EE 322 Advanced Analog Electronics, Spring 2010
Homework #9 solution

SS 12.48. It is required to design a third-order low-pass filter whose $|T|$ is equiripple in both the passband and the stopband (in the manner showin in Fig. 12.3, except that the response shown is for $N = 5$). The filter passband extends from $\omega = 0$ to $\omega = 1\,\text{rad/s}$ and the stopband edge is at $\omega = 1.2\,\text{rad/s}$. The following transfer function was obtained using filter design tables:

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$$

The actual filter is to have $\omega_p = 10^4\,\text{rad/s}$.

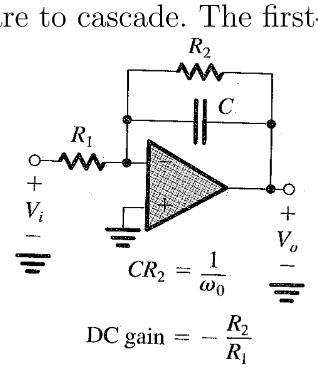
(a) Obtain the transfer function of the actual filter by replacing s by $s/10^4$.

(b) Realize this filter as the cascade connection of a first-order LP op-amp RC circuit of the type shown in Fig. 12.13(a) and a second-order LPN circuit of the type shown in Fig. 12.22(e). Each section is to have a dc gain of unity. Select appropriate component values. (Note: A filter with an equiripple response in both the passband and the stopband is known as an elliptic filter).

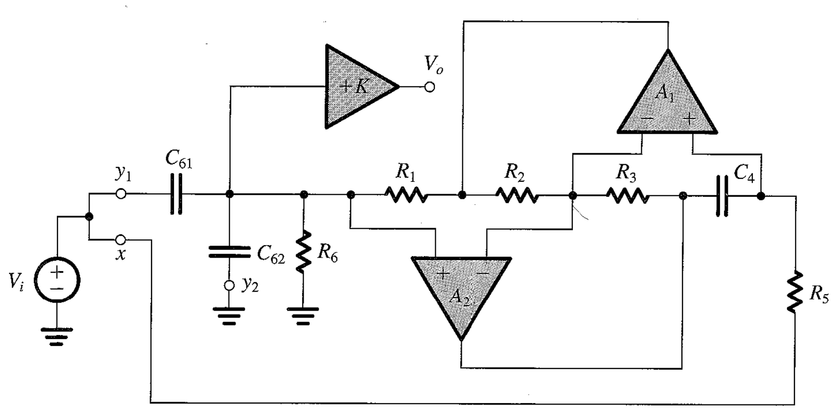
(a) Replacing s by $s/10^4$ we get

$$\begin{aligned} T(s) =& \frac{0.4508\left(\frac{s^2}{10^8} + 1.6996\right)}{\left(\frac{s}{10^4} + 0.7294\right)\left(\frac{s^2}{10^8} + \frac{s0.2786}{10^4} + 1.0504\right)} \\ =& \frac{4.508 \times 10^3\left(s^2 + 1.6996 \times 10^8\right)}{\left(s + 7.294 \times 10^3\right)\left(s^2 + s2.786 \times 10^3 + 1.0504 \times 10^8\right)} \end{aligned}$$

(b) Here are the two filters we are to cascade. The first-order LP filter



and the second-order LPN filter



(e) LPN, $\omega_n \geq \omega_0$

The DC gain of the third-order filter is 0.9778, but I will design each filter for unity DC gain as directed.

For the first-order filter we need

$$CR_2 = \frac{1}{\omega_0} \qquad\qquad \omega_0 = 7.294 \times 10^3\,\mathrm{s}^{-1}$$

Choose $C = 10\,\mathrm{nF}$ we get

$$R_2 = \frac{1}{\omega_0 C} = \frac{1}{7.294 \times 10^3 \times 10 \times 10^{-9}} = 13.7\,\mathrm{k}\Omega$$

For unity DC gain amplitude we need $R_1 = R_2 = 13.7\,\mathrm{k}\Omega$

Here are the properties of the 2nd-order LPN filter:

Low-pass notch (LPN)
Fig. 12.22(e)

$$T(s) = K\frac{C_{61}}{C_{61}+C_{62}} \times \frac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}} \qquad K = \text{DC gain}$$

$$\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2} \qquad C_{61}+C_{62} = C_6 = C$$

$$\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2} \qquad C_{61} = C(\omega_0/\omega_n)^2$$

$$Q = R_6\sqrt{\frac{C_{61}+C_{62}}{C_4}\frac{R_2}{R_1R_3R_5}} \qquad C_{62} = C - C_{61}$$

Comparing this to the transfer function let's begin by selecting $R_6 = 10\,\mathrm{k}\Omega$, and using the first-order term in the denominator. Then

$$C_{61} + C_{62} = \frac{1}{2.786 \times 10^3 \times R_6} = \frac{1}{2.786 \times 10^3 \times 10 \times 10^3} = 35.9\,\mathrm{nF}$$

which is a reasonable value, so we proceed. Next, if we look at the constant term in the denominator and choose $R_1 = R_2 = R_3 = R_5 = 10\,\mathrm{k}\Omega$, we get

$$
\begin{aligned}
C_4 =& \frac{R_2}{1.0504 \times 10^8 \times (C_{61} + C_{62})\, R_1 R_3 R_5} \\
=& \frac{10^4}{1.0504 \times 10^8 \times 35.9 \times 10^{-9} \times (10^4)^3} \\
=& 2.65\,\mathrm{nF}
\end{aligned}
$$

This is also an acceptable value, so we proceed. Next, we use the constant term in the parenthesis in numerator to determine C_{61}. We get

$$
\begin{aligned}
C_{61} =& \frac{R_2}{1.6996 \times 10^8 C_4 R_1 R_3 R_5} \\
=& \frac{10^4}{1.6996 \times 10^8 \times 2.65 \times 10^{-9} \times (10^4)^3} \\
=& 22.2\,\mathrm{nF}
\end{aligned}
$$

This is also a reasonable value, and fortunately less then $C_{61} + C_{62}$, so we can now determine $C_{62} = C_{61} + C_{62} - C_{61} = 35.9 - 22.2 = 13.7\,\mathrm{nF}$