EE 321 Analog Electronics, Fall 2011
Homework #6 solution

5.15. (a) Use the Ebers-Moll expressions in Eqs. 5.26 and 5.27 to show that the i_C-v_{CB} relationship sketch in Fig. 59. can be described by

$$i_C = \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right) e^{\frac{v_{BC}}{V_T}}$$

(b) Calculate and sketch i_C-v_{CB} curves for a transistor for which $I_S = 10^{-15}$ A, $\alpha_F \approx 1$, and $\alpha_R = 0.1$. Sketch graphs for $I_E = 0.1$ mA, 0.5 mA, and 1 mA. For each, give the values of v_{BC}, v_{BE}, and v_{CE} for which (a) $i_C = 0.5\alpha_F I_E$ and (b) $i_C = 0$.

(a) The Ebers-Moll equations are

$$i_E = \frac{I_S}{\alpha_F}\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

$$i_C = I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

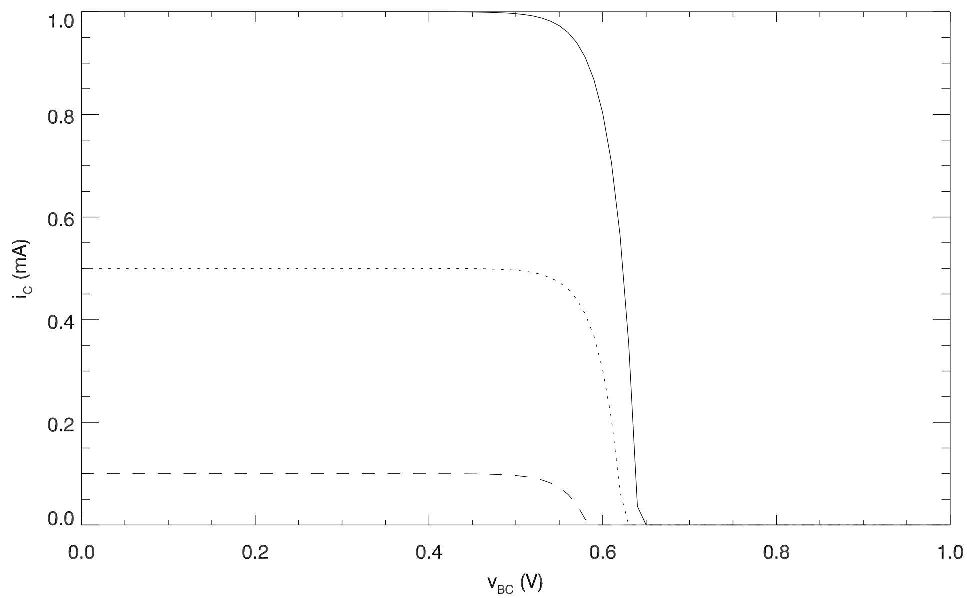
Eliminate $e^{\frac{v_{BE}}{V_T}}$ from the second equation by substituting the first equation into it. i_C is then a function of i_E (which the book assumes fixed biased so it calls it I_E) and v_{BC}. Re-arrange the first equation:

$$I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) = \alpha_F I_E + \alpha_F I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

Insert in the second equation

$$\begin{aligned} i_C =& \alpha_F I_E + \alpha_F I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) \\ =& \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right) e^{\frac{v_{BC}}{V_T}} \end{aligned}$$

(b) This plot shows i_C as a function of v_{CB}.



The solid curve is for $i_C = 1\,\text{mA}$, the dotted is for $i_C = 0.5\,\text{mA}$, and the dashed is for $i_C = 0.1\,\text{mA}$.

(a) The value of v_{CB} for which $i_C = 0.5\alpha_F I_E$ can be found from

$$0.5\alpha_F I_E = \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)e^{\frac{v_{BC}}{V_T}}$$

$$v_{BC} = V_T \ln\left(\frac{0.5\alpha_F I_E}{I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)}\right)$$

The values are tabulated here:

I_E	v_{BC}
0.1 mA	0.57 V
0.5 mA	0.61 V
1 mA	0.62 V

(b) The value for v_{CB} for which $i_C = 0$ can be found from

$$0 = \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)e^{\frac{v_{BC}}{V_T}}$$

$$v_{BC} = V_T \ln\left(\frac{\alpha_F I_E}{I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)}\right)$$

The values are tabulated here

I_E	v_{BC}
0.1 mA	0.58 V
0.5 mA	0.62 V
1 mA	0.64 V

5.20. For the circuits in Fig P5.20, assume that the transistors have very large β. Some measurements have been made on these circuits, with the results indicated in the figure. Find the values of the other labeled voltages and currents.

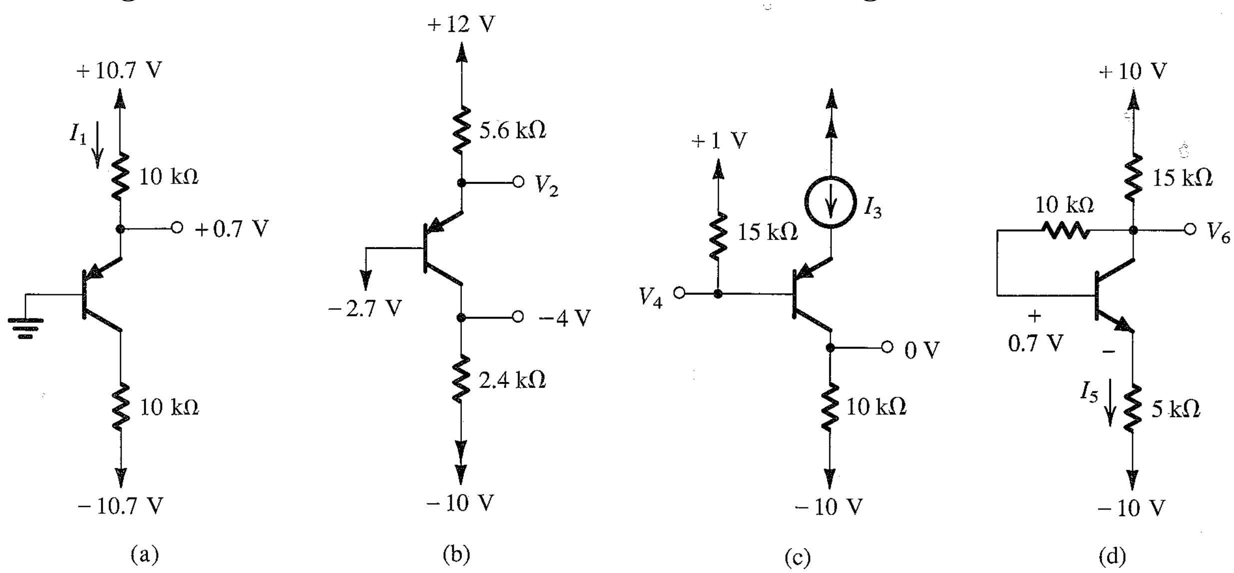


FIGURE P5.20

The hint that β is very large means that we can assume that $i_C = i_E$.

(a) $I_1 = \frac{V_{CC}-V_E}{R_E} = \frac{10.7-0.7}{10} = 10\,\text{mA}$

(b) We can see that the transistor must be on, so $V_2 = V_B + V_{BE} = -2.7 + 0.7 = -2\,\text{V}$

(c) The transistor is on because there is collector current flowing.

$$I_3 = I_E = I_C = \frac{V_C - V_{CC}}{R_C} = \frac{0+10}{10} = 1\,\text{mA}$$

Since β is very large there is no current flowing in the base, so $V_4 = V_{BB} = 1\,\text{V}$.

(d) Since β is very large there is no current flowing in the base and thus $V_B = V_C$, and we can write

$$V_{CC} - V_{EE} = I_5\left(R_C + R_E\right) + V_{BE}$$

and thus

$$I_5 = \frac{V_{CC} - V_{EE} - V_{BE}}{R_C + R_E} = \frac{10+10-0.7}{15+5} = 0.97\,\text{mA}$$

$$V_6 = V_{CC} - I_C R_C = 10 - 0.97 \times 15 = -4.6\,\text{V}$$

5.21. Measurements on the circuits of Fig. P5.21 produce labeled voltages as indicated. find the value of $\boldsymbol{\beta}$ for each transistor.

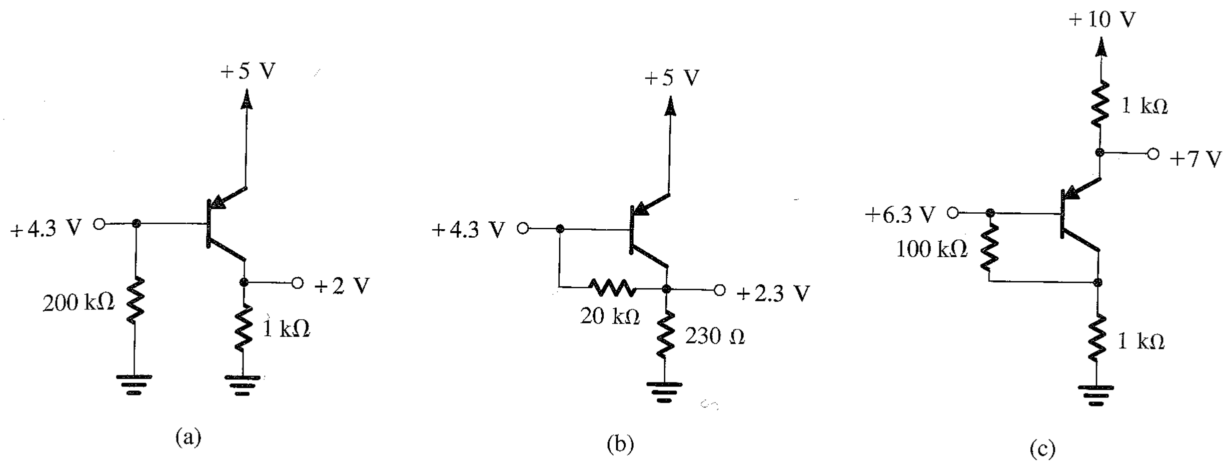


FIGURE P5.21

(a) $I_B = \frac{V_B}{R_B} = \frac{4.3}{200} = 21.5\,\mu\text{A}$, $I_C = \frac{V_C}{R_C} = \frac{2}{1} = 2\,\text{mA}$. We can verify that the transistor is in the active mode since $V_C < V_B$. The we get $\beta = \frac{I_C}{I_B} = \frac{2000}{21.5} = 93$.

(b) In this case we have $I_E = \frac{V_C}{R_C} = \frac{2.3}{0.23} = 10\,\text{mA}$. It is I_E because I_B and I_C are merged. We also have $I_B = \frac{V_B - V_C}{R_B} = \frac{4.3-2.3}{20} = 0.1\,\text{mA}$. We verify active mode since $V_C < V_B$, and we get $\beta = \frac{I_E}{I_B} - 1 = \frac{10}{0.1} - 1 = 99$.

(c) $I_E = \frac{V_{EE} - V_E}{R_E} = \frac{10-7}{1} = 3\,\text{mA}$. Then it must also be that $V_C = 0 + 3\,\text{V} = 3\,\text{V}$ since $R_C = R_E$ and we have I_E flowing through R_C. Then we have $I_B = \frac{V_B - V_C}{R_B} = \frac{6.3-3}{100} = 33\,\mu\text{A}$. We verify active mode operation and get $\beta = \frac{I_E}{I_B} - 1 = \frac{3}{0.033} - 1 = 90$.

5.24. for each of the circuits shown in Fig. P5.24, find the emiter, base, and collector voltages and currents. Use $\boldsymbol{\beta = 30}$, but assume $|\boldsymbol{V_{BE}}| = \mathbf{0.7\,V}$ independent of current level.

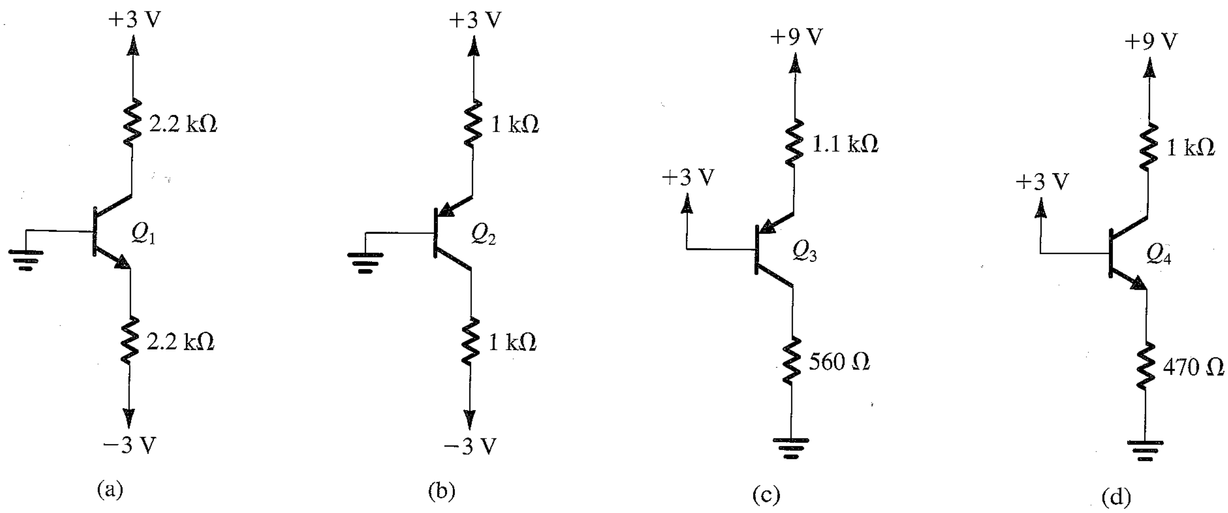


FIGURE P5.24

(a) First, $V_B = 0\,\text{V}$. The transistor must be on, so $V_E = V_B - V_{BE} = 0 - 0.7\,\text{V} = -0.7\,\text{V}$. Then $I_E = \frac{V_E - V_{EE}}{R_E} = \frac{-0.7+3}{2.2} = 1.0\,\text{mA}$. Next assume active mode and we get $I_C = \alpha I_E = \frac{\beta}{\beta+1} I_E = \frac{30}{30+1} \times 1.0 = 1.01\,\text{mA}$. Then $V_C = V_{CC} - I_C R_C = 3 - 1.01 \times 2.2 = 0.78\,\text{V}$. Active mode is verified. Then finally, $I_B = \frac{I_C}{\beta} = \frac{1.01}{30} = 33\,\mu\text{A}$.

(b) $V_B = 0\,\text{V}$. The transistor is on, so $V_E = 0.7\text{V}$. Then $I_E = \frac{V_{EE} - V_E}{R_E} = \frac{3-0.7}{1} = 2.3\,\text{mA}$. If we assume active mode then $I_C = \alpha I_E = \frac{30}{30+1} \times 2.3 = 2.2\,\text{mA}$. Then $V_C = V_{CC} + I_C R_C = -3 + 2.2 = -0.8\,\text{V}$, and active mode is verified. Finally, $I_B = \frac{I_C}{\beta} = \frac{2.2}{30} = 73, \mu\text{A}$.

(c) $V_B = 3\,\text{V}$, and the transistor is on, so $V_E = V_B + V_{EB} = 3 + 0.7 = 3.7\,\text{V}$. Then $I_E = \frac{V_{EE} - V_E}{R_E} = \frac{9-3.7}{1.1} = 4.8\,\text{mA}$. Assuming active mode we get $I_C = \alpha I_E = \frac{\beta}{\beta+1} I_E = \frac{30}{30+1} \times 4.8 = 4.6\,\text{mA}$. Then $V_C = R_C I_C = 0.56 * 4.6 = 2.6\,\text{V}$. Since $V_C < V_B$ active mode is verified. Finally, $I_B = \frac{I_C}{\beta} = \frac{4.6}{30} = 155\,\mu\text{A}$.

(d) $V_B = 3\,\text{V}$, and the transistor is on so $V_E = V_B - V_{BE} = 3 - 0.7 = 2.3\,\text{V}$. Then $I_E = \frac{V_E}{R_E} = \frac{2.3}{0.47} = 4.9\,\text{mA}$. Then, assuming active mode, $I_C = \alpha I_E = \frac{\beta}{\beta+1} I_E = \frac{30}{30+1} \times 4.9 = 4.7\,\text{mA}$. Then $V_C = V_{CC} - I_C R_C = 9 - 4.7 \times 1 = 4.3\,\text{V}$. Since $V_C > V_B$ active mode is verified. Then finally $I_B = \frac{I_C}{\beta} = \frac{4.3}{30} = 143\,\mu\text{A}$.