EE 322 Advanced Analog Electronics, Spring 2010
Homework #10 solution

SS 8.73. An amplifier has a dc gain of 10^5 and poles at 10^5 Hz, 3.16×10^5 Hz, and 10^6 Hz. Find the value of β and the corresponding closed-loop gain, for which a phase margin of 45° is obtained.

A phase margin of 45° corresponds to a amplifier phase of −135°. Assuming that the poles are widely spaced, that phase occurs exactly at the second pole. At the second pole the gain of the amplifier is down by a factor of $1/sqrt2$ relative to what it would be from the first pole. From the first pole it will be down by a factor of 1/3.16. Total amount down is thus a factor of 0.226, and the amplifier gain at that point is

$$A\,(\phi = -135^\circ) = 0.226 \times 10^5 = 2.26 \times 10^4$$

At that point, the value of β is

$$\beta = \frac{1}{\alpha} = 3.91 \times 10^{-5}$$

The corresponding closed-loop gain is

$$G_{\text{in}} = \frac{A}{1 + A\beta} = \frac{2.26 \times 10^4 e^{-j135^\circ}}{1 + e^{-j135^\circ}} = \frac{2.26 \times 10^4 \times e^{-j135^\circ}}{1 - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}} = \frac{2.26 \times 10^4 \times e^{-j135^\circ}}{0.293 - j0.707}$$

The amplitude is

$$|G_{\text{in}}| = \frac{2.26 \times 10^4}{\sqrt{0.293^2 + 0.707^2}} = 2.95 \times 10^4$$

(Larger than the open-loop gain at that frequency).

SS 8.75. For the amplifier described by Fig. 8.37 and with frequency-independent feedback, what is the minimum closed-loop voltage gain that can be obtained for phase margins of 90° and 45°?

Here is the figure with the readings labeled by the two sets of wide dotted lines.

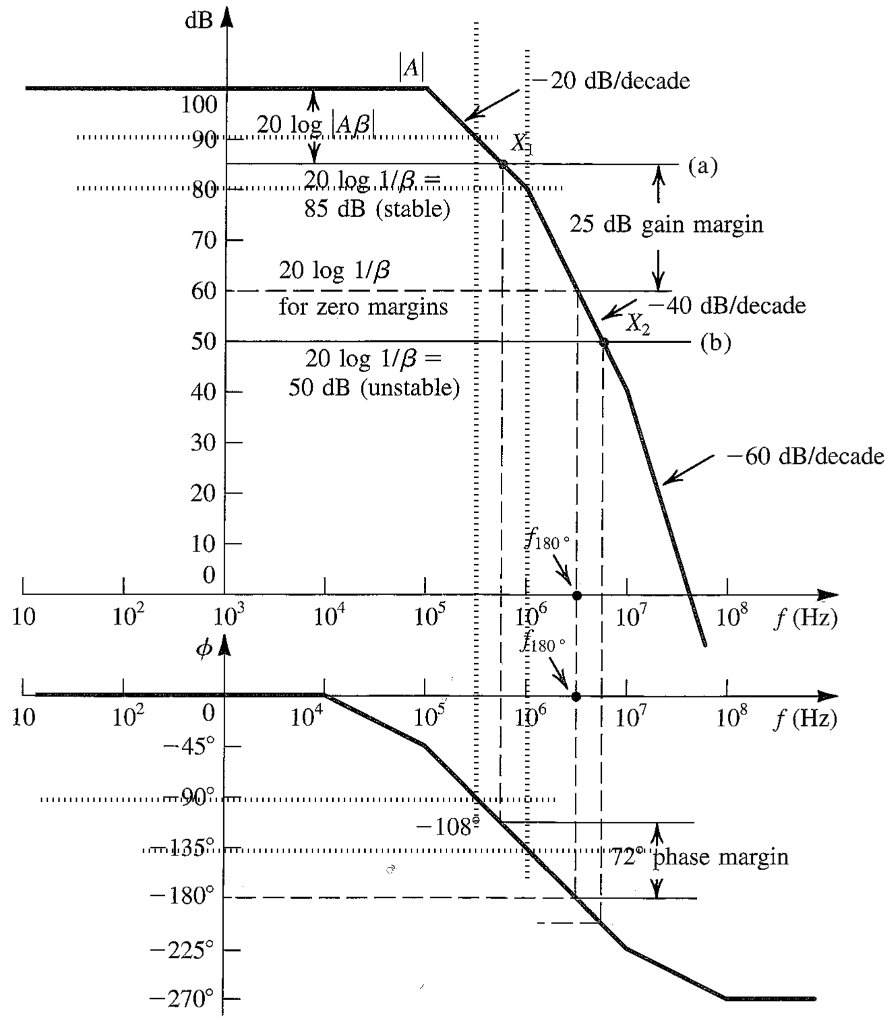

FIGURE 8.37 Stability analysis using Bode plot of $|A|$.

For a 90° phase marging the phase of A should be 90°. That corresponds to $|A| = 90\,\text{dB}$, and thus a minimum value of $1/\beta = 90\,\text{dB} = 31622.8$. The corresponding minimum closed loop gain is

$$G_{\min} = \frac{Ae^{-j90^\circ}}{1 + A\beta} = \frac{31622.8e^{-90^\circ}}{1 + e^{-j90^\circ}}$$

which has amplitude

$$|G_{\min}| = \frac{31622.8}{\sqrt{2}} = 2.2 \times 10^4$$

For a 45° phase margin the amplifier open-loop gain is $|A| = 80\,\text{dB} = 10^4$. That is also the value of the minimum value of $1/\beta$. The phase of the loop gain is now $-135°$. And the closed-loop gain is

$$G_{\min} = \frac{A}{1 + A\beta} = \frac{10^4 e^{-j135^\circ}}{1 + e^{-j135^\circ}} = \frac{10^4 e^{-j135^\circ}}{1 - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}} = \frac{10^4 e^{-j135^\circ}}{0.293 + j\frac{1}{\sqrt{2}}}$$

The minimum amplitude of the gain is then

$$|G_{\text{min}}| = \frac{10^4}{\sqrt{0.293^2 + \frac{1}{2}}} = 1.3 \times 10^4$$

SS 8.76. A multipole amplifer having a first pole at 2 MHz and a dc open-loop gain of 80 dB is to be compensated for closed-loop gains as low as unity by the introduction of a new dominant pole. At what frequency must the new pole be placed?

We want to move the second pole to unity open-loop gain, from a open-loop gain of 10^4 where it is now. To achieve this we need to insert a pole whose frequency is 10^4 times smaller than the existing pole. Its frequency should thus be

$$f_{\text{new}} = \frac{f_{\text{existing}}}{10^4} = \frac{2 \times 10^6}{10^4} = 200\,\text{Hz}$$