

EE 321 Analog Electronics, Fall 2009

Exam 1 September 28, 2009

Rules: This is a closed-book test. You may use pen, paper, and calculator. The exam will last 50 minutes. Problems 1-3 each count 30%, and problem 4 10%. None of the problems require long calculations.

(1) **CASCADING AMPLIFIERS.** Amplifier A has input resistance $R_{IA} = 10 \text{ k}\Omega$, gain $G_A = 10$, and output resistance $R_{OA} = 1 \text{ k}\Omega$. Amplifier B has $R_{IB} = 1 \text{ k}\Omega$, $G_B = 10$, $R_{OB} = 10 \text{ k}\Omega$. In what order should they be connected between a voltage source with output resistance $R_S = 1 \text{ k}\Omega$ and a load $R_L = 100 \Omega$ in order to produce the greatest voltage drop across the load?

For amplifiers 1, 2 cascaded with the source and the load, the expression for the overall gain is

$$\frac{V_o}{V_s} = \frac{R_{I1}}{R_{I1} + R_s} G_1 \frac{R_{I2}}{R_{I2} + R_{O1}} G_2 \frac{R_L}{R_{O2} + R_L}$$

Now make 1 = A, 2 = B, (SABL configuration) and we get

$$\frac{V_o}{V_s} = \frac{10}{10 + 1} \times 10 \times \frac{1}{1 + 1} \times 10 \times \frac{0.1}{0.1 + 10} = 0.45$$

Next, make 1 = B, and 2 = A, (SBAL configuration) and we get

$$\frac{V_o}{V_s} = \frac{1}{1 + 1} \times 10 \times \frac{10}{10 + 10} \times 10 \times \frac{0.1}{0.1 + 1} = 2.27$$

The SBAL configuration produces the largest voltage drop across R_L .

(2) **GAIN-BANDWIDTH PRODUCT.** An operational amplifier has a knee-frequency, $f_0 = 2 \text{ Hz}$, and a DC open-loop gain of $A_0 = 10^6$. What is the gain-bandwidth product of this amplifier? What is the bandwidth of a closed-loop amplifier built with this op-amp, which has a gain of $A = 10^3$? It is possible to improve the bandwidth by cascading several amplifiers, each with smaller gain, and thus larger bandwidth. To obtain a bandwidth of at least $f_B = 2 \times 10^4 \text{ Hz}$, what is the smallest number of identical amplifiers that would be needed, and what would be their gains? What is then the actual bandwidth obtained?

The gain-bandwidth product of the op-amp is

$$\text{GBWP} = f_0 \times A_0 = 2 \times 10^6 \text{ Hz}$$

Bandwidth of a closed-loop amplifier is

$$\text{BW} = \frac{\text{GBWP}}{\text{GAIN}} = \frac{2 \times 10^6}{10^3} = 2 \times 10^3 \text{ Hz}$$

How many amplifiers to achieve $\text{BW} \geq 2 \times 10^4 \text{ Hz}$:

Let's first try 2 amplifiers. Their gain would be $G = \sqrt{10^3} = 31.6$. Their bandwidth would be $\text{BW} = \frac{\text{GBWP}}{\text{GAIN}} = \frac{2 \times 10^6}{31.6} = 63 \text{ kHz}$. This is large enough, so two amplifiers is enough. The overall bandwidth is then 63 kHz.

(3) **BIAS CURRENT.** Using an op-amp, design an inverting amplifier with a input resistance of $10\text{ k}\Omega$, a gain of $A = -10^3$, and a grounded non-inverting terminal. If the data sheet for the op-amp specifies $I_B = 1\text{ }\mu\text{A}$, what is the expected effect of the bias current on the output? Adding a resistor to the non-inverting terminal can eliminate the effect of the bias current. Derive the expression for the value of this third resistor and determine its value.

Using the usual inverting amplifier terminology, we find that $R_1 = R_{\text{in}} = 10\text{ k}\Omega$. Then $R_2 = |A| \times R_1 = 10\text{ M}\Omega$.

With the non-inverting terminal grounded we have then at the inverting terminal

$$\frac{v_I}{R_1} + \frac{v_O}{R_2} = I_B$$

The effect on the output of the bias current can be found by setting $v_I = 0$, and $v_O = \Delta v_{OB}$,

$$\frac{\Delta v_{OB}}{R_2} = I_B$$

or

$$\Delta v_{OB} = I_B R_2 = 1 \times 10^{-6} \times 10 \times 10^6 = 10\text{ V}$$

If we add a resistor, R_3 , to the non-inverting terminal we get

$$v_+ = -R_3 I_B$$

and then current balance at the negative terminal is written

$$\frac{v_I - v_-}{R_1} + \frac{v_O - v_-}{R_2} = I_B$$

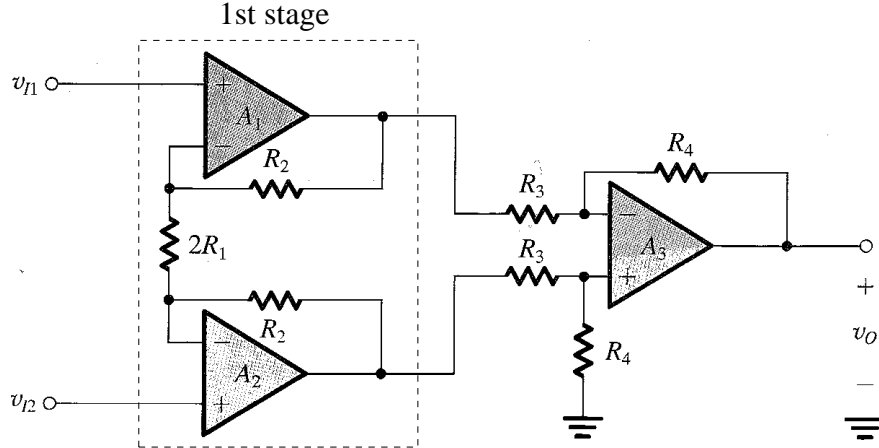
We want $v_o = 0$ when $v_I = 0$, so

$$-\frac{v_-}{R_1} - \frac{v_-}{R_2} = I_B$$

$$\frac{R_3}{R_1} I_B + \frac{R_3}{R_2} I_B = I_B$$

$$R_3 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = R_1 || R_2 = 10 \times 10^3 || 10 \times 10^6 = 10 \times 10^3 = 10\text{ k}\Omega$$

(4) **INSTRUMENTATION AMPLIFIER.** Explain why the first stage of the instrumentation amplifier has unity common-mode gain and a differential mode gain of $1 + \frac{R_2}{R_1}$. Hint: consider the current across $2R_1$ for common and differential mode inputs.



If we apply a common-mode signal on the input, $v_{I2} = v_{I1} = v_{ic}$, the voltages on the two positive input terminals are the same, $v_{+1} = v_{+2} = v_{ic}$. The voltage on the two negative terminals also have those values, because of the feedback configuration. Thus, no current flows across the resistor $2R_1$, and therefore no current across the two resistors R_2 either. The two outputs are therefore at the same voltages as the two negative terminals, and therefore at the same voltages as the positive terminals, the inputs. So, $v_{o1} = v_{o2} = v_{I1} = v_{I2} = v_{ic}$, and $v_{oc} = v_{ic}$. Thus, the common-mode gain is one.

For the differential mode we apply $v_{I1} = -\frac{v_{id}}{2}$, $v_{I2} = \frac{v_{id}}{2}$. These are also the voltages on the inverting terminals, so the current flowing upward on the figure through $2R_1$ is

$$i = \frac{\frac{v_{id}}{2} + \frac{v_{id}}{2}}{2R_1} = \frac{v_{id}}{2R_1}$$

The voltage difference between the two outputs is then the voltage drop that current creates across $2R_1 + 2R_2$,

$$v_{od} = (2R_1 + 2R_2) i = (2R_1 + 2R_2) \frac{v_{id}}{2R_1} = v_{id} \frac{R_1 + R_2}{R_1} = v_{id} \left(1 + \frac{R_2}{R_1} \right)$$

Thus the differential mode gain is $1 + \frac{R_2}{R_1}$.