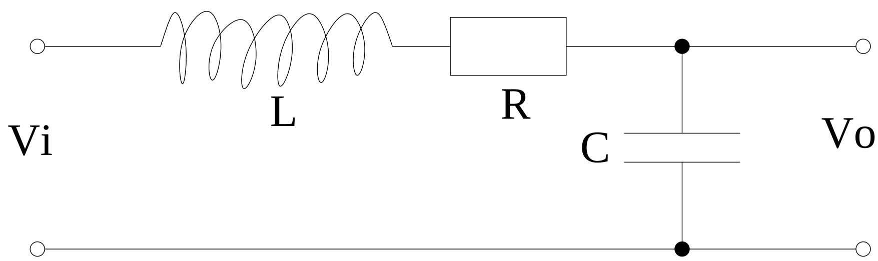
EE 322 Analog Electronics, Spring 2010
Exam 2 March 31, 2010
Solution

Rules: This is a open book test. You may use the textbooks as well as your notes. The exam will last 50 minutes. Each numbered problem counts equally toward your grade.

LCR circuit



1. **Derive the transfer function for this circuit.**
2. **What is the order and kind of filter is this? (e.g. high-, low-, etc...)**
3. **What is the natural frequency and Q value?**
4. **Extra credit: How is the filter modified if a inductor is added in parallel with the capacitor?**

1. The transfer function is found from

$$T(s) = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_2 = \frac{1}{sC} \qquad Z_1 = sL + R$$

Then

$$T(s) = \frac{\frac{1}{sC}}{sL + R + \frac{1}{sC}} = \frac{1}{s^2LC + sCR + 1} = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

2. This is a second-order low-pass filter.

3. The natural frequency is

$$\omega_0 = \frac{1}{LC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

The Q value is found from

$$\frac{\omega_0}{Q} = \frac{R}{L} \qquad Q = \frac{\omega_0 L}{R} = \frac{L}{\sqrt{LC}R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

4. If we add a inductor in series with the capacitor then

$$Z_2 = \frac{1}{\frac{1}{sL} + sC} = \frac{sL}{1 + s^2 LC}$$

and the transfer function becomes

$$\begin{aligned} T(s) =& \frac{\frac{sL}{1+s^2LC}}{\frac{sL}{1+s^2LC} + sL + R} = \frac{sL}{sL + (1 + s^2LC)(sL + R)} \\ =& \frac{sL}{sL + sL + R + s^3L^2C + s^2LCR} = \frac{sL}{s^3L^2C + s^2LCR + 2sL + R} \\ =& \frac{s\frac{C}{L}}{s^3 + s^2\frac{R}{L} + s\frac{1}{LC} + \frac{R}{L^2C}} \end{aligned}$$

This is a third-order filter, and it is low-pass, although it only goes to zero at 40 dB per decade due to the single zero located at zero frequency.

Filter implementation

Consider a filter with poles $p_1 = 10^4 e^{j120°}$ rad/s, and $p_2 = 10^4 e^{j180°}$ rad/s, and $p_3 = 10^4 e^{j240°}$ rad/s, and three zeros at inifinity.

5. **Is this one of our known standard filter types, and if so which one? What is the order of this filter? Is it a low-pass or high-pass filter? What is its 3 dB frequency?**

6. **Draw a circuit which implements this filter, using a bi-quadratic circuit followed by a passive RC circuit.**

7. **What should be the Q value of the second order portion of the circuit?**

8. **Use 1 nF capacitors throughout and a 100 kΩ value for the input and feedback resistors, and specify the rest of the resistors.**

9. **What is the maximum gain of the circuit, and at what frequency does it occur?**

5. The poles are located at a fixed radius and spaced evenly in angle with a half of the angle spacing near the imaginary axis. This is the recipe for the Butterworth filter. There are three poles, so it is a 3rd order filter. All three zeros are located at infinity so it is a low-pass filter.

Regarding the 3-dB frequency, if we look carefully at the expressions and plots defining the Butterworth filter (particularly Figures 12.8 and 12.9) we can see that the 3-dB frequency is the pass-band edge when $\epsilon = 1$. And in that case, $\omega_p = \omega_0 = 10^4\,\text{rad/s}$.

6. It is a biquad circuit as shown in figure 12.24a, with a low-pass RC filter attached to the V_{lp} output, with the output voltage taken across the capacitor to ground.

7. Consult the book and find that the real component of p_1 is $-\frac{\omega_0}{2Q}$, and thus

$$\cos(120°) = -\frac{1}{2Q}$$

$$Q = -\frac{1}{2\cos(120°)} = 1$$

8. We are told that $R_f = 100\,\text{k}\Omega$ and $R_2 = 100\,\text{k}\Omega$. Then $R_1 = R_f = 100\,\text{k}\Omega$. We also have that $R_3/R_2 = 2Q - 1$, such that $R_3 = (2Q-1)\,R_2 = R_2 = 100\,\text{k}\Omega$. Finally, $\omega_0 = \frac{1}{RC}$, so that $R = \frac{1}{\omega_0 C} = \frac{1}{10^4 \times 10 \times 10^{-9}} = 10\,\text{k}\Omega$.

9. Since this is a low-pass filter the maximum gain occurs at zero frequency. The maximum gain is

$$K = 2 - \frac{1}{Q} = 2 - 1 = 1$$