EE 322 Advanced Analog Electronics, Spring 2010
Homework #11 solution

SS 7.1. For a NMOS differential pair with a common-mode voltage v_{CM} applied, as shon in Fig. 7.2, let $V_{DD} = V_{SS} = 2.5\,\mathrm{V}$, $k'_n W/L = 3\,\mathrm{mA/V^2}$, $V_{tn} = 0.7\,\mathrm{V}$, $I = 0.2\,\mathrm{mA}$, $R_D = 5\,\mathrm{k\Omega}$, and neglect channel-length modulation.

- **(a) Find V_{OV} and V_{GS} for each transistor.**
- **(b) For $v_{CM} = 0$, find v_S, i_{D1}, i_{D2}, v_{D1}, and v_{D2}.**
- **(c) Repeat (b) for $v_{CM} = +1\,\mathrm{V}$.**
- **(d) Repeat (b) for $v_{CM} = -1\,\mathrm{V}$.**
- **(e) What is the highest value of v_{CM} for which Q_1 and Q_2 remain in saturation?**
- **(f) If the current source I requires a minimum voltage of $0.3\,\mathrm{V}$ to operate properly, what is the lowest value allowed for v_S and hence for v_{CM}?.**

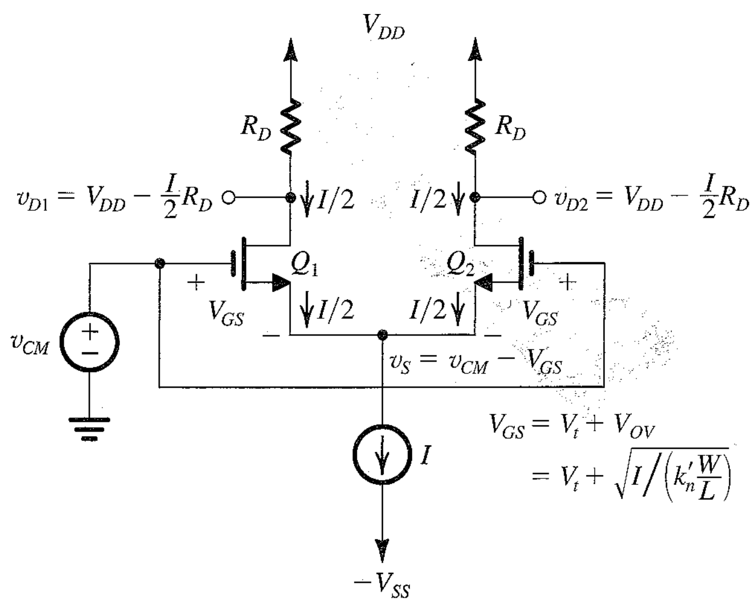

FIGURE 7.2 The MOS differential pair with a common-mode input voltage v_{CM}.

(a) Assuming saturation mode operation we get

$$\frac{I}{2} = k'_n \frac{W}{L} V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{I}{2}/k'_n\frac{W}{L}} = \sqrt{\frac{\frac{0.2}{2}}{3}} = 0.182\,\mathrm{V}$$

$$V_{GS} = V_{OV} + V_{tn} = 0.182 + 0.7 = 0.882\,\mathrm{V}$$

(b) In this case we have $v_G = 0\,\mathrm{V}$, and thus $v_S = v_G - V_{GS} = -0.882\,\mathrm{V}$. Also, $i_{D1} = i_{D2} = \frac{I}{2} = \frac{0.2}{2} = 0.1\,\mathrm{mA}$. And, $v_{D1} = v_{D2} = V_{DD} - R_D i_{D1} = 2.5 - 5 \times 10^3 \times 0.1 \times 10^{-3} = 2\,\mathrm{V}$.

(c) In this case we have $v_G = 1\,\mathrm{V}$, and $v_S = V_G - V_{GS} = 1 - 0.882 = 0.118\,\mathrm{V}$. $i_{D1} = i_{D2} = \frac{I}{2} = 0.1\,\mathrm{mA}$. $v_{D1} = v_{D2} = 2\,\mathrm{V}$. Since $v_D > v_G - V_t$, still in saturation mode.

(d) In this case we have $v_G = -1\,\mathrm{V}$, and $v_S = V_G - V_{GS} = -1 - 0.882 = -1.882\,\mathrm{V}$. Again, $i_{D1} = i_{D2} = \frac{I}{2} = 0.1\,\mathrm{mA}$. $v_{D1} = v_{D2} = 2\,\mathrm{V}$.

(e) The largest value for $v_G = v_{CM}$ is the one which make $v_G - V_t = v_D$. Given $v_D = 2\,\mathrm{V}$, we have $v_{G,\max} = v_D + V_t = 2 + 0.7 = 2.7\,\mathrm{V}$.

(f) In this case the minimum gate voltage is defined by $v_S = -V_{SS} + 0.3\,\mathrm{V}$, or $v_{G,\min} = -V_{SS} + 0.3\,\mathrm{V} + V_{GS} = -2.5 + 0.3 + 0.882 = -1.318\,\mathrm{V}$.

SS 7.3. For the differential amplifier specified in Problem 7.1, let $v_{G2} = 0$ and $v_{G1} = v_{id}$. Find the value of v_{id} that corresponds to each of the following situations:

(a) $i_{D1} = i_{D2} = 0.1\,\mathrm{mA}$.

(b) $i_{D1} = 0.15\,\mathrm{mA}$ and $i_{D2} = 0.05\,\mathrm{mA}$.

(c) $i_{D1} = 0.2\,\mathrm{mA}$ and $i_{D2} = 0$ (Q_2 just cuts off).

(d) $i_{D1} = 0.05\,\mathrm{mA}$ and $i_{D2} = 0.15\,\mathrm{mA}$.

(e) $i_{D1} = 0\,\mathrm{mA}$ (Q_1 just cuts off) and $i_{D2} = 0.2\,\mathrm{mA}$.

For each case find v_S, v_{D1}, v_{D2}, and $v_{D2} - v_{D1}$.

(a) This is simply $v_{id} = 0\,\mathrm{V}$, and we have the same values as in 7.1b, $v_S = -0.882\,\mathrm{V}$, $v_{D1} = v_{D2} = 2\,\mathrm{V}$, and $v_{D1} - v_{D2} = 0\,\mathrm{V}$.

(b) In this case we have

$$v_{D1} = V_{DD} - i_{D1} R_D = 2.5 - 5 \times 0.15 = 1.75\,\mathrm{V} \qquad v_{D2} = V_{DD} - i_{D2} R_D = 2.5 - 5 \times 0.05 = 2.25\,\mathrm{V}$$

$$v_{D1} - v_{D2} = 1.75 - 2.25 = -0.5\,\mathrm{V}$$

Since $v_{D2} - V_t = 2.25 - 0.7 = 1.55\,\mathrm{V} > v_{G2} = 0\,\mathrm{V}$, Q_2 is indeed in saturation and we can use

$$v_{OV2} = \sqrt{i_{D2}/k_n' \frac{W}{L}} = \sqrt{0.05/3} = 0.129\,\mathrm{V}$$

$$v_S = V_{G2} - V_{OV2} - V_t = 0 - 0.129 - 0.7 = -0.829\,\mathrm{V}$$

Assuming saturation we get

$$v_{OV1} = \sqrt{i_{D1}/k_n'\frac{W}{L}} = \sqrt{0.15/3} = 0.224\,\mathrm{V}$$

and

$$v_{id} = v_{G1} = v_S + V_t + v_{OV1} = -0.829 + 0.7 + 0.224 = 0.095\,\mathrm{V}$$

Verify saturation for Q_1 by noting that $v_{D1} = 1.75 > v_{G1} - V_t = 0.095 - 0.7 = -0.605\,\mathrm{V}$.

(c) In this case we have $v_{D1} = V_{DD} - i_{D1}R_D = 2.5 - 5 \times 0.2 = 1.5\,\mathrm{V}$, and $v_{D2} = V_{DD} = 2.5\mathrm{V}$. $v_{D1} - v_{D2} = 1.5 - 2.5 = -1, \mathrm{V}$. Also, $v_{GS2} = V_t$, so $v_S = -V_t = -0.7\,\mathrm{V}$. $v_{OV1} = \sqrt{i_{D1}/k_n'\frac{W}{L}} = \sqrt{0.2/3} = 0.258\,\mathrm{V}$. So $v_{id} = V_{G1} = v_S + V_t + v_{OV1} = -0.7 + 0.7 + 0.258 = 0.258\,\mathrm{V}$. Saturation mode verified for Q_1.

(d) In this case we have the reverse drain voltage of (b), $v_{D1} = 2.25\,\mathrm{V}$, $v_{D2} = 1.75\,\mathrm{V}$, and $v_{D1} - v_{D2} = 0.5\,\mathrm{V}$. Next, $v_{OV2} = \sqrt{i_{D2}/k_n'\frac{W}{L}} = \sqrt{0.15/3} = 0.224\,\mathrm{V}$, and $v_S = v_{G2} - v_{OV2} - V_t = 0 - 0.224 - 0.7 = -0.924\,\mathrm{V}$. Then $v_{OV1} = \sqrt{i_{D1}/k_n'\frac{W}{L}} = \sqrt{0.05/3} = 0.129$, so that $v_{id} = v_{G1} = v_S + V_t + v_{OV1} = -0.924 + 0.7 + 0.129 = -0.095\,\mathrm{V}$. And, clearly Q_1 is still in saturation mode.

(e) In this case we have the reverse drain voltages of (c), $v_{D1} = 2.5\,\mathrm{V}$, and $v_{D2} = 1.5\,\mathrm{V}$, and $v_{D1} - v_{D2} = 1\,\mathrm{V}$. Next $v_{OV2} = \sqrt{i_{D2}/k_n'\frac{W}{L}} = \sqrt{0.2/3} = 0.258\,\mathrm{V}$, and then $v_S = v_{G2} - V_t - v_{OV2} = 0 - 0.7 - 0.258 = -0.958\,\mathrm{V}$. And $v_{OV1} = 0\,\mathrm{V}$, so $v_{id} = v_{G1} = v_s + V_t = -0.958 + 0.7 = -0.258\,\mathrm{V}$.