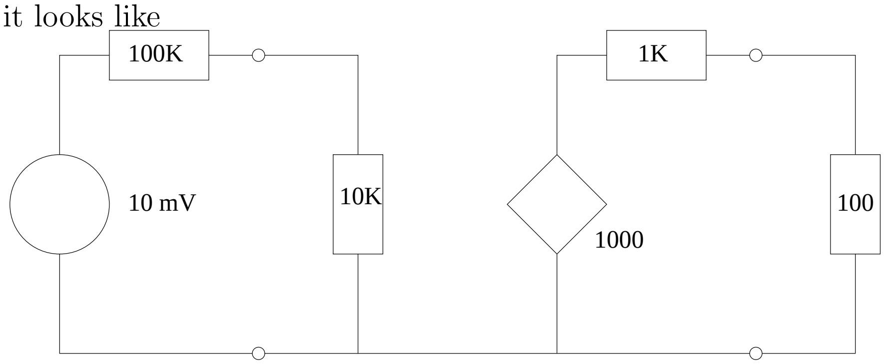
# EE 321 Analog Electronics, Fall 2011
# Homework #1 solution

**1.47. A 10 mV signal source having an internal resistance of 100 kΩ is connected to an amplifier for which the input resistance is 10 kΩ, the open-circuit voltage gain is 1000 V/V, and the output resistance is 1 kΩ. The amplifier is connected in turn to a 100 Ω load. What overall voltage gain results as measured from the source internaal voltage to the load? Where did all the gain go? What would the gain be if the source was connected directly to the load? What is the ratio of tese two gain? This ratio is a useful measure of the benefit the amplifier brings.**
This is what it looks like




The overall gain is

$$\frac{v_O}{v_S} = \frac{R_i}{R_s + R_i} A_{vo} \frac{R_L}{R_o + R_L}$$

Inserting values we get

$$\begin{aligned} G_{\text{amp}} =& \frac{v_O}{v_S} = \frac{10^4}{10^5 + 10^4} \times 1000 \times \frac{100}{1000 + 100} \\ =& 8.26 \end{aligned}$$

All the gain is lost in the voltage source output resistance and in the amplifier output resistance. If the source were connected directly to the load the gain would be

$$G_{\text{src}} = \frac{R_L}{R_s + R_L} = \frac{100}{10^5 + 100} = 0.0010$$

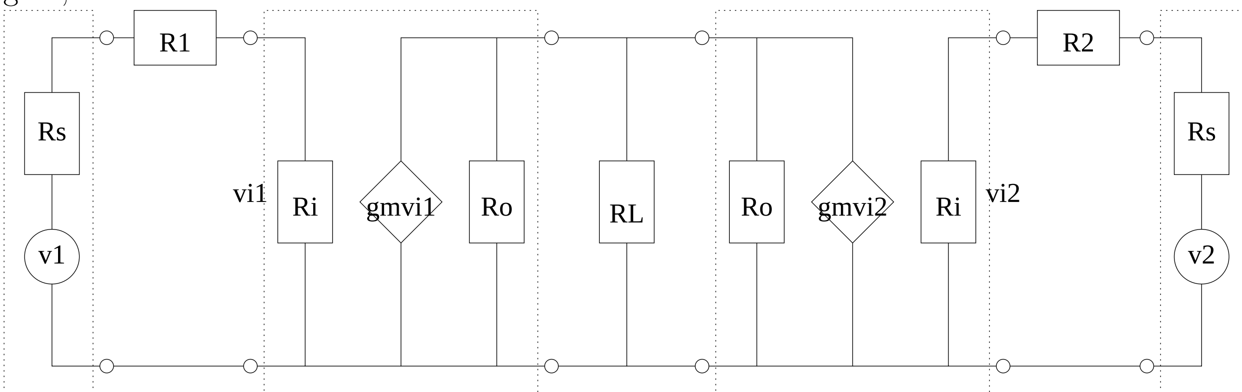
The ratio of the two (the gain by using the amplifier) is

$$\frac{G_{\text{amp}}}{G_{\text{src}}} = \frac{8.26}{0.001} = 8.26 \times 10^3$$

**1.57. A designer is required to provide, across a 10 kΩ load, the weighted sum, $v_o = 10v_1 + 20v_2$, of input signals $v_1$ and $v_2$, each having a source resistance of 10 kΩ. She has a number of transconductance amplifiers for which the input and output resistances are both 10 kΩ and $G = 20$ mA/V, together with a selection of suitable resistors. Sketch an appropriate amplifier topology with additional**

**resistors selected to provide the desired result. (Hint: In your design, arrange to add currents.**

In a transconductance amplifier an output current is proportional to the input voltage. We can add voltages by adding currents on the output. That can be done by putting two transconductance amplifiers in paralled across the output. We can then adjust the gain by adjusting the amount of voltage applied across the input, by placing a resistor either in parallel or in series across the input. If there is not enough gain in the amplifier to achieve the output formula we can cascade another set of amplifiers. Assuming that there is enough gain, the circuit could look like this:




We can solve this problem with superposition. The relationship between the output voltage, $v_O$, across $R_L$, and the input voltage $v_1$, is

$$\frac{v_O}{v_1} = \frac{R_i}{R_S + R_1 + R_i} g_m R_O || R_O || R_L$$

We know the values of all the constant except $R_1$, and we want to choose it such that the ratio $\frac{v_O}{v_1} = 10$. Rearranging it we get

$$\begin{aligned} R_1 =& \frac{v_1}{v_O} R_i g_m \left(R_O || R_O || R_L\right) - R_s - R_i \\ =& 0.1 \times 10 \times 10^3 \times 20 \times 10^{-3} \times \left(10^4 || 10^4 || 10^4\right) - 20 \times 10^3 \\ =& 46.7\,\mathrm{k\Omega} \end{aligned}$$

Similarly we can find $R_2$ from

$$\begin{aligned} R_2 =& \frac{v_2}{v_O} R_i g_m \left(R_O || R_O || R_L\right) - R_s - R_i \\ =& 0.05 \times 10 \times 10^3 \times 20 \times 10^{-3} \left(10^4 || 10^4 || 10^4\right) - 20 \times 10^3 \\ =& 13.3\,\mathrm{k\Omega} \end{aligned}$$

If instead we place $R_1$ and $R_2$ in parallel with the input the expression for $\frac{v_O}{v_1}$ becomes

$$\frac{v_O}{v_1} = \frac{R_1 || R_i}{R_S + R_1 || R_i} g_m \left(R_O || R_O || R_L\right)$$

$$\left(R_S + R_1 || R_i\right) \frac{v_O}{v_1} = R_1 || R_i g_m \left(R_O || R_O || R_L\right)$$

$$R_1||R_i\left[\frac{v_O}{v_1}-g_m\left(R_O||R_O||R_L\right)\right]=-R_S\frac{v_O}{v_1}$$

$$\begin{aligned}R_1||R_i=&\frac{R_s\frac{v_O}{v_1}}{g_m\left(R_O||R_O||R_L\right)-\frac{v_O}{v_1}}\\=&\frac{10\times10^3\times10}{20\times10^{-3}\times\frac{1}{3\times10^{-4}}-10}\\=&1.76\,\mathrm{k\Omega}\end{aligned}$$

In which case

$$\begin{aligned}R_1=&\frac{1}{\frac{1}{R_1||R_i}-\frac{1}{R_i}}\\=&\frac{1}{\frac{1}{1.76\times10^3}-\frac{1}{10^4}}\\=&2.14\,\mathrm{k\Omega}\end{aligned}$$

Similarly,

$$\begin{aligned}R_2||R_i=&\frac{R_s\frac{v_O}{v_2}}{g_m\left(R_O||R_O||R_L\right)-\frac{v_O}{v_2}}\\=&\frac{10\times10^3\times20}{20\times10^{-3}\times\frac{1}{3\times10^{-4}}-20}\\=&4.29\,\mathrm{k\Omega}\end{aligned}$$

and

$$\begin{aligned}R_2=&\frac{1}{\frac{1}{R_2||R_i}-\frac{1}{R_i}}\\=&\frac{1}{\frac{1}{4.29\times10^3}-\frac{1}{10^4}}\\=&7.51\,\mathrm{k\Omega}\end{aligned}$$

**1.58. Figure P1.58 shows a transconductance amplifier whose output is fed back to its input. Find the input resistance $R_{\text{in}}$ of the resulting one-port network. (Hint: Apply a test voltage $v_x$ between the two input terminals, and find the current $i_x$ drawn from the source. Then $R_{\text{in}}=v_x/i_x$.**

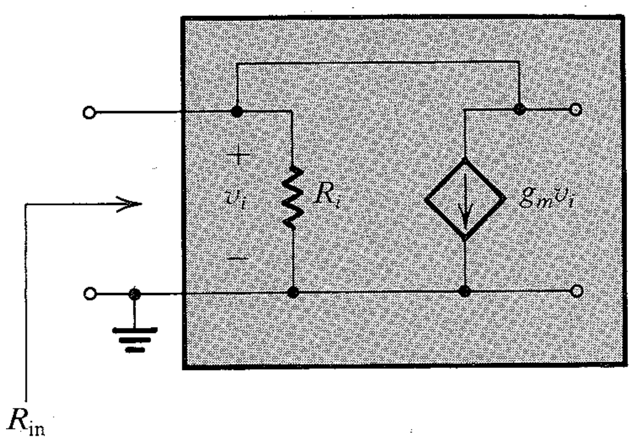



**FIGURE P1.58**

If we apply $v_x$ on the input, then we apply $v_i$ across $R_i$. The current through $R_i$ is then

$$i_{\text{in}} = \frac{v_x}{R_i}$$

The current through the current source (from top to bottom) is

$$i_{\text{out}} = g_m v_x$$

The sum of those currents must be supplied by the source, so

$$i_x = \frac{v_x}{R_i} + g_m v_x = v_x \left( g_m + \frac{1}{R_i} \right)$$

The input resistance is then

$$R_{\text{in}} = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{R_i}} = \frac{R_i}{1 + g_m R_i}$$

The input resistance is thus smaller with the feedback than without the feedback.