EE 322 Advanced Analog Electronics, Spring 2010
Homework #3 solution

SS 13.5. In a particular oscillator characterized by the structure of Fig 13.1, the frequency-selective network exhibits a loss of 20 dB and a phase shift of 180° at ω_0. What is the minimum gain and the phase shift that the amplifier must havfe for oscillation to begin?

The amplifier must have a minimum gain of 20 dB (a factor of 10), and a phase shift of 180°.

SS 13.13. For the circuit in Fig P13.13 find $L(s)$, $L(j\omega)$, the frequency for zero phase loop phase, and R_2/R_1 for oscillation.

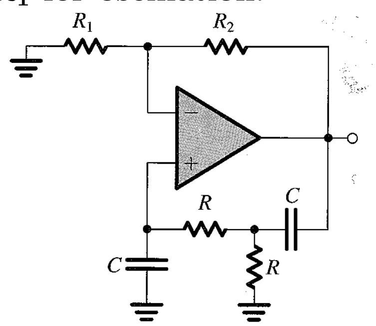

FIGURE P13.13

We have

$$\frac{V_+}{V_X} = \frac{Z_C}{Z_C + R}$$

$$\frac{V_X}{V_O} = \frac{(Z_C + R)\,||R}{(Z_C + R)\,||R + Z_C} = \frac{\frac{1}{\frac{1}{Z_C+R}+\frac{1}{R}}}{\frac{1}{\frac{1}{Z_C+R}+\frac{1}{R}} + Z_C} = \frac{1}{1 + \left(\frac{1}{Z_C+R} + \frac{1}{R}\right) Z_C}$$

$$\frac{V_O}{V_+} = 1 + \frac{R_2}{R_1}$$

The loop gain is the product of these three factors,

$$\begin{aligned}
L(s) =& \frac{V_X}{V_O}\frac{V_+}{V_O}\frac{V_O}{V_+} = \left(1 + \frac{R_2}{R_1}\right)\frac{Z_C}{Z_C + R}\frac{1}{1 + \left(\frac{1}{Z_C+R} + \frac{1}{R}\right) Z_C} \\
&= \left(1 + \frac{R_2}{R_1}\right)\frac{Z_C}{Z_C + R + \left(1 + \frac{Z_C+R}{R}\right) Z_C} \\
&= \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + \frac{R}{Z_C} + 1 + \frac{Z_C+R}{R}} \\
&= \left(1 + \frac{R_2}{R_1}\right)\frac{1}{3 + \frac{R}{Z_C} + \frac{Z_C}{R}} \\
&= \left(1 + \frac{R_2}{R_1}\right)\frac{1}{3 + sRC + \frac{1}{sRC}}
\end{aligned}$$

and thus

$$L(j\omega) = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3 + j\omega RC + \frac{1}{j\omega RC}}$$

We want the complex portion to be zero to get zero phase, so we have

$$j\omega_0 RC = -\frac{1}{j\omega_0 RC} = \frac{j}{\omega_0 RC}$$

or

$$\omega_0 = \frac{1}{RC}$$

At that frequency we have

$$1 = \left(1 + \frac{R_2}{R_1}\right)\frac{1}{3}$$

or

$$R_2 = 2R_1$$

SS 13.14. Repeat problem 13.13 for the circuit shown in Figure P13.14.

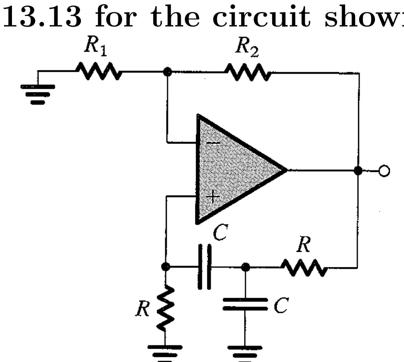

FIGURE P13.14

$$\frac{V_+}{V_X} = \frac{R}{R + Z_C}$$

$$\frac{V_X}{V_O} = \frac{(R+Z_C)\,||Z_C}{(R+Z_C)\,||Z_C + R} = \frac{\frac{1}{\frac{1}{R+Z_C}+\frac{1}{Z_C}}}{\frac{1}{\frac{1}{R+Z_C}+\frac{1}{Z_C}} + R} = \frac{1}{1 + \left(\frac{1}{R+Z_C} + \frac{1}{Z_C}\right) R}$$

$$\frac{V_O}{V_+} = 1 + \frac{R_2}{R_1}$$

The loop gain is

$$
\begin{aligned}
L(s) =& \frac{V_+}{V_X}\frac{V_X}{V_O}\frac{V_O}{V_+} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{R}{R+Z_C}\frac{1}{1+\left(\frac{1}{R+Z_C}+\frac{1}{Z_C}\right)R} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{1}{\frac{R+Z_C}{R}+1+\frac{R+Z_C}{Z_C}} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{Z_C}{R}+1+\frac{R}{Z_C}+1} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{1}{3+\frac{Z_C}{R}+\frac{R}{Z_C}}
\end{aligned}
$$

Now it is clear that it is the same problem with the same solution,

$$
\omega_0 = \frac{1}{RC} \qquad R_2 = 2R_1
$$

SS 13.18. For the circuit in Fig P13.18, break the loop at node X and find the loop gain (working backward for simplicity to find V_X in terms of V_O). For $R = 10\,\text{k}\Omega$, find C and R_f to obtain sinusoidal oscillations at 10 kHz.

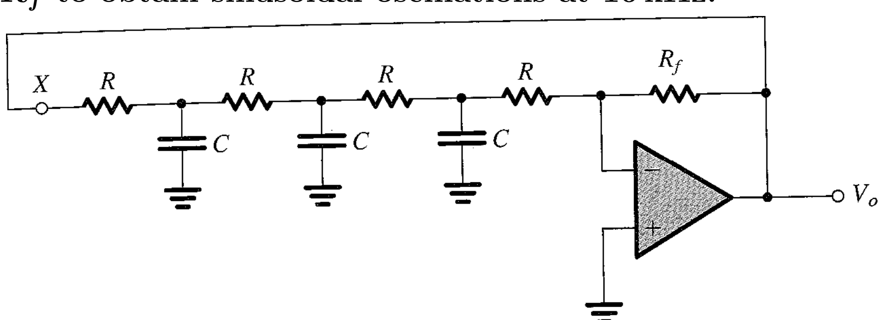

FIGURE P13.18

I am going to use the terminology of I_i is the current through the ith resistor R (positive toward the left direction) to the left of the inverting input. V_i is the voltage at the left side of the ith resistor, and I_{xi} is the current through the ith capacitor (toward ground) to the left of the inverting input. In that case we have

$$
I_1 = \frac{V_o}{R_f}
$$

$$
V_1 = -RI_1 = -V_o\frac{R}{R_f}
$$

$$
I_{x1} = \frac{V_1}{Z_C} = -V_o\frac{R}{Z_C R_f} = -\frac{V_o}{R_f}\frac{R}{Z_C}
$$

$$
I_2 = I_1 - I_{x1} = \frac{V_o}{R_f} + \frac{V_o}{R_f}\frac{R}{Z_C} = \frac{V_o}{R_f}\left(1+\frac{R}{Z_C}\right)
$$

$$V_2 = V_1 - I_2 R = -V_o \frac{R}{R_f} - V_o \frac{R}{R_f}\left(1 + \frac{R}{Z_C}\right) = -V_o \frac{R}{R_f}\left(2 + \frac{R}{Z_C}\right)$$

$$I_{2x} = \frac{V_2}{Z_C} = -\frac{V_o}{R_f}\frac{R}{Z_C}\left(2 + \frac{R}{Z_C}\right) = -\frac{V_o}{R_f}\left(2\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right)$$

$$I_3 = I_2 - I_{2x} = \frac{V_o}{R_f}\left(1 + \frac{R}{Z_C}\right) + \frac{V_o}{R_f}\left(2\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right) = \frac{V_o}{R_f}\left(1 + 3\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right)$$

$$\begin{aligned} V_3 =& V_2 - I_3 R = -V_o \frac{R}{R_f}\left(2 + \frac{R}{Z_C}\right) - V_o \frac{R}{R_f}\left(1 + 3\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right) \\ =& -V_o \frac{R}{R_f}\left(3 + 4\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right) \end{aligned}$$

$$I_{3x} = \frac{V_3}{Z_C} = -\frac{V_o}{R_f}\frac{R}{Z_C}\left(3 + 4\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right) = -\frac{V_o}{R_f}\left(3\frac{R}{Z_C} + 4\frac{R^2}{Z_C^2} + \frac{R^3}{Z_C^3}\right)$$

$$\begin{aligned} I_4 =& I_3 - I_{3x} = \frac{V_o}{R_f}\left(1 + 3\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right) + \frac{V_o}{R_f}\left(3\frac{R}{Z_C} + 4\frac{R^2}{Z_C^2} + \frac{R^3}{Z_C^3}\right) \\ =& \frac{V_o}{R_f}\left(1 + 6\frac{R}{Z_C} + 5\frac{R^2}{Z_C^2} + \frac{R^3}{Z_C^3}\right) \end{aligned}$$

$$\begin{aligned} V_X =& V_4 = V_3 - I_4 R \\ =& -V_o \frac{R}{R_f}\left(3 + 4\frac{R}{Z_C} + \frac{R^2}{Z_C^2}\right) - V_o \frac{R}{R_f}\left(1 + 6\frac{R}{Z_C} + 5\frac{R^2}{Z_C^2} + \frac{R^3}{Z_C^3}\right) \\ =& -V_o \frac{R}{R_f}\left(4 + 10\frac{R}{Z_C} + 6\frac{R^2}{Z_C^2} + \frac{R^3}{Z_C^3}\right) \end{aligned}$$

Now, inserting $Z_C = \frac{1}{j\omega C}$ we get

$$V_X = -V_o \frac{R}{R_f}\left(4 + 10j\omega RC - \omega^2 R^2 C^2 - j\omega^3 R^3 C^3\right)$$

Since $V_X = V_o$ the loop gain is

$$L(j\omega) = -\frac{R}{R_f}\left(4 + 10j\omega RC - \omega^2 R^2 C^2 - j\omega^3 R^3 C^3\right)$$

At resonance its phase must be zero, so its imaginary component must be zero.

$$10j\omega_0 RC - j\omega_0^3 R^3 C^3 = 0$$

$$10\omega_0 RC = \omega_0^3 R^3 C^3$$

$$\omega_0 = \frac{\sqrt{10}}{RC}$$

Next, the real component of the loop gain must be unity, so

$$1 = -\frac{R}{R_f}\left(4 - \omega_0^2 R^2 C^2\right) = -\frac{R}{R_f}\left(4 - 10\right) = 6\frac{R}{R_f}$$

$$\frac{R_f}{R} = 6$$

Now to find the value of C,

$$C = \frac{\sqrt{10}}{R\omega_0} = \frac{\sqrt{10}}{10 \times 10^3 \times 2 \times \pi \times 10 \times 10^3} = 5\,\mathrm{nF}$$

and the value of R_f,

$$R_f = 6R = 60\,\mathrm{k\Omega}$$