EE 321 Analog Electronics, Fall 2013
Homework #5 solution

3.26. For the circuit shown in Fig. P3.26, both diodes are identical, conducting 10 mA at 0.7 V, and 100 mA at 0.8 V. Find the value of R for which $V = 80$ mV.

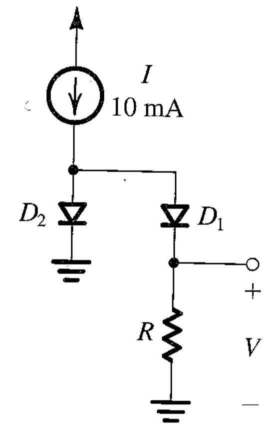


FIGURE P3.26

V_1 is the voltage across diode 1, V_2 across diode 2, I_1 is current through diode 1, and I_2 through diode 2. Then we have (using exponential model)

$$I = I_1 + I_2 \quad I_2 = I_s \exp\left(\frac{V_2}{nV_T}\right) \quad I_1 = I_s \exp\left(\frac{V_2 - V}{nV_T}\right) = I_s \exp\left(\frac{V_2}{nV_T}\right) \exp\left(-\frac{V}{nV_T}\right)$$

or alternatively,

$$I_2 = I_1 \exp\left(\frac{V}{nV_T}\right)$$

and thus

$$I = I_1 + I_2 = I_1 \left[1 + \exp\left(\frac{V}{nV_T}\right)\right]$$

and

$$I_1 = \frac{I}{1 + \exp\left(\frac{V}{nV_T}\right)}$$

and

$$R = \frac{V}{I_1} = \frac{V}{I}\left[1 + \exp\left(\frac{V}{nV_T}\right)\right]$$

Now we only need to determine nV_T. Note that

$$\log\left(\frac{I_D}{I_S}\right) = \frac{V}{nV_T}$$

and

$$\log\left(\frac{I_{Da}}{I_S}\right) - \log\left(\frac{I_{Db}}{I_S}\right) = \frac{V_a - V_b}{nV_T}$$

$$\log\left(\frac{I_{Da}}{I_{Db}}\right) = \frac{V_a - V_b}{nV_T}$$

$$nV_T = \frac{V_a - V_b}{\log\left(\frac{I_{Da}}{I_{Db}}\right)}$$

Now inserting $V_a = 0.8\,\mathrm{V}$, $I_a = 100\,\mathrm{mA}$, $V_b = 0.7\,\mathrm{V}$, and $I_b = 10\,\mathrm{mA}$, we get

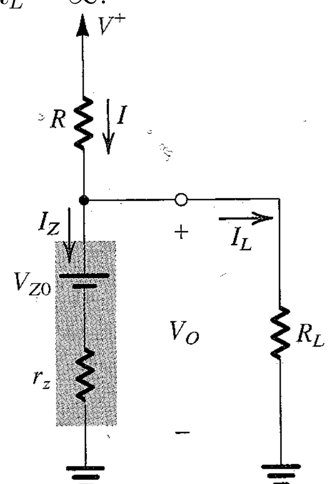
$$nV_T = \frac{0.1}{\log 10} = 43.4\,\mathrm{mV}$$

Inserting that in the expression for R, we get

$$R = \frac{80}{10}\left[1 + \exp\left(\frac{80}{43.4}\right)\right] = 58.5\,\Omega$$

3.66. A shunt regulator utilizing a zener diode with an incremental resistance of $5\,\Omega$ is fed through an $82 - \Omega$ resistor. If the raw supply changes by $1.3\,\mathrm{V}$, what is the corresponding change in the regulated output voltage?

The circuit looks like this, with $R_L = \infty$.



The output is across the Zener diode, so across the supply and r_z. The relationship between the input and output voltages is

$$v_O = V_{Z0} + (V - V_{Z0})\,\frac{r_z}{R + r_z}$$

To get the regulation, the ratio of change in the output for a change in the output we take the derivative of the output with respect to the input.

$$\frac{dv_O}{dV} = \frac{r_z}{R + r_z}$$

For some change in the input voltage, $\Delta V = 1.3\,\mathrm{V}$, the output is

$$\begin{aligned}\Delta v_O =& \Delta V \frac{r_z}{R + r_z} \\ =& 1.3 \times \frac{5}{82+5} \\ =& 0.075\,\mathrm{V}\end{aligned}$$

3.74. Consider a half-wave rectifier circuit with a triangular wave input of $5-\mathrm{V}$ peak-to-peak amplitude and zero average and with $R = 1\,\mathrm{k}\Omega$. Assume that the diode can be represented by the piecewise-linear model $V_{D0} = 0.65\,\mathrm{V}$ and $r_D = 20\,\Omega$. Find the aveage value of v_o.

The relationship between the input and the output is

$$v_o = \begin{cases} (v_I - V_{D0}) \frac{R}{R+r_D} & v_I \geq v_{D0} \\ 0 & v_I < v_{D0} \end{cases}$$

If the period of the signal is T, and the input voltage is $v_I = V \sin\left(\frac{2\pi t}{T}\right)$, then the diode is turned on between times t_1 and t_2, where

$$\sin\left(\frac{2\pi t_1}{T}\right) = \frac{V_{D0}}{V} \quad 0 \leq t_1 \leq \frac{T}{4}$$

$$t_1 = \frac{T}{2\pi} \sin^{-1} \frac{V_{D0}}{V} = \sin^{-1} \frac{0.65}{5} \frac{T}{2\pi} = \frac{0.130369}{2\pi} T$$

and

$$t_2 = \frac{T}{2} - t_1 = \frac{T}{2} - \frac{0.130369}{2\pi} T = \left(\frac{1}{2} - \frac{0.130369}{2\pi}\right) T = \frac{\pi - 0.130369}{2\pi} T$$

The average value of v_O is found by integrating it over the period and dividing by the period,

$$\begin{aligned}\langle v_o \rangle =& \frac{1}{T} \int_0^T v_o\,dt \\ =& \frac{1}{T} \int_{t_1}^{t_2} v_o\,dt \\ =& \frac{1}{T} \int_{t_1}^{t_2} \left[V \sin\left(\frac{2\pi t}{T}\right) - V_{D0}\right] \frac{R}{R+r_D}\,dt \\ =& \frac{1}{T} \frac{R}{R+r_D} \left[V \int_{t_1}^{t_2} \sin\left(\frac{2\pi t}{T}\right)\,dt - (t_2 - t_1)\,V_{D0}\right]\end{aligned}$$

Change integration variable, $x = 2\pi\frac{t}{T}$, $dx = \frac{2\pi}{T}\,dt$, $x_1 = 2\pi\frac{t_1}{T} = 0.130369$, $x_2 = 2\pi\frac{t_2}{T} = \pi - 0.130369$, we get

$$
\begin{aligned}
\langle v_o \rangle =& \frac{1}{T}\frac{R}{R+r_D}\left[V\frac{T}{2\pi}\int_{x_1}^{x_2}\sin x\,dx - (t_2-t_1)\,V_{D0}\right]\\
=& \frac{1}{T}\frac{R}{R+r_D}\left[V\frac{T}{2\pi}\int_{x_1}^{x_2}\sin x\,dx - \frac{T}{2\pi}(x_2-x_1)\,V_{D0}\right]\\
=& \frac{1}{2\pi}\frac{R}{R+r_D}\left[V\int_{x_1}^{x_2}\sin x\,dx - (x_2-x_1)\,V_{D0}\right]\\
=& \frac{1}{2\pi}\frac{R}{R+r_D}\left[V\left[-\cos x\right]_{x_1}^{x_2} - (x_2-x_1)\,V_{D0}\right]
\end{aligned}
$$

Now inserting all the numbers

$$
\begin{aligned}
\langle v_o \rangle =& \frac{1}{2\pi}\frac{10^3}{20+10^3}\left[5\times(\cos 0.130369-\cos(\pi-0.130369))-0.65\times(\pi-2\times 0.130369)\right]\\
=&1.2549\,\mathrm{V}
\end{aligned}
$$

3.78. A full-wave bridge rectifier circuit with a 1− kΩ load operates from a 120−V (rms) 60− Hz household supply through a 10-to-1 step-down transformer having a single secondary winding. It uses four diodes, each of which can be modeled to have a 0.7 − V drop for any current. What is the peak value of the rectified voltage across the load? For what fraction of the cycle does each diode conduct? What is the average voltage across the load? What is the average current through the load?

The peak value of the rectified voltage across the load is

$$V_O = V_I - 2v_D$$

where $V_I = 120\,\mathrm{V}\frac{\sqrt{2}}{10} = 16.97\,\mathrm{V}$, so

$$V_O = 16.97 - 2\times 0.7 = 15.57\,\mathrm{V}$$

Each dioded conducts for a fraction, f, of time which is equal to the time the input voltage is greater than $2v_D$, or the fraction of the time that a sinusoid exceeds the value $2v_D/V_I$.

$$
\begin{aligned}
f =& \frac{1}{2\pi}\left(\pi - 2\sin^{-1}\left(\frac{2v_D}{V_I}\right)\right)\\
=& \frac{1}{2\pi}(\pi - 2\times 0.0825924)\\
=&0.4737\\
=&47.37\%
\end{aligned}
$$

(the answer in the book is incorrect). The average voltage across the load is found by integrating the output voltage over a period and dividing by the period,

$$\langle v_o \rangle = \frac{1}{T}\int_0^T v_o\,dt$$

The relationship between the input and the output voltage is

$$v_o = \begin{cases} |v_I| - 2V_D & |v_I| > 2V_D \\ 0 & |v_I| \le 2V_D \end{cases}$$

where $v_I = V_I \sin\frac{2\pi t}{T}$. Because of the symmetry we can just integrate over the half period corresponding to the positive peak in v_I,

$$\begin{aligned} \langle v_o \rangle =& \frac{2}{T}\int_0^{\frac{T}{2}} v_o\,dt \\ =& \frac{2}{T}\int_{t_1}^{t_2} V_I \sin\left(\frac{2\pi t}{T}\right) - 2V_D\,dt \end{aligned}$$

Now change variables, $x = \frac{2\pi t}{T}$, and thus $dx = \frac{2\pi}{T}dt$, $x_1 = \frac{2\pi t_1}{T}$, and $x_2 = \frac{2\pi t_2}{T}$,

$$\begin{aligned} \langle v_o \rangle =& \frac{2}{T}\frac{T}{2\pi}\int_{x_1}^{x_2} V_I \sin\, x - 2V_D\,dx \\ =& \frac{1}{\pi}\left[V_I\left[-\cos x\right]_{x_1}^{x_2} - 2\left(x_2 - x_1\right)V_D\right] \end{aligned}$$

Now,

$$x_1 = \sin^{-1}\left(\frac{2V_D}{V_I}\right) = \sin^{-1}\left(\frac{2\times 0.7}{16.97}\right) = 0.0825924$$

and $x_2 = \pi - x_1 = \pi = -0.0825924$, and we can insert

$$\langle v_o \rangle = \frac{1}{\pi}\left[16.97\times 2\times\cos(0.0825924) - 2\times 0.7\times(\pi - 2\times 0.0825924)\right] = 9.44\,\mathrm{V}$$

The average current is simply the average voltage divided by the load resistance,

$$\langle i_o \rangle = \frac{1}{R}\langle v_o \rangle = \frac{9.44}{10^3} = 9.44\,\mathrm{mA}$$

3.91. The op amp in the precision rectifier circuit of Fig P3.91 is ideal with output saturation levels of $\pm 12\,\mathrm{V}$. Assume that when conducting the diode exhibits a constant voltage drop of $0.7\,\mathrm{V}$. Find v_-, v_a, and v_A for:

(a) $v_I = +1\,\mathrm{V}$

(b) $v_I = +2\,\mathrm{V}$

(c) $v_I = -1\,\mathrm{V}$

(d) $\boldsymbol{v_I = -2\,\mathrm{V}}$

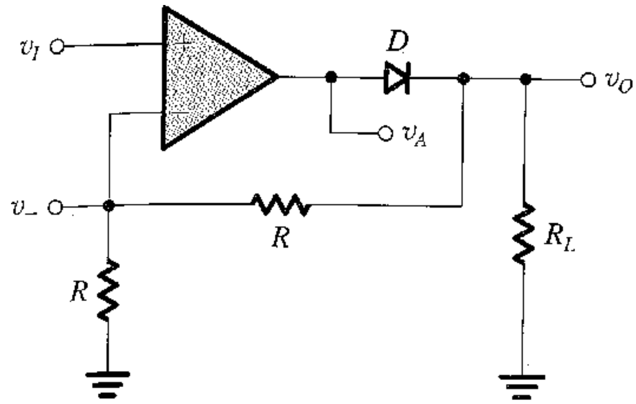


FIGURE P3.91

(a) When $v_I > v_D$, the op-amp will attempt to output current to raise v_- to v_I by raising its output voltage. Therefore I expect the diode to be conducting. In that case we have,

$$i_- = \frac{v_-}{R} = \frac{v_D}{R}$$

and thus

$$v_o = 2Ri_- = 2R\frac{v_I}{R} = 2v_I$$

and

$$v_A = v_O + V_D$$

Inserting values we get

$$v_- = v_I = 1\,\mathrm{V}$$

$$v_o = 2v_I = 2 \times 1 = 2\,\mathrm{V}$$

$$v_A = v_o + V_D = 2 + 0.7 = 2.7\,\mathrm{V}$$

(b) In this case the derivation is exactly the same as for case (a), so

$$v_- = v_I = 2\,\mathrm{V}$$

$$v_o = 2v_I = 4\,\mathrm{V}$$

$$v_A = v_o + V_D = 4 + 0.7 = 4.7\,\mathrm{V}$$

(c) In this case, the op-amp output will attempt to draw current by lowering its voltage. It cannot draw current so the op-amp output will go to negative rail. There is no current anywhere else in the circuit so $v_- = v_o = 0$. Thus,

$$v_- = 0\,\mathrm{V}$$

$$v_o = 0\,\mathrm{V}$$

$$v_A = -12\,\mathrm{V}$$

(d) In this case the situation is identical to case (c), with the same voltages.

3.92. The op-amp in the circuit of Fig P3.92 is ideal with saturation levels of $\pm 12\,\mathrm{V}$. The diodes exhibit a constant $0.7\,\mathrm{V}$ drop when conducting. Find v_-, v_A, and v_o for:

(a) $v_I = +1\,\mathrm{V}$

(b) $v_I = +2\,\mathrm{V}$

(c) $v_I = -1\,\mathrm{V}$

(d) $v_I = -2\,\mathrm{V}$

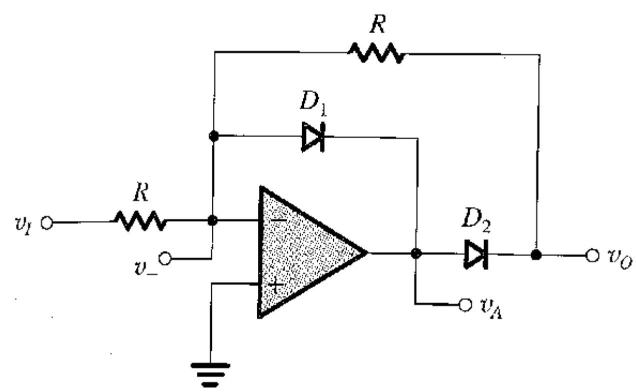


FIGURE P3.92

(a) In this case the input voltage is above ground, and the op-amp will attempt to adjust by drawing current in. It can draw current through D_1, and then D_2 will not be conducting. Thus, $v_A = -V_D = -0.7\,\mathrm{V}$, $v_- = 0\,\mathrm{V}$. For the ground we realize that no current flows through the loop containing ground, and thus $v_o = v_- = 0\,\mathrm{V}$.

(b) This case is the same as case (a). The op-amp is simply drawing twice as much current through D_1. The voltages are the same as case (a).

(c) In this case the input is below ground and the op-amp will attempt to compensate by supplying current, raising its output voltage. In this case diode D_2 is conducting, and the op-amp current will rise until $v_- = 0$. At that point, $v_o = -v_I = 1\,\mathrm{V}$, and $v_A = v_o + V_D = 1.7\,\mathrm{V}$.

(d) This case is similar to case (c). The op-amp will output current through D_2 to make $v_- = 0$, and then $v_o = -v_I = 2\,\mathrm{V}$, and $v_A = v_o + V_D = 2.7\,\mathrm{V}$.

5.15. (a) Use the Ebers-Moll expressions in Eqs. 5.26 and 5.27 to show that the i_C-v_{CB} relationship sketch in Fig. 59. can be described by

$$\boldsymbol{i_C = \alpha_F I_E - I_S \left(\frac{1}{\alpha_R} - \alpha_F\right) e^{\frac{v_{BC}}{V_T}}}$$

(b) Calculate and sketch $\boldsymbol{i_C}$-$\boldsymbol{v_{CB}}$ curves for a transistor for which $\boldsymbol{I_S = 10^{-15}}$ A, $\boldsymbol{\alpha_F \approx 1}$, and $\boldsymbol{\alpha_R = 0.1}$. Sketch graphs for $\boldsymbol{I_E = 0.1}$ mA, 0.5 mA, and 1 mA. For each, give the values of $\boldsymbol{v_{BC}}$, $\boldsymbol{v_{BE}}$, and $\boldsymbol{v_{CE}}$ for which (a) $\boldsymbol{i_C = 0.5\alpha_F I_E}$ and (b) $\boldsymbol{i_C = 0}$.

(a) The Ebers-Moll equations are

$$i_E = \frac{I_S}{\alpha_F}\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

$$i_C = I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

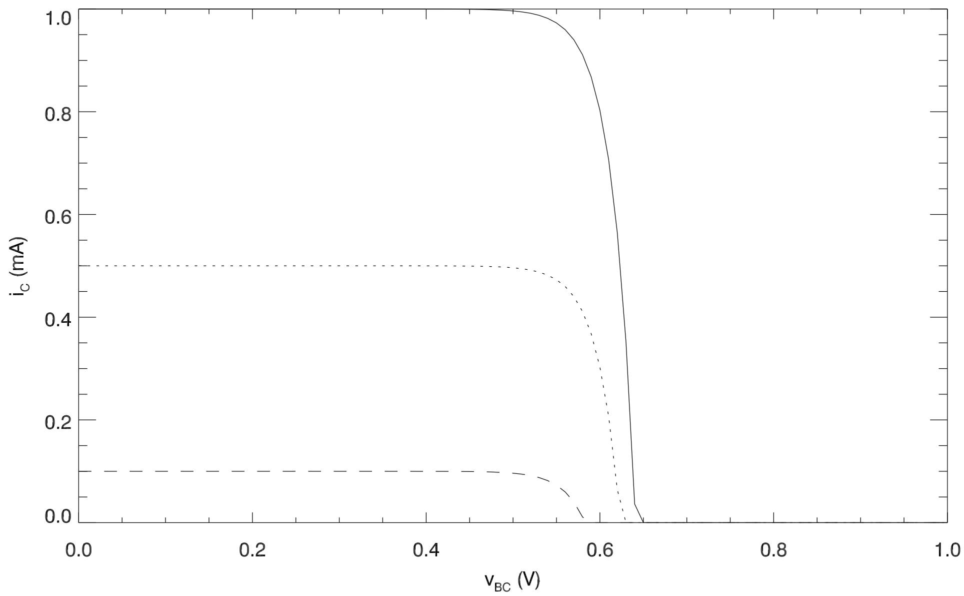
Eliminate $e^{\frac{v_{BE}}{V_T}}$ from the second equation by substituting the first equation into it. i_C is then a function of i_E (which the book assumes fixed biased so it calls it I_E) and v_{BC}. Re-arrange the first equation:

$$I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) = \alpha_F I_E + \alpha_F I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

Insert in the second equation

$$\begin{aligned} i_C =& \alpha_F I_E + \alpha_F I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) - \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) \\ =& \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right) e^{\frac{v_{BC}}{V_T}} \end{aligned}$$

(b) This plot shows i_C as a function of v_{CB}.



The solid curve is for $i_C = 1\,\text{mA}$, the dotted is for $i_C = 0.5\,\text{mA}$, and the dashed is for $i_C = 0.1\,\text{mA}$.

(a) The value of v_{CB} for which $i_C = 0.5\alpha_F I_E$ can be found from

$$0.5\alpha_F I_E = \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right) e^{\frac{v_{BC}}{V_T}}$$

$$v_{BC} = V_T \ln\left(\frac{0.5\alpha_F I_E}{I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)}\right)$$

The values are tabulated here:

I_E	v_{BC}
0.1 mA	0.57 V
0.5 mA	0.61 V
1 mA	0.62 V

(b) The value for v_{CB} for which $i_C = 0$ can be found from

$$0 = \alpha_F I_E - I_S\left(\frac{1}{\alpha_R} - \alpha_F\right) e^{\frac{v_{BC}}{V_T}}$$

$$v_{BC} = V_T \ln\left(\frac{\alpha_F I_E}{I_S\left(\frac{1}{\alpha_R} - \alpha_F\right)}\right)$$

The values are tabulated here

I_E	v_{BC}
0.1 mA	0.58 V
0.5 mA	0.62 V
1 mA	0.64 V

5.20. For the circuits in Fig P5.20, assume that the transistors have very large β. Some measurements have been made on these circuits, with the results indicated in the figure. Find the values of the other labeled voltages and currents.

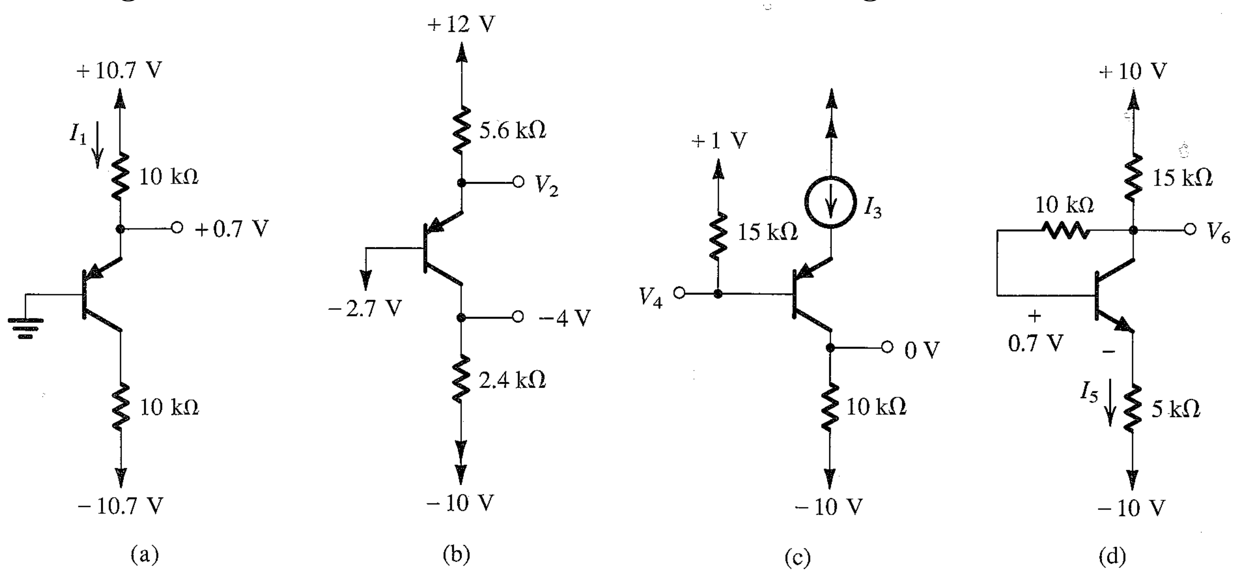


FIGURE P5.20

The hint that β is very large means that we can assume that $i_C = i_E$.

(a) $I_1 = \frac{V_{CC}-V_E}{R_E} = \frac{10.7-0.7}{10} = 10\,\text{mA}$

(b) We can see that the transistor must be on, so $V_2 = V_B + V_{BE} = -2.7 + 0.7 = -2\,\text{V}$

(c) The transistor is on because there is collector current flowing.

$$I_3 = I_E = I_C = \frac{V_C - V_{CC}}{R_C} = \frac{0+10}{10} = 1\,\text{mA}$$

Since β is very large there is no current flowing in the base, so $V_4 = V_{BB} = 1\,\text{V}$.

(d) Since β is very large there is no current flowing in the base and thus $V_B = V_C$, and we can write

$$V_{CC} - V_{EE} = I_5\,(R_C + R_E) + V_{BE}$$

and thus

$$I_5 = \frac{V_{CC} - V_{EE} - V_{BE}}{R_C + R_E} = \frac{10 + 10 - 0.7}{15 + 5} = 0.97\,\text{mA}$$

$$V_6 = V_{CC} - I_C R_C = 10 - 0.97 \times 15 = -4.6\,\mathrm{V}$$