EE 322 Advanced Analog Electronics, Spring 2010 Homework #8 solution

HH 9.5. Show that these choices of filter components actually give a loop gain of magnitude 1.0 at $f_2 = 2.0$ Hz.

The magnitude of the loop gain expression on page 649 is

$$|G_{\text{loop}}| = K_P \sqrt{\frac{1 + \omega^2 R_4^2 C_2^2}{1 + \omega^2 (R_3 C_2 + R_4 C_2)^2}} \frac{K_{\text{VCO}}}{\omega} K_{\text{div}}$$

Using $K_P = 1.59$, $\omega = 2\pi f_2 = 4\pi$, $R_4 = 330 \text{ k}\Omega$, $C_2 = 1 \,\mu\text{F}$, $R_3 = 4.3 \,\text{M}\Omega$, $K_{\text{VCO}} = 1.13 \times 10^5$, and $K_{\text{div}} = \frac{1}{1024}$ we get

$$|G_{\text{loop}}| = 1.59 \times \sqrt{\frac{1 + (4\pi \times 330 \times 10^3 \times 1 \times 10^{-6})^2}{1 + [4\pi \times (4.3 \times 10^6 \times 1 \times 10^{-6} + 330 \times 10^3 \times 1 \times 10^{-6})]^2}} \times \frac{1.13 \times 10^5}{4\pi} \times \frac{1}{1024}$$
$$= 1.02$$

Close enough given that we are specifying most quantities to two significant digits.

SS 12.49. Design the KHN circuit of Fig. 12.24(a) to realize a bandpass filter with a center frequency of 1 kHz and a 3 - dB bandwidth of 50 Hz. Use 10 nF capacitors. Give the complete circuit and specify all component values. What value of center-frequency gain is obtained?



The center frequency is $\omega = \frac{1}{RC}$. Given that C = 10 nF, we get $R = \frac{1}{\omega C} = \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-9}} = 15.9 \text{ k}\Omega$. We also know that $R_f = R_1$, so let's choose $R_f = R_1 = 100 \text{ k}\Omega$. Lastly we need to choose R_2 and R_3 which sets the bandwidth. First we find Q, which is the inverse of the fractional bandwidth,

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{10^3}{50} = 20$$

Next, choose $R_2 = 10 \text{ k}\Omega$, and we have and we have

$$R_3 = R_2 \left(2Q - 1 \right) = 39 \, R_2 = 390 \, \mathrm{k}\Omega$$

The K gain parameter is then

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{20} = 1.95$$

Finally, the gain function is

$$T_{\rm bp}(s) = -\frac{K\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

For $s = j\omega_0$ we get

$$T_{\rm bp}(j\omega_0) = -\frac{jK\omega_0^2}{-\omega_0^2 + j\frac{\omega_0^2}{Q} + \omega_0^2} = -KQ = -1.95 \times 20 = -39$$