EE 321 Analog Electronics, Fall 2011
Homework #8 solution

5.103. An npn BJT with grounded emitter is operated with $V_{BE} = 0.700\,\mathrm{V}$, at which the collector current is $1\,\mathrm{mA}$. A $10\,\mathrm{k\Omega}$ resistor connects the collector to $+15\,\mathrm{V}$ supply. What is the resulting collector voltage V_C? Now, if a signal applied to the base raised v_{BE} to $705\,\mathrm{mV}$, find the resulting total collector current i_C and total collector voltage v_C using the exponential i_C-v_{BE} relationship. For this situation, what are v_{be} and v_c? Calculate the voltage gan v_c/v_{be}. Compare with the value obtained using the small-signal approximation, that is $-g_m R_C$.

The output voltage is $V_C = V_{CC} - I_C R_C = 15 - 1 \times 10 = 5\,\mathrm{V}$. Next we rais the base voltage. First we want to find the value of I_C from the bias point, and

$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$

$$I_S = \frac{i_C}{e^{\frac{v_{BE}}{V_T}}} = \frac{1 \times 10^{-3}}{e^{\frac{0.7}{25 \times 10^{-3}}}} = 6.91 \times 10^{-16}\,\mathrm{A}$$

Next for $v_{BE} = 0.705\,\mathrm{V}$:

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} = 6.91 \times 10^{-16} \times e^{\frac{0.705}{25 \times 10^{-3}}} = 1.22\,\mathrm{mA}$$

$$v_C = V_{CC} - i_C R_C = 15 - 1.22 \times 10 = 2.8\,\mathrm{V}$$

For this situation $v_{be} = 5 \times 10^{-3}$, and $v_c = 2.8 - 5 = -2.2\,\mathrm{V}$, and the gain is $v_c/v_{be} = -440$. The gain using the linear approximation is

$$A_{vo} = -\frac{I_C R_C}{V_T} = -\frac{1 \times 10}{25 \times 10^{-3}} = 400$$

5.104. A transitor with $\beta = 120$ is biased to operate at a DC collector current of $1.2\,\mathrm{mA}$. Find the values of g_m, r_π, and r_e. Repeat for a bias current of $120\,\mu\mathrm{A}$.

$$g_m = \frac{I_C}{V_T} = \frac{1.2}{25} = 0.048\,\Omega^{-1}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{0.048} = 2500\,\Omega$$

$$r_e = \frac{\alpha}{g_m} = \frac{\beta}{\beta+1}\frac{1}{g_m} = \frac{120}{121}\frac{1}{0.048} = 20.7\,\Omega$$

If instead the transistor is biased at $I_C = 120\,\mu\mathrm{A}$, we get

$$g_m = \frac{I_C}{V_T} = \frac{0.12}{25} = 0.0048\,\Omega^{-1}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{0.0048} = 25\,\mathrm{k\Omega}$$

$$r_e = \frac{\alpha}{g_m} = \frac{\beta}{\beta+1}\frac{1}{g_m} = \frac{120}{121}\frac{1}{0.0048} = 207\,\Omega$$

5.110. The following table summarizes some of the basic attributes of a number of BJTs of different types, operating as amplifiers under various conditions. Provide the missing entries.

Transistor	a	b	c	d	e	f	g
α	1.000					0.90	
β		100		∞			
I_C (mA)	1.00		1.00				
I_E (mA)		1.00				5	
I_B (mA)			0.020				1.10
g_m (mA/V)							700
r_e (Ω)				25	100		
r_π (Ω)					10.1 kΩ		

(*Note:* Isn't it remarkable how much two parameters can reveal?)

I will just provide the explicit equations for the first column, (a):

$$\beta = \frac{\alpha}{1-\alpha}$$

$$I_E = \frac{I_C}{\alpha}$$

$$I_B = \frac{I_C}{\beta}$$

$$g_m = \frac{I_C}{V_T}$$

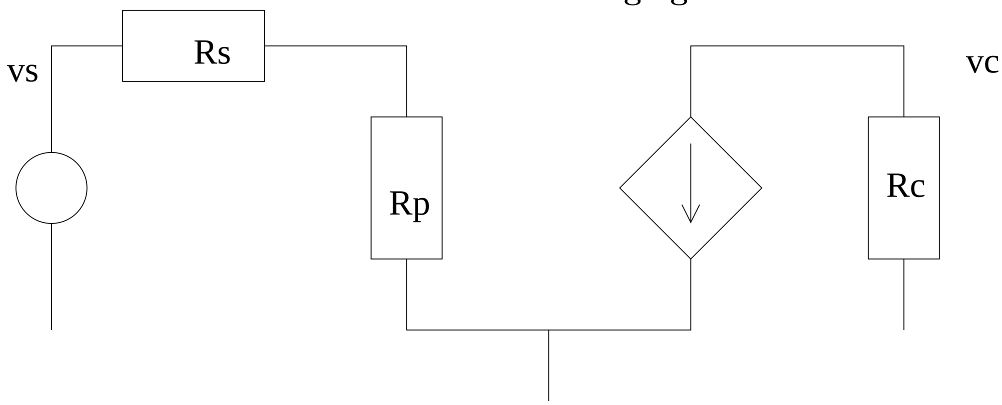
$$r_e = \frac{\alpha}{g_m}$$

$$r_\pi = \frac{\beta}{g_m}$$

Transistor	a	b	c	d	e	f	g
α	**1.000**	0.99	0.98	1	0.99	**0.9**	0.984
β	∞	**100**	50	$\boldsymbol{\infty}$	100	9	62.8
I_C (mA)	**1.00**	0.99	**1.00**	1	0.25	4.5	69.1
I_E (mA)	1.00	**1.00**	1.02	1	0.253	**5**	70.2
I_B (mA)	0	0.0099	**0.02**	0	0.0025	0.5	**1.10**
g_m (mA/V)	40	39.6	40	40	9.9	180	**700**
r_e (Ω)	25	25	24.5	**25**	**100**	5	1.41
r_π (Ω)	∞	2530	1250	∞	**10.1K**	50	89.7

5.114. A biased BJT operates as a grounded-emitter amplifier between a signal source, with a source resistance of 10 kΩ, connected to the base and a 10 kΩ

load connected as a collector resistance R_C. In the corresponding model, g_m is 40 mA/V and r_π is 2.5 kΩ. Draw the complete amplifier model using the hybrid-π BJT equivalent circuit. Calculate the overall voltage gain v_c/v_s. What is the value of BJT β implied by the values of the model parameters? To what value must β be increased to double the overall voltage gain?



$$v_o = -g_m R_C v_i = -g_m R_C \frac{r_\pi}{R_s + r_\pi} v_s = -0.04 \times 10 \times 10^3 \times \frac{2.5}{10 + 2.5} = -80$$

The value of β can be found from

$$r_\pi = \frac{\beta}{g_m}$$

or

$$\beta = r_\pi g_m = 2.5 \times 10^3 \times 0.04 = 100$$

To double the gain we would want to double the factor

$$\frac{r_\pi}{R_s + r_\pi}$$

It is currently equal to

$$\frac{2.5}{10 + 2.5} = 0.2$$

To double it we would need to change r_π:

$$\frac{r_\pi}{R_s + r_\pi} = 0.4$$

$$r_\pi = \frac{0.4}{0.6} R_s = 0.67 R_s = 6.7\,\mathrm{k\Omega}$$

The factor increase in β is the same as the factor increase in r_π:

$$\beta_{\text{new}} = \beta_{\text{old}} \frac{r_{\pi\,\text{new}}}{r_\pi\ \text{old}} = 100 \times \frac{6.7}{2.5} = 268$$