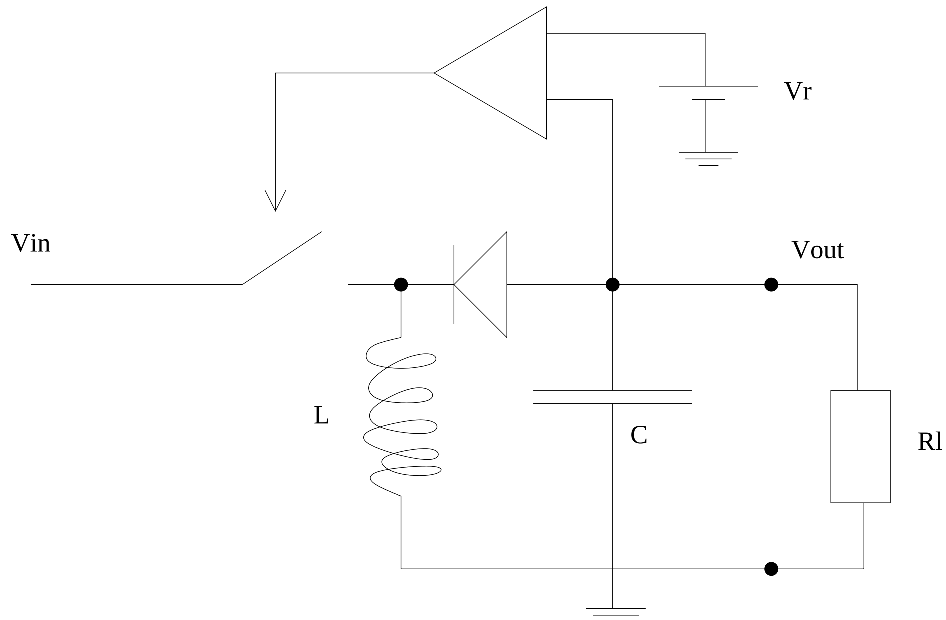
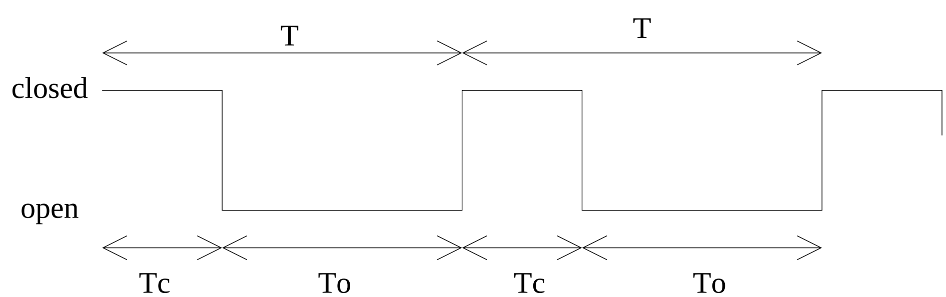
EE 322 Advanced Analog Electronics, Spring 2008
Handout 1
More on the Inverting Switching Regulator

The inverting switching regulator looks like this



Where we have included the the load resistor, R_L in the illustration. We wish to derive the relationship between input and output voltage as a function of switching frequency, switch duty cycle, and load resistance. We will assume that the circuit is operating in a steady state, meaning that the input voltage is constant, the output voltage is constant, and the current drawn by the load is constant. This also implies that the switch operates at a high enough frequency that the voltage across the capacitor stay constant during a period.

A period of length T consist of a length of time, T_C, where the switch is closed, and a length of time, T_o, where the switch is open.



When the switch is closed the voltage V_{in} is applied across the the inductor, and the current through the inductor rises as function of time, t, since the start of the closed period.

$$V_{\text{in}} = tL\frac{dI_L}{dt}$$

such that the amount of current flowing while the switch is closed is

$$I_L(t) = I_L(0) + t\frac{V_{\text{in}}}{L}$$

The amount of energy stored in the inductor at any given time is

$$U_L = \frac{1}{2}LI^2$$

The amount of energy added to the inductor while the switch is closed is thus

$$\begin{aligned}
\Delta U_{L,\text{closed}} =& \frac{L}{2}\left(I_L(T_C)^2 - I_L(0)^2\right) \\
=& \frac{L}{2}\left(\left(I_L(0) + T_C\frac{V_{\text{in}}}{L}\right)^2 - I_L(0)^2\right) \\
=& \frac{L}{2}\left(2I_L(0)T_C\frac{V_{\text{in}}}{L} + T_C^2\frac{V_{\text{in}}^2}{L^2}\right) \\
=& I_L(0)V_{\text{in}}T_C + \frac{V_{\text{in}}^2T_C^2}{2L}
\end{aligned}$$

It is interesting to note that if we already have a significant current flowing through the inductor at the beginning of a closed switch cycle, the amount of energy deposited in the inductor is larger than if there were only a small current or no current flowing in the inductor at the beginning of the cycle.

While the switch is open, T_o, all of this energy must be removed again in order to satisfy the steady state condition. We now have

$$V_{\text{out}} = L\frac{dI_L}{dt}$$

and thus during time since T_C,

$$I_L(t) = I_L(T_C) + t\frac{V_{\text{out}}}{L}$$

During steady state condition it must be true that the current through the inductor at the end of the open cycle equals the current through the conductor at the beginning of the closed cycle,

$$\begin{aligned}
I_L(0) =& I_L(T_C + T_O) \\
=& I_L(T_C) + T_O\frac{V_{\text{out}}}{L} \\
=& I_L(0) + T_C\frac{V_{\text{in}}}{L} + T_O\frac{V_{\text{out}}}{L}
\end{aligned}$$

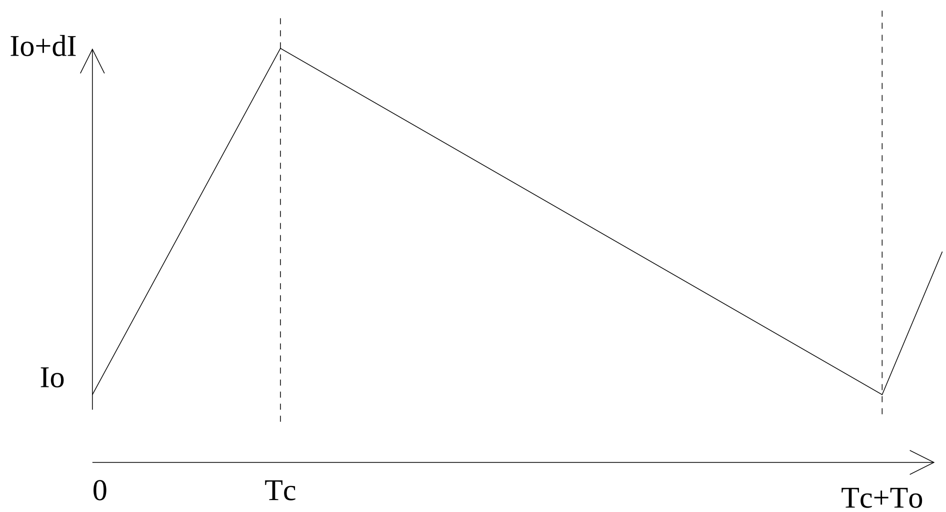
which gives us the result

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{T_C}{T_O}$$

In other words, the ratio of voltages is given by the ratios of the duration of time the switch is closed to the open.

Continuous and Discontinuous operation

We have made an implicit assumption in the above derivation. We have assumed that current will flow continuously through the inductor and still satisfy the condition that the current is the same at the beginning and at the end of the cycle. Let's examine that assumption more closely. We have no control over the rate of current change. It is fixed by V_{in} and V_{out}. In the figure below is an example of how the current changes during one period when $|V_{\text{out}}| < |V_{\text{in}}|$.



As you can see, there is always some current flowing greater than $I_L(0)$. Remember that the current can only flow in one direction. Because V_{in} is positive, and because there is a diode in the circuit, the current through the inductor can only flow in the positive direction towards ground. Even if the minimum current is zero, there is still a non-zero current flowing on average, because the time T_C is determined by the ratio of input and output voltages.

OK, let us examine what this minimum current is. I will examine it from the point of view of the load. The minimum load current is the amount of energy charged up in the inductor (when the beginning current is zero), divided by the length of the entire cycle, $T_C + T_O$, and divided by the (negative of the) output voltage.

$$I_{\text{min}} = \frac{V_{\text{in}}^2 T_C^2}{2L} \frac{1}{-V_{\text{out}}} \frac{1}{T_C + T_O}.$$

After some manipulations, including using $T = T_C + T_O$, and selecting the current direction to make it positive, we can get

$$I_{\text{min}} = -\frac{V_{\text{out}}}{2L} \frac{T}{\left(1 - \frac{V_{\text{out}}}{V_{\text{in}}}\right)^2}$$

If we use $f = (T_C + T_o)^{-1}$ we can write

$$I_{\min} = -\frac{V_{\text{out}}}{2L} f^{-1} \left(1 - \frac{V_{\text{out}}}{V_{\text{in}}}\right)^{-2}$$

In order for the load to draw this minimum current, the load resistance must be $R_L = -V_{\text{out}}/I_{\min}$. We can see that if we increase the frequency of the switching, the minimum current goes down, and that as we increase the magnitude of V_{out} the minimum current also goes down.

The next natural question is what happens if the load resistance is different from that which is required to draw the current $I_{\min}$. Remember that the current $I_{\min}$ is the output current which must be drawn by the load in order for the current through the inductor to be exactly zero at the beginning of each cycle.

Suppose the load resistor is smaller than this value. The load resistor will want to draw more current from the capacitor. This means that more energy must be transferred to the inductor each time the switch is closed. But the amount of time the switch is closed is fixed by the ratio $V_{\text{out}}/V_{\text{in}}$. However we remember that more energy can be transferred during the same time if we increase the initial current in the inductor.

If the current demand of the load is suddenly increased, the feedback circuit will respond by increasing T_C slightly such that at the end of each cycle the current is slightly larger than at the beginning, until the current is large enough that the required amount of energy can be transferred during each T_C, at which time T_C will be adjusted back such that the current is the same at the beginning and end of each cycle.

When the load is drawing enough current that there is always current flowing through the inductor, the regulator is operating in so-called **continuous mode**. This is the case in steady state operation if there is current flowing through the inductor at the beginning of the cycle.

Next let us imagine that the load suddenly decreases its current demand from the regulator (i.e. the effective load resistance increases). What happens is that the output voltage (voltage across the capacitor) will rise, and the feedback circuit will respond by decreasing the size of T_C slightly such that the current through the inductor is slightly smaller at the end of each cycle than it is at the beginning, until the current reaches a level corresponding to the current demand of the load. However, if the current demand of the load is less than $I_{\min}$, the circuit will enter into **discontinuous mode**. In discontinuous mode there is no current flowing in the inductor at the beginning of the the cycle, and the time the switch is closed is shorter than the time needed to charge the inductor such that it returns to zero current at the end of the cycle. What happens in this situation is that the inductor is discharged before the end of the cycle, and no current flows for the remainder of the cycle. The current does not reverse because of the diode. It is therefore possible to transfer a very small amount of energy by reducing T_C and having the inductor discharge for only part of T_O.

To summarize, for large current loads T_C is the same as for the minimum continuous current load, $I_{\min}$, and the current at the beginning of the cycle is adjusted up such that the required amount of energy is transferred each cycle. For small current loads less than $I_{\min}$ T_C is reduced such that energy is transferred during only part of the cycle. The three different scenarios are illustrated in the following figure. The solid curve represents continuous mode, the dashed curve discontinuous mode, and dotted curve the borderline case. Notice that the slope of the currents is the same in all cases.