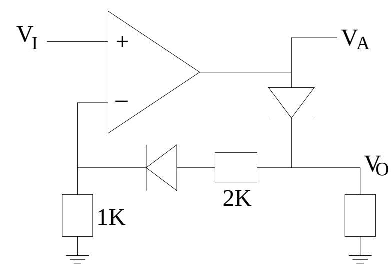
EE 321 Analog Electronics, Fall 2009
Exam 2 October 21, 2009

Rules: This is a closed-book test. You may use pen, paper, and calculator. The exam will last 50 minutes. Each problem counts equally toward your grade. None of the problems require long calculations.

(1) DIODE CIRCUIT. For the following circuit, assume 0.7 V across a forward biased diode, and op-amp rail limits of $L_{\pm} = \pm 9\,\text{V}$. (a) Give expressions for V_A and V_O as a function of V_I. (b) plot V_A, and V_O as a function of V_I in the $[-5\,\text{V}; 5\,\text{V}]$ interval.



(a)

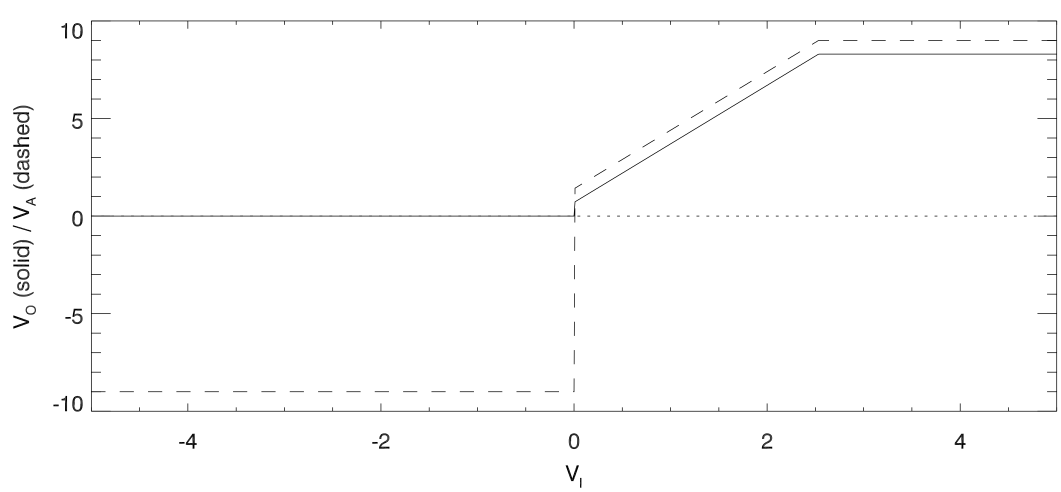
$$V_O = \begin{cases} 0 & V_I \le 0 \\ 3V_I + V_D & 0 < V_I \end{cases}$$

but with clipping $V_O < L_+ - V_D$

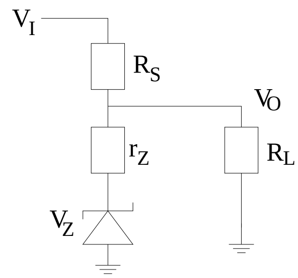
$$V_A = \begin{cases} L_- & V_I \le 0 \\ 3V_I + 2V_D & 0 < V_I \end{cases}$$

but with clipping $V_O < L_+$

(b) The plot is here:



(2) **ZENER VOLTAGE REGULATOR. The following circuit (with the Zener modeled as an ideal Zener in series with a small resistance r_Z) has $V_I = 10\,\text{V}$, $R_S = 1\,\text{k}\Omega$, $r_Z = 50\,\Omega$, and $V_Z = 5.6\,\text{V}$, (a) what is the largest value of V_o? (b) For what value of R_L does regulation cease (current through Zener goes to zero).**



(a) The largest value of V_O occurs when all the curent passes through r_z, which corresponds to $R_L \to \infty$.

$$V_{\text{O,max}} = V_Z + r_z I_Z$$

where

$$I_Z = \frac{V_I - V_Z}{R_S + r_z}$$

Inserting

$$V_{\text{O,max}} = V_Z + r_Z \frac{V_I - V_Z}{R_s + r_z}$$

Now inserting values we get

$$V_{\text{O,max}} = 5.6 + 50\frac{10 - 5.6}{10^3 + 50} = 5.81\,\text{V}$$

(b) Regulation ceases when the voltage drop across R_L is reduced to V_Z, and no current flows through the Zener diode,

$$IR_L = V_Z$$

with

$$I = \frac{V_I}{R_s + R_L}.$$

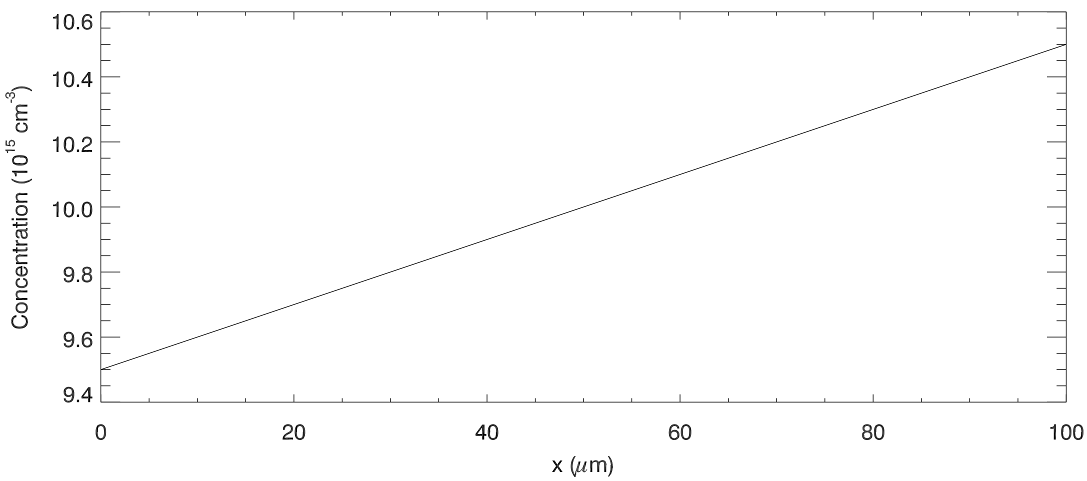
Inserting

$$V_I \frac{R_L}{R_s + R_L} = V_Z$$

$$R_L = \frac{V_Z R_s}{V_I - V_Z} = \frac{5.6 \times 10^3}{10 - 5.6} = 1273\,\Omega$$

(3) SEMICONDUCTOR. A bar of n-type semiconductor of length $L = 100\,\mu$m has a electron concentration which increases linearly from $9.5 \times 10^{15}\,\text{cm}^{-3}$ at the left end ($x = 0$) to $1.05 \times 10^{16}\,\text{cm}^{-3}$ at the right end ($x = L$). Ignore minority carriers. (a) Plot the electron concentration as a function of position in the semiconductor. (b) What is the size and direction of the diffusion current in the material? (c) The gradient can be maintained by applying a opposing drift current. What size and polarity of voltage should be applied? (To simplify the math you may assume constant number density across the slab).

(a) Plot of electron number density



(b) The electron diffusion current in the positive x-direction is

$$J_n = qD_n \frac{dn}{dx}$$

where

$$\frac{dn}{dx} = \frac{n(x = L) - x(x = 0)}{L} = \frac{10.5 \times 10^{15} - 9.5 \times 10^{15}}{100 \times 10^{-4}} = 1.0 \times 10^{17}\,\text{cm}^{-4}$$

Inserting we get

$$J_n = 1.602 \times 10^{-19} \times 34 \times 1.0 \times 10^{17} = 0.54\,\text{A/cm}^2$$

which is a current along the positive x-axis.

(c) We can create an opposing drift current by imposing an opposite electric field. We want a positive current, so we apply a electric field which is directed in the positive x-direction. Ignore minority carriers, so we get

$$J_{\text{drift}} = qn\mu_n E$$

and assume $n = 10^{16}\,\text{cm}^{-3}$ everywhere. This current must cancel the diffusion current so we have

$$-J_n = J_{\text{drift}} = qn\mu_n E$$

or

$$E = -\frac{J_n}{qn\mu_n} = -\frac{0.54}{1.602 \times 10^{-19} \times 10^{16} \times 1350} = -0.25\,\text{V/cm}$$

which is a voltage drop of

$$V = EL = -0.25 \times 100 \times 10^{-4} = -2.5 \times 10^{-3} V = -25\,\text{mV}$$

where the polarity is such that the higher potential is at $x = L$.

EXERCISE

3.34 A diode has $N_A = 10^{17}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $L_p = 5\ \mu\text{m}$, $L_n = 10\ \mu\text{m}$, $A = 2500\ \mu\text{m}^2$, D_p (in the n region) $= 10\ \text{cm}^2/\text{V}\cdot\text{s}$, and D_n (in the p region) $= 18\ \text{cm}^2/\text{V}\cdot\text{s}$. The diode is forward biased and conducting a current $I = 0.1$ mA. Calculate: (a) I_S; (b) the forward-bias voltage V; (c) the component of the current I due to hole injection and that due to electron injection across the junction; (d) τ_p and τ_n; (e) the excess hole charge in the n region Q_p, and the excess electron charge in the p region Q_n, and hence the total minority stored charge Q, as well as the transit time τ_T; and (f) the diffusion capacitance.

Ans. (a) 2×10^{-15} A; (b) 0.616 V; (c) 91.7 μA, 8.3 μA; (d) 25 ns, 55.6 ns; (e) 2.29 pC, 0.46 pC, 2.75 pC, 27.5 ns; (f) 110 pF

Junction Capacitance The depletion-layer or junction capacitance under forward-bias conditions can be found by replacing V_R with $-V$ in Eq. (3.57). It turns out, however, that the accuracy of this relationship in the forward-bias region is rather poor. As an alternative, circuit designers use the following rule of thumb:

$$C_j \simeq 2C_{j0} \tag{3.70}$$

3.7.6 Summary

For easy reference, Table 3.2 provides a listing of the important relationships that describe the physical operation of pn junctions.

TABLE 3.2 Summary of Important Equations for *pn*-Junction Operation

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic *Si* at $T = 300$ K)
Carrier concentration in intrinsic silicon (/cm^3)	$n_i^2 = BT^3 e^{-E_G/kT}$	$B = 5.4 \times 10^{31}/(\text{K}^3\text{cm}^6)$ $E_G = 1.12$ eV $k = 8.62 \times 10^{-5}$ eV/K $n_i = 1.5 \times 10^{10}/\text{cm}^3$
Diffusion current density (A/cm^2)	$J_p = -qD_p \frac{dp}{dx}$ $J_n = qD_n \frac{dn}{dx}$	$q = 1.60 \times 10^{-19}$ coulomb $D_p = 12\ \text{cm}^2/\text{s}$ $D_n = 34\ \text{cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{drift} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$ $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$
Resistivity ($\Omega\cdot$cm)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q$ $\simeq 25.8$ mV
Carrier concentration in n-type silicon (/cm^3)	$n_{n0} \simeq N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in p-type silicon (/cm^3)	$p_{p0} \simeq N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W_{dep} = x_n + x_p$ $= \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$	$\varepsilon_s = 11.7\varepsilon_0$ $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm
Charge stored in depletion layer (coulomb)	$q_J = q\frac{N_A N_D}{N_A + N_D} A W_{dep}$	
Depletion capacitance (F)	$C_j = \frac{\varepsilon_s A}{W_{dep}}$, $C_{j0} = \frac{\varepsilon_s A}{W_{dep}\vert_{V_R=0}}$ $C_j = C_{j0} \Big/ \left(1 + \frac{V_R}{V_0}\right)^m$ $C_j \simeq 2C_{j0}$ (for forward bias)	$m = \frac{1}{3}$ to $\frac{1}{2}$
Forward current (A)	$I = I_p + I_n$ $I_p = Aqn_i^2 \frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$ $I_n = Aqn_i^2 \frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$	
Saturation current (A)	$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
Minority-carrier lifetime (s)	$\tau_p = L_p^2/D_p \quad \tau_n = L_n^2/D_n$	$L_p, L_n = 1\ \mu\text{m}$ to $100\ \mu\text{m}$ $\tau_p, \tau_n = 1$ ns to 10^4 ns
Minority-carrier charge storage (coulomb)	$Q_p = \tau_p I_p \quad Q_n = \tau_n I_n$ $Q = Q_p + Q_n = \tau_T I$	
Diffusion capacitance (F)	$C_d = \left(\frac{\tau_T}{V_T}\right) I$	

3.8 SPECIAL DIODE TYPES[8]

In this section, we discuss briefly some important special types of diodes.

[8] This section can be skipped with no loss in continuity.