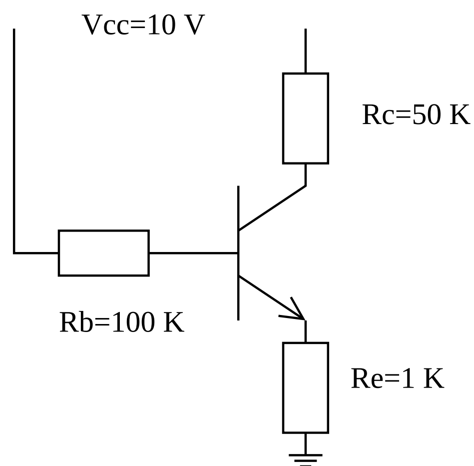
EE 321 Analog Electronics, Fall 2011
Exam 2 November 7, 2011
Solution

Rules: This is a closed-book exam. You may use only your brain, a calculator and pen/paper. Each numbered question counts equally toward your grade.

Note: The questions are designed to test your conceptual understanding, not your ability to do many pages of math. If you find yourself doing long calculations there is a high probability that you are doing something wrong.

DC analysis

1. **Find all the voltages and currents in this circuit. Assume $\beta = 100$**



First let's guess active mode. In that case we have

$$V_{CC} = I_B R_B + V_{BE} + (\beta + 1) I_B R_E$$

and thus

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1) R_E} = \frac{10 - 0.7}{100 + 101 \times 1} = 46.3\,\mu\text{A}$$

then

$$I_C = \beta I_B = 100 \times 46.3 = 4.63\,\text{mA}$$

and

$$V_C = V_{CC} - I_C R_C = 10 - 50 \times 4.63 = -221.5\,\text{V}$$

This is not consistent with the active mode assumption, so the transistor must be saturation mode. In saturation mode we assume that $V_{CE} = 0.2\,\text{V}$. Write down some new equations

$$\begin{aligned} V_B =& V_{CC} - I_B R_B \\ =& V_{BE} + I_E R_E \\ =& V_{CC} - I_C R_C - V_{CE} + V_{BE} \\ I_E =& I_B + I_C \end{aligned}$$

If I use mA for currents and kΩ for resistances I can omit units and insert the values directly, like this

$$\begin{aligned} V_B =& 10 - 100\, I_B \\ =& 0.7 + I_E \\ =& 10.5 - 50\, I_C \\ I_E =& I_B + I_C \end{aligned}$$

Insert the last equation into the 2nd equation to get

$$V_B = 0.7 + I_B + I_C$$

and eliminate I_C between this and the 3rd equation

$$I_C = V_B - 0.7 - I_B$$

$$I_C = \frac{10.5 - V_B}{50}$$

$$V_B - 0.7 - I_B = 0.21 - \frac{V_B}{50}$$

$$V_B - I_B = 0.91 - \frac{V_B}{50}$$

Next eliminate V_B between this equation and the first equation

$$V_B + \frac{V_B}{50} = 0.91 + I_B$$

$$1.02\, V_B = 0.91 + I_B$$

$$V_B = 0.8922 + 0.9804\, I_B$$

$$10 - 100\, I_B = 0.8922 + 0.9804 I_B$$

$$100.9804\, I_B = 9.1078$$

$$I_B = \frac{9.1078}{100.9804} = 0.0902\,\text{mA} = 90.2\,\text{mA}$$

Then

$$V_B = V_{CC} - I_B R_B = 10 - 0.0902 \times 100 = 0.98\,\text{V}$$

And

$$I_E = V_B - 0.7 = 0.98 - 0.7 = 0.28\,\text{mA}$$

and

$$V_E = I_E R_E = 0.28 \times 1 = 0.28\,\text{V}$$

and

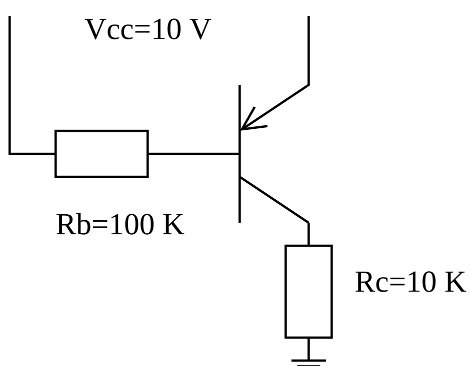
$$I_C = \frac{10.5 - V_B}{50} = \frac{10.5 - 0.98}{50} = 0.19\,\text{mA}$$

and

$$V_C = V_{CC} - I_C R_C = 10 - 0.19 \times 50 = 0.5\,\text{V}$$

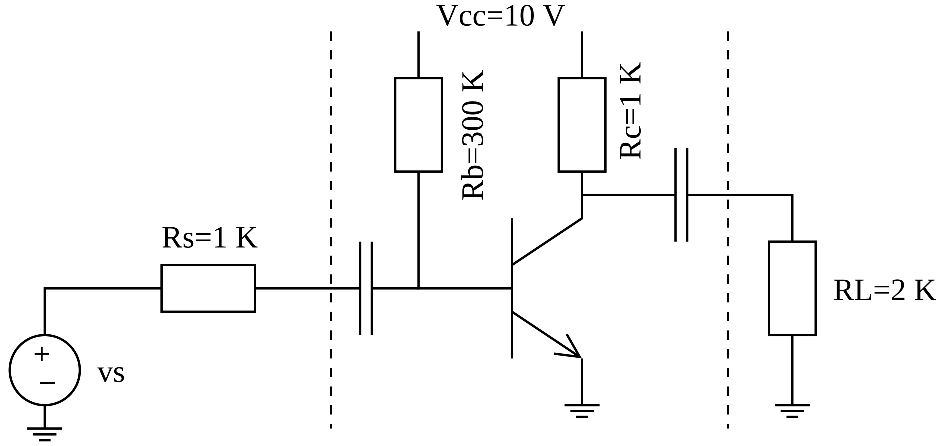
Verify that $V_{CE} = 0.5 - 0.28 = 0.22\,\text{V}$, close to what we started with. The 0.02 difference is probably a rounding error.

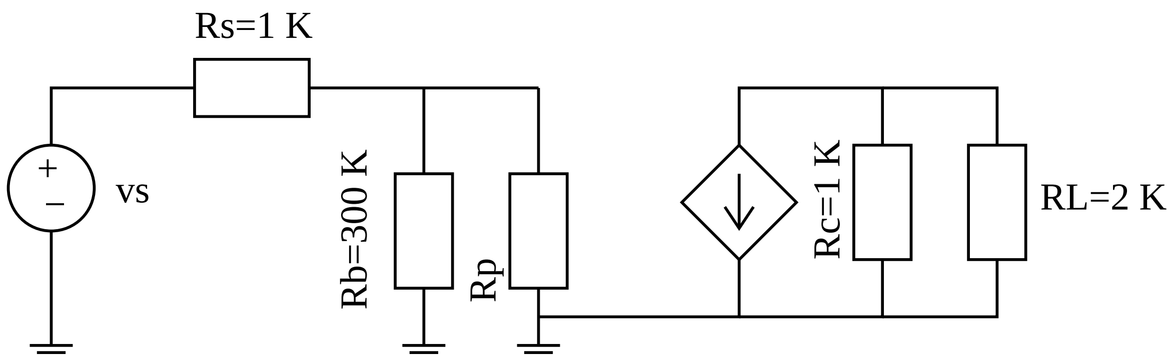
2. **Find all the voltages and currents in this circuit. Assume $\boldsymbol{\beta} = 50$**



This transistor is clearly in cutoff mode, which means that $I_C = I_B = I_E =$, and then $V_B = V_E = V_{CC} = 10\,\text{V}$, and $V_C = 0\,\text{V}$.

3. Draw the small-signal model of this amplifier, including the values of all resistors. Compute R_{in}, R_{out}, A_{vo}, A_v, G_{vo}, and G_v. Assume $\beta = 100$.





The base bias current is

$$V_{BE} + I_B R_B = V_{CC}$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{10 - 0.7}{300 \times 10^3} = 31\,\mu\text{A}$$

and then

$$r_\pi = \frac{V_T}{I_B} = \frac{25 \times 10^{-3}}{31 \times 10^{-6}} = 806\,\Omega$$

The input resistance

$$R_{\text{in}} = r_\pi || R_B = 806 || 300 \times 10^3 = 804\,\Omega$$

The output resistance

$$R_{\text{out}} = R_C = 1\,\text{k}\Omega$$

Open output gain from input

$$A_{vo} = \alpha \frac{R_C}{r_e} = \frac{\beta}{\beta+1} \frac{R_C}{r_e} = \beta \frac{R_C}{r_\pi} = 100 \times \frac{1000}{806} = 124$$

Gain from input

$$A_v = A_{vo} \frac{R_L}{R_L + R_{\text{out}}} = 124 \times \frac{2 \times 10^3}{2 \times 10^3 + 806} = 88.4$$

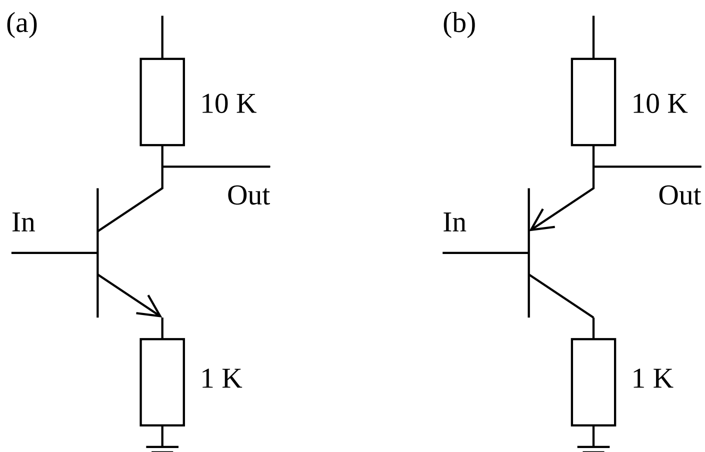
Open output gain from source

$$G_{vo} = \frac{R_{\text{in}}}{R_{\text{in}} + R_S} A_{vo} = \frac{804}{804 + 1 \times 10^3} \times 124 = 55.3$$

Gain from source

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_S} A_v = \frac{804}{804 + 1 \times 10^3} \times 88.4 = 39.4$$

4. **What is the small-signal gain, A_{vo}, for these two circuits in active mode? In each case, what does A_v become if you add a AC-coupled $10\,\text{k}\Omega$ load resistor? I have deliberately left out some information so you will need to make the usual reasonable assumptions.**



(a) For the common-emitter amplifier with emitter resistance we have

$$A_{vo} = -\frac{R_C}{R_E} = -\frac{10}{1} = -10$$

The output resistance is also $R_{\text{out}} = R_C = 10\,\text{k}\Omega$. Thus if we attach R_L to the output we should expect the gain to be cut in half, so

$$A_v = -5$$

(b) For the emitter follower amplifier the gain is approximately unity, so

$$A_{vo} = 1$$

The output resistance is also very small, similar to r_e, which is usually much smaller than the load resistance we propose to attach. Thus the gain will not change, and

$$A_v = 1$$