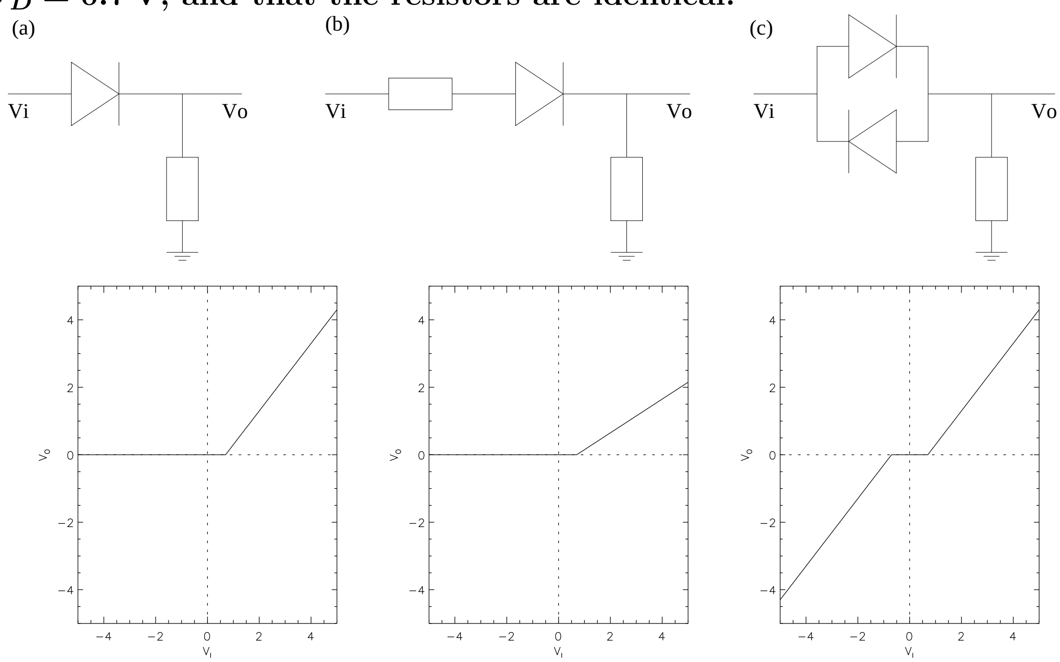
EE 321 Analog Electronics, Fall 2011
Exam 1 October 3, 2011

Rules: This is a closed-book exam. You may use only your brain, a calculator and pen/paper. Each numbered question counts equally toward your grade.

Note: The questions are designed to test your conceptual understanding. Thus, except for question 2, which requires some derivation, the solutions to all the problems are literally 1- or 2-line calculations. If you get stuck on long calculations you are doing something wrong.

Diode circuits

1. **Carefully plot the output as a function of the input for $-5\,\mathrm{V} < V_{\mathrm{in}} < 5\,\mathrm{V}$ for the following circuits. By "carefully" is meant that voltage pairs can be read off your plot. Assume the fixed voltage drop diode model with $V_D = 0.7\,\mathrm{V}$, and that the resistors are identical.**



Finite open-loop gain

2. **Show that the expression for the closed-loop gain of a non-inverting amplifier with finite open-loop gain is**

$$v_o = \frac{Av_i}{1 + \frac{A}{\frac{R_2}{R_1}+1}}$$

Show that it reduces to the well-known expression for a ideal op-amp

Begin with

$$v_o = A\left(v_i - v_-\right) \qquad R_2 I = v_o - v_- \qquad R_1 I = v_-$$

Next we eliminate v_- and I between these equations. First eliminate I between the last two equations to get

$$\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1}$$

Isolate v_-

$$\frac{v_o}{R_2} = v_- \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v_o = v_- \left(\frac{R_2}{R_1} + 1\right)$$

$$v_- = \frac{v_o}{\frac{R_2}{R_1} + 1}$$

Next insert this into the first equation

$$v_o = A\left(v_i - \frac{v_o}{\frac{R_2}{R_1} + 1}\right)$$

$$v_o\left(1 + \frac{A}{\frac{R_2}{R_1} + 1}\right) = Av_i$$

$$\frac{v_o}{v_i} = \frac{A}{1 + \frac{A}{\frac{R_2}{R_1}+1}} = \left(\frac{R_2}{R_1} + 1\right)\frac{1}{\frac{1}{A}\left(\frac{R_2}{R_1} + 1\right) + 1}$$

If we let $A \to \infty$ then the fraction factor goes to unity.

3. **If A is a low-pass filter with $\omega_0 = 10\,\mathrm{s}^{-1}$, and $A_0 = 10^5$, what is the gain-bandwidth product, in Hz? And then what is the bandwidth of an amplifier design with a gain of 10^3, again in Hz?**

$$\mathrm{GBWP} = \omega_0 A_0 = 10 \times 10^5 = 10^6\,\mathrm{s}^{-1} = \frac{10^6}{2\pi}\,\mathrm{Hz} = 159\,\mathrm{kHz}$$

$$\mathrm{BW} = \frac{\mathrm{GBWP}}{G} = \frac{159 \times 10^3}{10^3} = 159\,\mathrm{Hz}$$

Offset and bias current

An op-amp has $V_{\mathrm{OS}} = 1\,\mathrm{mV}$, $I_B = 1\,\mu\mathrm{A}$, and $I_{OS} = 100\,\mathrm{nA}$.

4. **For a inverting amplifier, what is the largest value of the feedback resistor allowed to keep the output below 10 mV (consider only the bias current).**

$$R_{2,\max} I_B = 10\,\mathrm{mV}$$

$$R_{2,\max} = \frac{10\,\mathrm{mV}}{I_B} = \frac{10 \times 10^{-3}}{1 \times 10^{-6}} = 10\,\mathrm{k}\Omega$$

5. **Using the maximum value for the feedback, what value of the input resistor will produce a gain ($\boldsymbol{A_{vo}}$) of -10. What is the size of the output due to $\boldsymbol{V_{\mathrm{OS}}}$?**

$$A_{vo} = -\frac{R_2}{R_1}$$

$$R_1 = -\frac{R_2}{A_{vo}} = -\frac{10^3}{-10} = 1\,\mathrm{k}\Omega$$

The output due to V_{OS} is simply V_{OS} amplified by the non-inverting gain, so

$$v_O\left(V_{OS}\right) = 1 \times 11 = 11\,\mathrm{mV}$$

6. **What would be $\boldsymbol{G_{vo}}$ if attaching a source with output resistance $\boldsymbol{R_S} = 100\,\mathrm{k}\Omega$, and what is a simple way to restore the original gain of -10?**

The gain would be

$$G_{vo} = A_{vo}\frac{R_1}{R_1 + R_s} = -10 \times \frac{10^3}{100 \times 10^3} = -0.1$$

A simple way to restore the gain is to place a buffer/follower between he source and R_1.

Zener diode regulator

Assume a piecewise linear model for a Zener diode, with $\boldsymbol{V_{Z0}} = 5.6\,\mathrm{V}$ and $\boldsymbol{r_Z} = 5\,\Omega$.

7. **Using a 10 V supply, design a regulator (choose resistor value in series with Zener), which outputs exactly $\boldsymbol{V_{Z0}}$ to a load resistor $\boldsymbol{R_L} = 100\,\Omega$.**

This requires voltage division like this:

$$V\frac{R_L}{R + R_L} = V_{Z0}$$

$$R = \frac{V}{V_{Z0}}R_L - R_L = 100 \times \left(\frac{10}{5.6} - 1\right) = 78.6\,\Omega$$

8. **By how much does the output voltage change if we change to $R_L = 200\,\Omega$? In setting up the problem you should simplify the problem by assuming that V_Z is essentially unchanged, otherwise you will run out of time mired in longer calculations. You will need to obtain the change in current through the Zener diode**

   The quick approximation is that the output voltage changes very little, so that the new R_L carries half the current. The rest if carried through the Zener, and that current has a small voltage increase associated with it,

$$\Delta V = I r_Z = \frac{1}{2}\frac{V_{Z0}}{R_L} r_z = \frac{1}{2}\frac{5.6}{100} \times 5 = 0.14\,\mathrm{V}$$

9. **When no load is attached, what is the change in output voltage for each volt change in the supply voltage?**

   The voltage above V_{Z0} is voltage divided between r_z and R, and thus

$$v_O = V_{Z0} + (v_I - v_{Z0})\frac{r_z}{r_z + R}$$

   and then

$$\frac{dv_O}{dv_I} = \frac{r_z}{r_z + R} = \frac{5}{5 + 78.6} = 0.060\,\frac{\mathrm{V}}{\mathrm{V}}$$