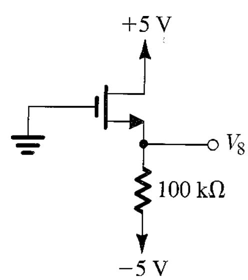
$$C = AV_t^2 - V = 0.2 \times 1^2 - 5 = -4.8\,\mathrm{V}$$

$$v_{GS} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$v_{GS} = 3.62\,\mathrm{V} \;\text{ or }\; v_{GS} = -6.6\,\mathrm{V}$$

The first potential solution is the correct one as we assumed $v_{GS} > V_t$.

(h) Figure P3.43h



(h)

In this case we have

$$v_{DS} + i_D R_D = 2V \qquad V_8 = i_D R - V = -v_{GS}$$

since $v_{DS} = v_{GS} + V > v_{GS} - V_t$, it is operating in the saturation region

$$i_D = \frac{k_n'}{2} \frac{W}{L} (v_{GS} - V_t)^2$$

Inserting the expression for i_D in terms of v_{GS} above,

$$i_D = \frac{V - v_{GS}}{R}$$

we get

$$\frac{V - v_{GS}}{R} = \frac{k_n'}{2}\frac{W}{L}\left(v_{GS} - V_t\right)^2$$

This is identical to the expression in problem (f) above, so the solution is the same, $v_{GS} = 1.43\,\text{V}$, and

$$V_8 = -v_{GS} = -1.43\,\text{V}$$

4.44. For each of the circuits shown in Fig P4.44, find the labeled node voltages. The NMOS transistors have $V_t = 1\,\text{V}$, and $k_n' \frac{W}{L} = 2\,\text{mA/V}^2$. Assume $\lambda = 0$.

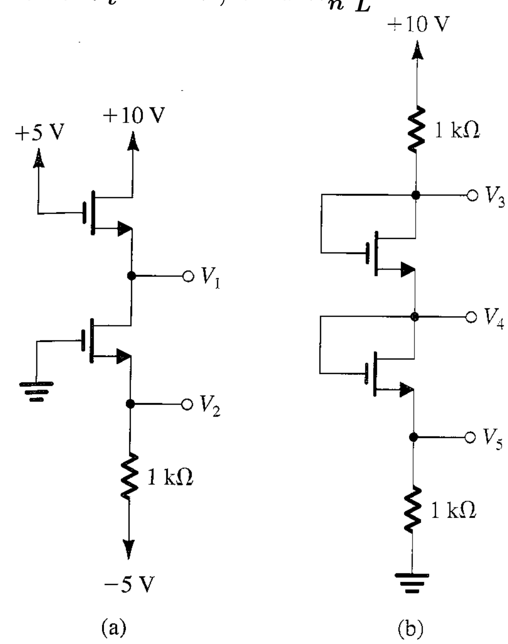


FIGURE P4.44

(a) For Figure P4.44a

We have $v_{DS1} = 2V - V_1$, and $v_{GS1} = V - V_1$, where $V = 5\,\text{V}$. Thus, transistor 1 is operating in saturation. Also notice that $V_2 = i_D R - V$. If we assume that transistor 2 is also operating in saturation then $v_{GS1} = v_{GS2}$. And since $v_{GS2} = -V_2$, we have

$$-v_{GS1} = i_D R - V = R\frac{k_n'}{2}\frac{W}{L}\left(v_{GS1} - V_t\right)^2 - V$$

Choose $A = R\frac{k_n'}{2}\frac{W}{L}$, we get

$$-v_{GS1} = Av_{GS1}^2 + AV_t^2 - 2Av_{GS1}V_t - V$$

$$Av_{GS1}^2 + AV_t^2 - 2Av_{GS1}V_t + v_{GS1} - V = 0$$

or

$$Av_{GS1}^2 + Bv_{GS} + C = 0$$

where

$$A = R\frac{k_n'}{2}\frac{W}{L} = \frac{1 \times 10^3}{2} \times 2 \times 10^{-3} = 1\,\frac{1}{\mathrm{V}}$$

$$B = 1 - 2AV_t = 1 - 2 = -1 \qquad C = AV_t^2 - V = -4\,\mathrm{V}$$

$$v_{GS1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{1 \pm \sqrt{1 + 4 \times 4}}{2}$$

$$v_{GS1} = 2.56\,\mathrm{V} \text{ or } v_{GS1} = -1.56\,\mathrm{V}$$

The second cannot be a solution, because we assumed $v_{GS1} > V_t$ to be conducting. In the first case we get

$$V_1 = V - v_{GS1} = 5 - 2.56 = 2.44\,\mathrm{V}$$

which puts the second transistor in saturation mode, so that assumption is OK. Thus,

$$V_2 = -v_{GS2} = -v_{GS1} = -2.56\,\mathrm{V}$$

(b) For Figure P4.44b

Both transistors are operationg in saturation, and are identical. We have

$$v_{DS1} = v_{DS2} = v_{DS} = v_{GS1} = v_{GS2} = v_{GS}$$

Thus,

$$2v_{GS} + 2i_D R = V$$

($V = 10\,\mathrm{V}$). We also have

$$i_D = \frac{k_n'}{2}\frac{W}{L}\left(v_{GS} - V_t\right)^2$$

Thus

$$\frac{V - 2v_{GS}}{2R} = i_D = \frac{k'_n}{2}\frac{W}{L}(v_{GS} - V_t)^2$$

Choosing $A = \frac{k'_n}{2}\frac{W}{L}$, we get

$$\frac{V - 2v_{GS}}{2R} = Av_{GS}^2 + AV_t^2 - 2Av_{GS}V_t$$

$$Av_{GS}^2 + AV_t^2 - 2Av_{GS}V_t + \frac{v_{GS}}{R} - \frac{V}{2R} = 0$$

$$Av_{GS}^2 + Bv_{GS} + C = 0$$

where

$$A = \frac{k'_n}{2}\frac{W}{L} = 1 \times 10^{-3}\,\frac{\text{A}}{\text{V}^2} \qquad B = -2AV_t + \frac{1}{R} = -1 \times 10^{-3}\,\Omega^{-1}$$

$$C = AV_t^2 - \frac{V}{2R} = -4 \times 10^{-3}\,\text{A}$$

$$v_{GS} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{1 \times 10^{-3} \pm \sqrt{1 \times 10^{-6} + 4 \times 10^{-3} \times 4 \times 10^{-3}}}{2 \times 10^{-3}}$$

$$v_{GS} = 2.56\,\text{V} \;\text{ or }\; v_{GS} = -1.56\,\text{V}$$

The first solution is the correct one because we assumed $v_{GS} > V_t$. In that case,

$$i_D = \frac{k'_n}{2}\frac{W}{L}(v_{GS} - V_t)^2 = 1 \times 10^{-3}(2.56 - 1)^2 = 2.43 \times 10^{-3}\,\text{A}$$

and

$$\begin{aligned} V_3 &= V - i_D R = 10 - 2.43 = 7.57\,\text{V} \\ V_4 &= V_3 - v_{GS} = 7.57 - 2.56 = 5.01\,\text{V} \\ V_5 &= V_4 - v_{GS} = 5.01 - 2.56 = 2.45\,\text{V} \end{aligned}$$

V_5 should also be $V_5 = 0 + i_D R = 2.43\,\text{V}$, which is close to within a few rounding errors.