EE 321 Analog Electronics, Fall 2011
Homework #5 solution

3.37. Find the parameters of a piecewise-linear model of a diode for which $v_D = 0.7\,\mathrm{V}$ at $i_D = 1\,\mathrm{mA}$ and $n = 2$. The model is to fit exactly at 1 mA and 10 mA. Calculate the error in millivolts in predicting v_D using a piecewise-linear model at $i_D = 0.5,\ 5$ and, 14 mA

We have $i_D = I_S e^{\frac{v_D}{nV_T}}$, so

$$I_S = i_D e^{-\frac{v_D}{nV_T}} = 1\times10^{-3}\times e^{-\frac{0.7}{2\times25.2\times10^{-3}}} = 0.93\,\mathrm{nA}$$

Next,

$$v_D = nV_T\ln\left(\frac{i_D}{I_S}\right) = 2\times25.2\times10^{-3}\times\ln\frac{10\times10^{-3}}{0.93\times10^{-9}} = 0.82\,\mathrm{V}$$

Then we have

$$r_D = \frac{v_{D10}-v_{D1}}{i_{D10}-i_{D1}} = \frac{0.82-0.7}{10-1} = 13\,\Omega$$

and

$$v_{D0} = v_{D1} - r_D i_{D1} = 0.7 - 13\times1\times10^{-3} = 0.69\,\mathrm{V}$$

We can now calculate v_D based on the piecewise linear model as

$$v_D = v_{D0} + i_D r_D$$

or based on the exponential model

$$v_D = nV_T\ln\frac{i_D}{I_S}$$

and those are tabulated here

i_D	v_D (exp)	v_D (lin)	error
0.5	0.67	0.70	0.03
5	0.78	0.76	0.02
14	0.83	0.87	0.04

3.54. In the circuit shown in Fig. P3.54, I is a DC current and v_s is a sinusoidal signal. Capacitors C_1 and C_2 are very large; their function is to couple the signal to and from the diode but block the DC current from flowing into the signal source or the load (not shown). Use the diode small-signal model to show that the signal component of the output voltage is

$$v_o = v_s\frac{nV_T}{nV_T + IR_s}$$

If $v_s = 10\,\mathrm{mV}$, find v_o for $I = 1$ ma, 0.1 mA, and 1 μA. Let $R_s = 1\,\mathrm{k}\Omega$ and $n = 2$. At what value of I does v_o become one-half of v_s? Note that this circuit functions

as a signal attenuator whith the attenuation factor controlled by the value of the DC current I.

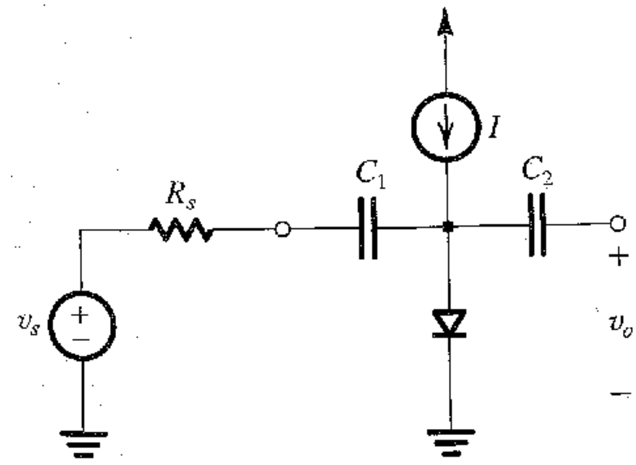

FIGURE P3.54

The signal portion is transferred from input to output according to a voltage division between R_S and r_D,

$$v_o = v_s \frac{r_D}{R_s + r_D}$$

where

$$r_D = \frac{dv_D}{di_D}_{i_D=I} = \left(\frac{di_D}{dv_D}\right)^{-1}_{i_d=I} = \left(\frac{I}{nV_T}\right)^{-1} = \frac{nV_T}{I}$$

and thus

$$v_o = v_s \frac{\frac{nV_T}{I}}{R_S + \frac{nV_T}{I}} = \frac{nV_T}{IR_s + nV_T}$$

Find v_o for several values of I, and $v_s = 10\,\text{mV}$, $R_s = 1\,\text{k}\Omega$, and $n = 2$. It is tabulated below

I (mA)	v_s (mV)
1	0.48
0.1	3.4
10^{-3}	9.8

Value of I for which $v_o = \frac{v_s}{2}$:

$$\frac{nV_T}{nV_T + IR_s} = \frac{1}{2}$$

$$I = \frac{nV_T}{R_s} = \frac{2 \times 25.2 \times 10^{-3}}{1 \times 10^3} = 5.0 \times 10^{-5}\,\text{A} = 50\,\mu\text{A}$$

3.59 Consider the voltage-regulator crictuit show in Fig. P3.59. The value of R is selected to obtain an output voltage V_o (across the diode) of 0.7 V.

(a) **Use the diode small-signal model to show that the change in output voltage corresponding to a change of 1 V in V^+ is**

$$\frac{\Delta V_o}{\Delta V^+} = \frac{nV_T}{V^+ + nV_T - 0.7}$$

This quantity is known as the line regulation and is usually expressed in mV/V.

(b) **Generalize the expression above to the case of m diodes connected in seris and the value of R adjusted so that the voltage across each diode is 0.7 V (and $V_o = 0.7\,m$ V).**

(c) **Calculate the value of line regulation for the case $V^+ = 10$ V (nominally) and (i) $m = 1$, and (ii) $m = 3$. Use $n = 2$.**

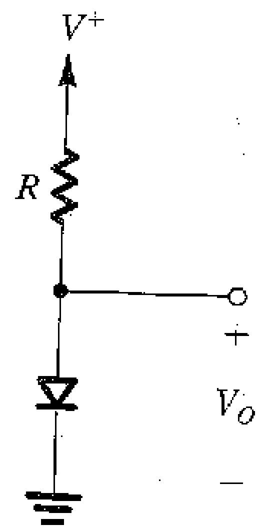

FIGURE P3.59

(a) We can write

$$i_D = \frac{V^+ - v_D}{R} = I_S e^{\frac{v_D}{nV_T}}$$

and we are interested in finding $\frac{dv_D}{dV^+}$. we can re-arrange

$$V^+ = R\left(\frac{v_D}{R} + I_S e^{\frac{v_D}{nV_T}}\right) = v_D + RI_S e^{\frac{v_D}{nV_T}}$$

and then compute

$$\frac{dV^+}{dv_D} = 1 + \frac{RI_S}{nV_T} e^{\frac{v_D}{nV_T}}$$

Now, we are interested in evaluating this at a bias point $v_D = V_D$, for which

$$I_D = I_S e^{\frac{V_D}{nV_T}}$$

so we can insert that and get

$$\frac{dV^+}{dv_D} = 1 + \frac{RI_D}{nV_T}$$

at that bias point we have $RI_D = V^+ - V_D$, so we can write

$$\frac{dV^+}{dv_D} = 1 + \frac{V^+ - V_D}{nV_T}$$

and we are asked to compute

$$\frac{dv_D}{dV^+} = \frac{1}{1 + \frac{V^+ - V_D}{nV_T}} = \frac{nV_T}{nV_T + V^+ - V_D}$$

With the bias point $V_D = 0.7\,\mathrm{V}$ this is identical to the expression we were asked to compute.

(b) In this case we just write

$$i_D = \frac{V^+ - mv_D}{R} = I_S e^{\frac{v_D}{nV_T}}$$

re-arrange

$$V^+ = R\left(\frac{mv_D}{R} + I_S e^{\frac{v_D}{nV_T}}\right) = mv_D + RI_S e^{\frac{v_D}{nV_T}} = v_O + RI_S e^{\frac{v_O}{mnV_T}}$$

and compute

$$\frac{dV^+}{dv_o} = 1 + \frac{RI_S}{mnV_T} e^{\frac{v_O}{mnV_T}} = 1 + \frac{RI_S}{mnV_T} e^{\frac{v_D}{nV_T}}$$

Now note that at the bias point, $v_D = V_D = \frac{V_O}{m}$, we can substitute

$$I_D = I_S e^{\frac{v_D}{nV_T}}$$

$$\frac{dV^+}{dv_o} = 1 + \frac{RI_D}{mnV_T} = 1 + \frac{V^+ - V_O}{mnV_T} = 1 + \frac{V^+ - mV_D}{mnV_T}$$

Finally we can compute

$$\frac{dv_o}{dV^+} = \frac{1}{1 + \frac{V^+ - mV_D}{mnV_T}} = \frac{mnV_T}{mnV_T + V^+ - mV_D}$$

Again we use $V_D = 0.7\,\mathrm{V}$.

(c) (i)

$$\frac{dv_o}{dV^+} = \frac{2 \times 25.2 \times 10^{-3}}{2 \times 25.2 \times 10^{-3} + 10 - 0.7} = 0.0054$$

(ii)

$$\frac{dv_o}{dV^+} = \frac{3 \times 2 \times 25.2 \times 10^{-3}}{3 \times 2 \times 25.2 \times 10^{-3} + 10 - 3 \times 0.7} = 0.019$$

3.61 Design a diode voltage regulator to supply 1.5 V to a 150 Ω load. Use two diodes specified to have a 0.7 V drop at a current of 10 mA and $n = 1$. The diodes are to be connected to a +5 V supply through a resistor R. Specify the value of R. What is the diode current with the load connected? What is the increase resulting in the output voltage when the load is disconnected? What change results if the load resistance is reduced to 100 Ω? To 75 Ω? To 50 Ω?

First we compute I_S from the 10 mA point as

$$I_S = \frac{i_D}{e^{\frac{v_D}{nV_T}}} = \frac{10 \times 10^{-3}}{e^{\frac{0.7}{25.2 \times 10^{-3}}}} = 8.64 \times 10^{-15}\,\mathrm{A}$$

Next compute the amount of current which is required to produce a 0.75 V drop across one diode.

$$i_D = I_S e^{\frac{v_D}{V_T}} = 8.64 \times 10^{-15} e^{\frac{0.75}{25.2 \times 10^{-3}}} = 72.8\,\mathrm{mA}$$

We have this current through the resistor, as well as the curent through the 150 Ω load resistor. The current through the load is

$$i_L = \frac{v_L}{R_L} = \frac{1.5}{150} = 10\,\mathrm{mA}$$

and the size of the resistor can then be found from

$$V = R\,(i_D + i_L) + 2v_D$$

$$R = \frac{V - 2v_D}{i_D + i_L} = \frac{5 - 1.5}{72.8 + 10} = 42.3\,\Omega$$

If the load is disconnected let's assume that the additional small 10 mA goes through the diodes. Let's compute the diode resistance, r_D,

$$r_D = \left(\frac{di_D}{dv_D}\right)^{-1}_{i_D = 72.8\,\mathrm{mA}} = \left(\frac{I_D}{nV_T}\right) = \frac{nV_T}{I_D} = \frac{25.2 \times 10^{-3}}{72.8 \times 10^{-3}} = 0.35\,\Omega$$

The change in voltage is then

$$\Delta v_O = \Delta i_D r_D$$

If the load is disconnected, $\Delta i_D = 10\,\text{mA}$, and $\Delta v_D = 10 \times 10^{-3} \times 0.35 = 3.5\,\text{mV}$

If the load resistance is reduced to $100\,\Omega$, $\Delta i_D = \frac{1.5}{150} - \frac{1.5}{100} = -5\,\text{mA}$, and then $\Delta v_D = -5 \times 10^{-3} \times 0.35 = -1.8\,\text{mV}$.

If the load resistance is reduced to $75\,\Omega$, $\Delta i_D = \frac{1.5}{150} - \frac{1.5}{75} = -10\,\text{mA}$, and then $\Delta v_D = -3.5\,\text{mV}$.

If the load resistance is reduced to $50\,\Omega$, $\Delta i_D = \frac{1.5}{150} - \frac{1.5}{50} = -20\,\text{mA}$, and then $\Delta v_D = -7\,\text{mV}$.