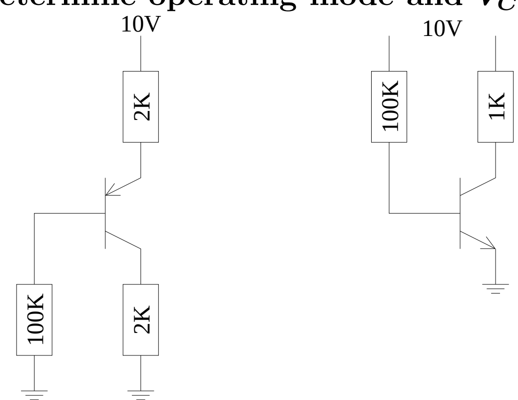
EE 321 Analog Electronics, Fall 2010
Exam 4 December 8, 2010
Solution

Rules: This is a closed-book exam. You may use one small note card previously prepared and the attached formula sheets. The exam will last 50 minutes. Each problem counts equally toward your grade. **IMPORTANT: Use symbolic math or loose points: name the resistors and voltages using the usual terminology and only insert numerical values at the end**. You may assume throughout that $V_A \to \infty$, and $\beta = 100$.

1. BJT DC analysis. Determine operating mode and V_C for these two circuits.



(a) It appears to not be in cutoff mode. Begin by assuming active mode. In that case we have

$$V_{EE} = (\beta + 1)\, I_B R_E + V_{EB} + I_B R_B$$

or

$$I_B = \frac{V_{EE} - V_{EB}}{(\beta + 1)\, R_E + R_B} = \frac{10 - 0.7}{101 \times 2 + 100} = 30.8\,\text{nA}$$

In that case

$$V_B = I_B R_B = 100 \times 30.8 \times 10^{-3} = 3.08\,\text{V}$$

and

$$V_C = \beta I_B R_C = 100 \times 30.8 \times 10^{-3} \times 2 = 6.16\,\text{V}$$

This is not consistent with active mode so it must be in saturation mode where we assume $V_{EC} = 0.2\,\text{V}$. Instead we set it up as

$$I_B R_B + V_{EB} = I_C R_C + V_{EC} = V_{EE} - (I_B + I_C)\, R_C$$

Insert the first into the second to eliminate I_B and we get

$$I_C R_C + V_{EC} = V_{EE} - \left(\frac{I_C R_C + V_{EC} - V_{EB}}{R_B} + I_C \right) R_C$$

$$I_C \left(R_C + \frac{R_C^2}{R_B} + R_C \right) = V_{EE} - \frac{R_C}{R_B} (V_{EC} - V_{EB})$$

$$I_C = \frac{V_{EE} - \frac{R_C}{R_B}(V_{EC} - V_{EB})}{R_C + \frac{R_C^2}{R_B} + R_C} = \frac{10 - \frac{2}{100}(0.2 - 0.7)}{2 + \frac{2^2}{100} + 2} = 2.48\,\text{mA}$$

and then

$$V_C = I_C R_C = 4.96\,\text{V}$$

(b) This one is also not in cutoff. Assume active mode and get

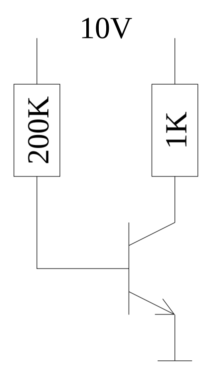
$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{10 - 0.7}{100} = 93\,\text{nA}$$

and then

$$V_C = V_{CC} - \beta I_B R_C = 10 - 100 \times 0.093 \times 1 = 0.7\,\text{V}$$

This is consistent with active mode so the problem is solved.

2. BJT amplifier. For the following amplifier circuit, with input at the base and output at the collector, what are V_C, R_{in}, R_{out}, and A_{vo}? If a source with output resistance $R_{\text{sig}} = 1\,\text{k}\Omega$ is attached and a load of $R_L = 1\,\text{k}\Omega$ is attached, what is the overall voltage gain, G_v?



We can see that, comparing with 1B, this must be in active mode. Then

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{10 - 0.7}{200} = 46.5\,\text{nA}$$

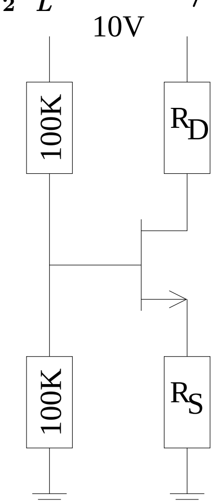
Then $I_C = \beta I_B = 4.65\,\text{mA}$, $V_C = V_{CC} - I_C R_C = 10 - 4.65 = 5.35\,\text{V}$, and $g_m = \frac{I_C}{V_T} = \frac{4.65}{25} = 0.186\,\Omega^{-1}$, and $r_\pi = \frac{\beta}{g_m} = \frac{100}{0.186} = 537\,\Omega$. Then

$$R_{\text{in}} = R_B || r_\pi = 536\,\Omega$$

And $R_{\text{out}} = R_C = 1\,\text{k}\Omega$. $A_{vo} = -g_m R_C = -0.186 \times 1 \times 10^3 = -186$. The expression ofr the overall voltage gain is

$$G_v = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}} A_{vo} \frac{R_L}{R_L + R_{\text{out}}} = -\frac{536}{1000 + 536} \times 186 \times \frac{1}{1+1} = -32.5$$

3. MOSFET DC design. For the following circuit, pick R_S and R_D such that $I_D = 1\,\text{mA}$, and $V_D = 7\,\text{V}$. Use $\frac{k_n'}{2}\frac{W}{L} = 1\,\text{mA/V}^2$.



Since $V_G = 5\,\text{V}$, we are designing for saturation mode, so

$$I_D = \frac{k_n'}{2}\frac{W}{L}(V_{GS} - V_t)^2$$

or

$$V_{GS} = \sqrt{\frac{I_D}{\frac{k_n'}{2}\frac{W}{L}}} + V_t = \sqrt{\frac{1}{1}} + 1 = 2\,\text{V}$$

In that case $V_S = V_G - V_{GS} = 5 - 2 = 3\,\text{V}$, and $R_S = \frac{V_S}{I_S} = \frac{3}{1} = 3\,\text{k}\Omega$. We can easily see that then we also have $R_D = 3\,\text{k}\Omega$.

4. MOSFET amplifier. For the circuit you designed in problem 3, what are A_{vo}, R_{in}, and R_{out}? If a source with output resistance $20\,\text{k}\Omega$ is a attached to the input and a load $R_L = 1\,\text{k}\Omega$ is attached to the output, what is the overall voltage gain G_v?

First compute g_m

$$g_m = \frac{2I_D}{V_{GS} - V_t} = \frac{2 \times 10^{-3}}{2 - 1} = 2 \times 10^{-3}\,\Omega^{-1}$$

$$A_{vo} = -\frac{g_m R_D}{1 + g_m R_S} = -\frac{3}{1+3} = -0.857$$

The input resistance is $R_{\text{in}} = R_1 || R_2 = 100 || 100 = 50\,\text{k}\Omega$. The output resistance is $R_{\text{out}} = R_D = 3\,\text{k}\Omega$. The overall gain is

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_{vo} \frac{R_L}{R_L + R_{\text{out}}} = -\frac{50}{50+20} \times 0.857 \times \frac{1}{1+3} = -0.153$$