EE 321 Analog Electronics, Fall 2013
Homework #8 solution

5.110. The following table summarizes some of the basic attributes of a number of BJTs of different types, operating as amplifiers under various conditions. Provide the missing entries.

Transistor	a	b	c	d	e	f	g
α	1.000					0.90	
β		100		∞			
I_C (mA)	1.00		1.00				
I_E (mA)		1.00				5	
I_B (mA)			0.020				1.10
g_m (mA/V)							700
r_e (Ω)				25	100		
r_π (Ω)					10.1 kΩ		

(*Note:* Isn't it remarkable how much two parameters can reveal?)

I will just provide the explicit equations for the first column, (a):

$$\beta = \frac{\alpha}{1-\alpha}$$

$$I_E = \frac{I_C}{\alpha}$$

$$I_B = \frac{I_C}{\beta}$$

$$g_m = \frac{I_C}{V_T}$$

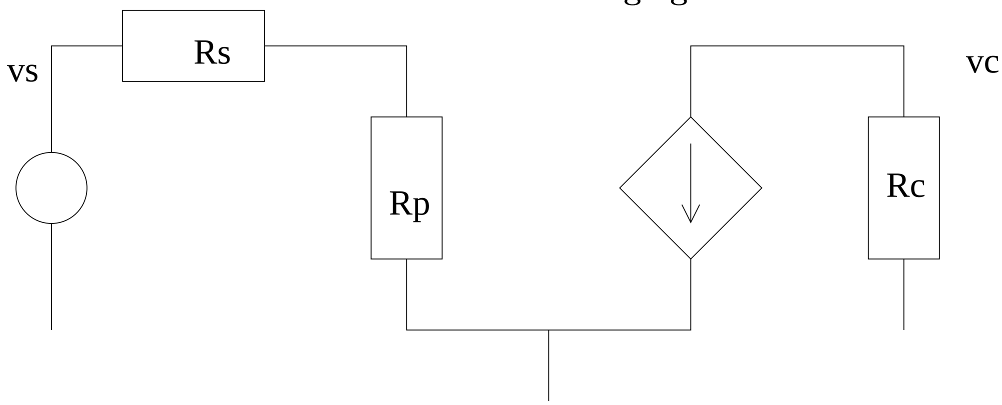
$$r_e = \frac{\alpha}{g_m}$$

$$r_\pi = \frac{\beta}{g_m}$$

Transistor	a	b	c	d	e	f	g
α	**1.000**	0.99	0.98	1	0.99	**0.9**	0.984
β	∞	**100**	50	$\boldsymbol{\infty}$	100	9	62.8
I_C (mA)	**1.00**	0.99	**1.00**	1	0.25	4.5	69.1
I_E (mA)	1.00	**1.00**	1.02	1	0.253	**5**	70.2
I_B (mA)	0	0.0099	**0.02**	0	0.0025	0.5	**1.10**
g_m (mA/V)	40	39.6	40	40	9.9	180	**700**
r_e (Ω)	25	25	24.5	**25**	**100**	5	1.41
r_π (Ω)	∞	2530	1250	∞	**10.1K**	50	89.7

5.114. A biased BJT operates as a grounded-emitter amplifier between a signal source, with a source resistance of 10 kΩ, connected to the base and a 10 kΩ

load connected as a collector resistance R_C. In the corresponding model, g_m is 40 mA/V and r_π is 2.5 kΩ. Draw the complete amplifier model using the hybrid-π BJT equivalent circuit. Calculate the overall voltage gain v_c/v_s. What is the value of BJT β implied by the values of the model parameters? To what value must β be increased to double the overall voltage gain?



$$v_o = -g_m R_C v_i = -g_m R_C \frac{r_\pi}{R_s + r_\pi} v_s = -0.04 \times 10 \times 10^3 \times \frac{2.5}{10 + 2.5} = -80$$

The value of β can be found from

$$r_\pi = \frac{\beta}{g_m}$$

or

$$\beta = r_\pi g_m = 2.5 \times 10^3 \times 0.04 = 100$$

To double the gain we would want to double the factor

$$\frac{r_\pi}{R_s + r_\pi}$$

It is currently equal to

$$\frac{2.5}{10 + 2.5} = 0.2$$

To double it we would need to change r_π:

$$\frac{r_\pi}{R_s + r_\pi} = 0.4$$

$$r_\pi = \frac{0.4}{0.6} R_s = 0.67 R_s = 6.7\,\mathrm{k\Omega}$$

The factor increase in β is the same as the factor increase in r_π:

$$\beta_{\text{new}} = \beta_{\text{old}} \frac{r_{\pi\,\text{new}}}{r_\pi\ \text{old}} = 100 \times \frac{6.7}{2.5} = 268$$

5.115. For the circuit shown in Fig. P5.115, draw a complete small-signal equivalent circuit utilizing an appropriate T model for the BJT (use $\alpha = 0.99$). Your

circuit should show the values of all components, including the model parameters. What is the input resistance R_{in}? Calculate the overall gain v_o/v_{sig}.

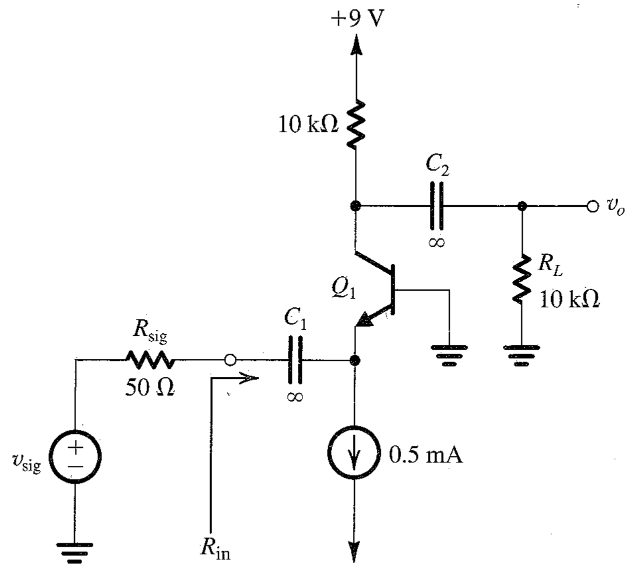
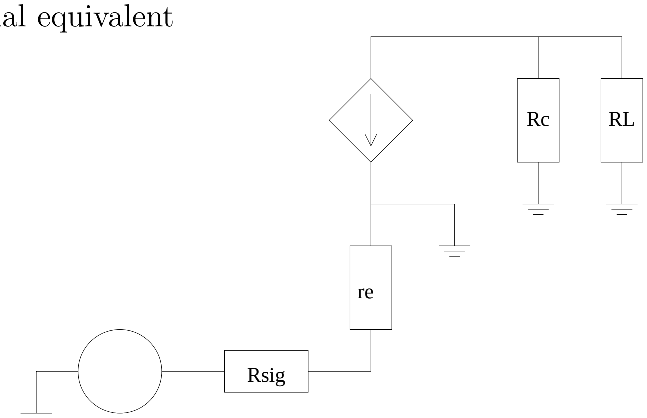


FIGURE P5.115

Here is the small-signal equivalent



where

The overall voltage can be found from

$$v_o = -\left(R_C || R_L\right) i_c$$

$$i_c = \alpha i_e$$

$$i_e = -\frac{v_{sig}}{R_{sig} + r_e}$$

so

$$
\begin{aligned}
\frac{v_o}{v_{\text{sig}}} &= (R_C||R_L)\,\alpha\frac{1}{R_{\text{sig}}+r_e} \\
&= (10||10)\,0.99\frac{1}{50+50} \\
&=49.5
\end{aligned}
$$

5.128. A common-emitter amplifier of the type shown in Fig 5.60(a) is biased to operate at $I_C = 0.2\,\text{mA}$ and has a collector resistance $R_C = 24\,\text{k}\Omega$. The transistor has $\beta = 100$ and a large V_A. The signal source is directly coupled to the base and C_{c1} and R_B are eliminated. Find R_{in}, the voltage gain A_{vo}, and R_o. Use these results to determine the overall voltage gain when a $10\,\text{k}\Omega$ resistor is connected to the collector and the source resistance $R_{\text{sig}} = 10\,\text{k}\Omega$.

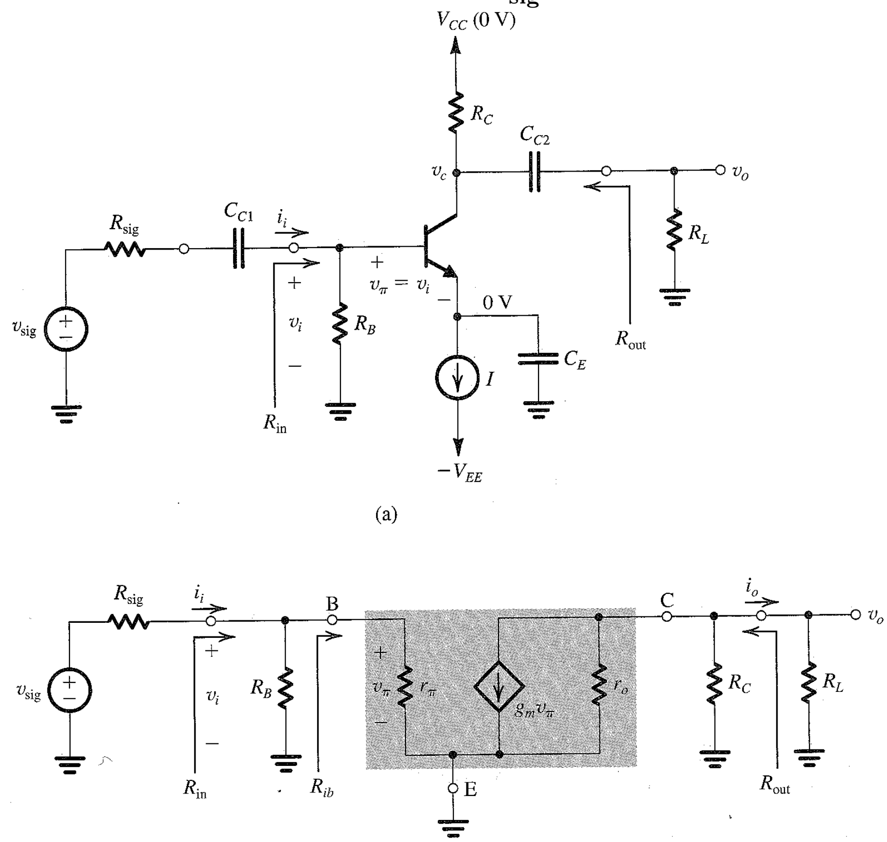


FIGURE 5.60 **(a)** A common-emitter amplifier using the structure of Fig. 5.59. **(b)** Equivalent circuit obtained by replacing the transistor with its hybrid-π model.

This is a common-emitter amplifier with $R_B = \infty$, so the input resistance is

$$
R_{\text{in}} = r_\pi = \frac{\beta V_T}{I_C} = \frac{100\times 25}{0.2} = 12.5\,\text{k}\Omega
$$

$$
A_{vo} = -g_m R_C = -\frac{I_C}{V_T}R_C = -\frac{0.2}{25}10\times 10^3 = -80
$$

$$R_O = R_C = 10\,\mathrm{k}\Omega$$

The overall voltage gain, v_o/v_{sig} is then

$$\begin{aligned} G_v =& \frac{v_o}{v_{\mathrm{sig}}} = \frac{r_\pi}{R_{\mathrm{sig}} + r_\pi} A_{vo} \frac{R_L}{R_L + R_O} \\ =& \frac{12.5}{10 + 12.5} (-80) \frac{10}{10 + 10} \\ =& -22.2 \end{aligned}$$

5.130. For the common-emitter amplifier shown in Figure P5.130, let $V_{CC} = 9\,\mathrm{V}$, $R_1 = 27\,\mathrm{k}\Omega$, $R_2 = 15\,\mathrm{k}\Omega$, $R_E = 1.2\,\mathrm{k}\Omega$, and $R_C = 2.2\,\mathrm{k}\Omega$. The transistor has $\beta = 100$, and $V_A = 100\,\mathrm{V}$. Calculate the dc bias current I_E. If the amplifier operates between a source for which $R_{\mathrm{sig}} = 10\,\mathrm{k}\Omega$ and a load of $2\,\mathrm{k}\Omega$, replace the transistor with its bybrid-π model, and find the values of R_{in}, the voltage gain v_o/v_{sig}, and the current gain i_o/i_i.

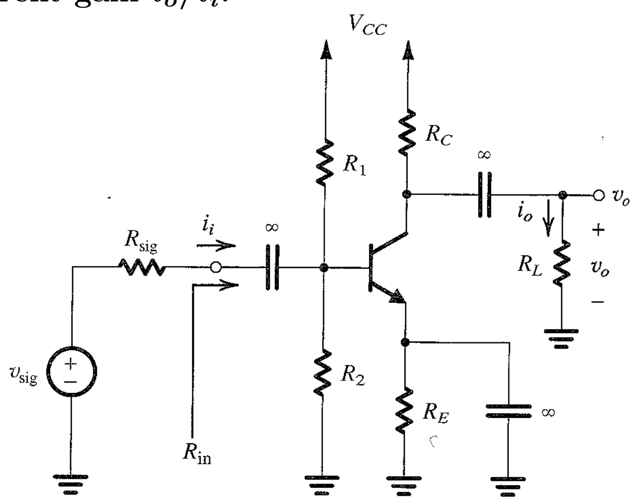


FIGURE P5.130

We have

$$V_B = (i_1 - i_B)\, R_2 \qquad V_B = V_{BE} + (\beta + 1)\, R_E i_B \qquad V_B = V_{CC} - i_1 R_1$$

and we can eliminate i_D and i_B from the first equation.

$$V_B = \left(\frac{V_{CC} - V_B}{R_1} - \frac{V_B - V_{BE}}{(\beta + 1)\, R_E} \right) R_2$$

$$V_B \left[1 + \frac{R_2}{R_1} + \frac{R_2}{(\beta + 1)\, R_E} \right] = V_{CC} \frac{R_2}{R_1} + V_{BE} \frac{R_2}{(\beta + 1)\, R_E}$$

$$V_B = \frac{V_{CC} \frac{R_2}{R_1} + V_{BE} \frac{R_2}{(\beta+1) R_E}}{1 + \frac{R_2}{R_1} + \frac{R_2}{(\beta+1) R_E}} = \frac{9 \times \frac{15}{27} + 0.7 \times \frac{15}{101 \times 1.2}}{1 + \frac{15}{27} + \frac{15}{101 \times 1.2}} = 3.03\,\mathrm{V}$$

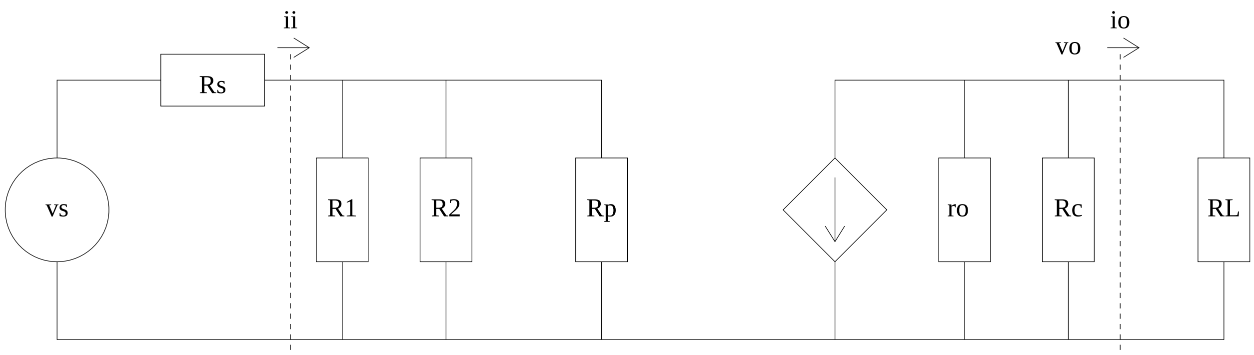
Note that for $(\beta+1)\,R_E \gg R_2$ and $V_{CC} \gg V_{BE}$ the expression for V_B reduces to the voltage divider expression,

$$V_B = V_{CC}\frac{\frac{R_2}{R_1}}{1+\frac{R_2}{R_1}} = V_{CC}\frac{R_2}{R_1+R_2} = 9 \times \frac{15}{15+27} = 3.21\,\mathrm{V}$$

I will proceed with the result from the fully correct expression. Next, the emitter current is

$$I_E = \frac{V_E}{R_E} = \frac{V_B - V_{BE}}{R_E} = \frac{3.03-0.7}{1.2} = 1.94\,\mathrm{mA}$$

The small-signal model looks like this



The input resistance is

$$R_{\mathrm{in}} = R_1||R_2||r_\pi$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{\beta V_T}{\alpha I_E} = \frac{(\beta+1)\,V_T}{I_E} = \frac{101 \times 25}{1.94} = 1302\,\Omega$$

such that

$$R_{\mathrm{in}} = R_1||R_2||r_\pi = 27||15||1.302 = 1.15\,\mathrm{k}\Omega$$

The voltage gain is

$$G_v = -\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_s}A_v = -\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_s}g_m\,(R_C||r_o||R_L) = -\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_s}\frac{\beta I_E}{(\beta+1)\,V_T}\,(R_C||r_o||R_L)$$

where

$$r_o = \frac{V_A}{I_C} = \frac{V_A}{\alpha I_E} = \frac{(\beta+1)\,V_A}{\beta I_E} = \frac{101 \times 100}{100 \times 1.94 \times 10^{-3}} = 52.1\,\mathrm{k}\Omega$$

and then

$$
\begin{aligned}
G_v &= -\frac{R_{\text{in}}}{R_{\text{in}}+R_s}\frac{\beta I_E}{(\beta+1)\,V_T}\,(R_C||r_o||R_L) \\
&= -\frac{1.14}{1.14+10}\frac{100\times 1.94}{101\times 25}\,(2.2||52.1||2) \\
&= -\,8.07
\end{aligned}
$$

5.131. Using the topology of Fig. P5.130, design an amplifier to operate between a 10 kΩ source and a 2 kΩ load with a gain v_o/v_{sig} of −8. The power supply available is 9 V. Use an emitter current of approximately 2 mA and a current of about one-tenth of that in the voltage divider that feeds tbe base, with the dc voltage at the base about one third of the supply. The transistor available has $\beta = 100$ and $V_A = 100$ V. Use standard 5% resistors (See Appendix G)

This is the same circuit as before, except choose the nearest 5% resistors. In that case choose $R_1 = 27\,\text{k}\Omega$, $R_2 = 15\,\text{k}\Omega$, $R_E = 1.2\,\text{k}\Omega$, $R_C = 2.2\,\text{k}\Omega$. Well, it turns out the problem is identical to P5.130. That was easy.