EE 322 Advanced Analog Electronics, Spring 2010
Homework #6 solution

SS 12.9. A third-order low-pass filter has transmission zeros at $\omega = 2\,\mathrm{rad/s}$ and $\omega = \infty$. Its natural modes are at $s = -1$ and $s = -0.5 \pm j0.8$. The DC gain is unity. Find $T(s)$.

We construct the transfer function by multiplying terms $s - z$ in the numerator (except for zeros at infinity), and terms $s - p$ in the denominator. Thus we get

$$T(s) = \frac{s-2}{(s+1)(s+0.5-j0.8)(s+0.5+j0.8)}$$

SS 12.17. Contrast the attenuation provided by a fifth-order Chebyshev filter at $\omega_s = 2\omega_p$ to that provided by a Butterworth filter of equal order. For both, $A_{\max} = 1\,\mathrm{dB}$. Sketch $|T|$ for both filters on the same axes.

For the Butterworth filter we have

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}} \qquad \epsilon^2 = 10^{\frac{A_{\max}}{10}} - 1$$

Now for $A_{\max} = 1\,\mathrm{dB}$ we get $\epsilon^2 = 0.2590$, and for $\omega = 2\omega_p$ we get

$$|T(j2\omega)| = \frac{1}{\sqrt{1+0.2590\times 2^{10}}} = 0.061$$

For the Chebyshev filter we have (for $\omega > \omega_p$)

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\cosh^2\left[N\cosh^{-1}\left(\frac{\omega}{\omega_p}\right)\right]}}$$

with the same expression for ϵ as before. Inserting we get

$$|T(j2\omega_p)| = \frac{1}{\sqrt{1+0.2590\cosh^2\left[5\cosh^{-1}2\right]}} = 0.103$$