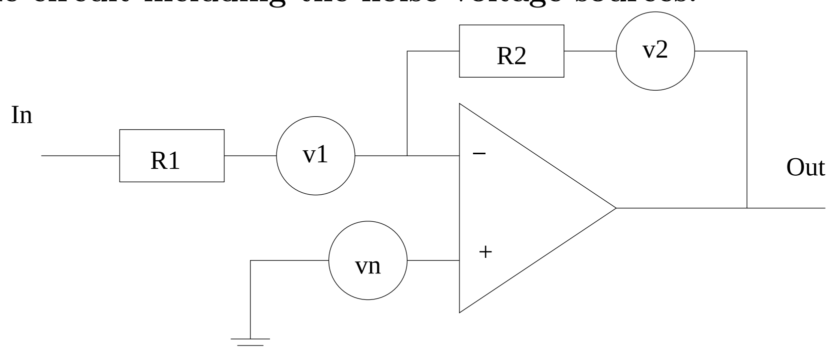
EE 322 Analog Electronics, Spring 2011
Exam 3 April 20, 2011
solution

Noise

Consider a normal inverting amplifier using an op-amp. Assume the usual noise on the two resistors and noise on one of the inputs of the op-amp (it doesn't matter which input you place the noise on).

1. **Draw the circuit including the noise voltage sources.**



2. **Write expressions for the mean-squared output signal voltage in terms of the mean-squared input signal voltage, and for the mean-squared output noise in terms of the resistances and the root power spectrum (e_n) of the op-amp, and the bandwidth B.**

$$V_{\text{out}} = -\frac{R_2}{R_1} V_{\text{in}}$$

so

$$\langle V_{\text{out}}^2 \rangle = \left(\frac{R_2}{R_1}\right)^2 \langle V_{\text{in}}^2 \rangle$$

and

$$v_{\text{out}} = -\frac{R_2}{R_1} v_1 + \left(\frac{R_2}{R_1} + 1\right) v_n + v_2$$

so that

$$\langle v_{\text{out}}^2 \rangle = \left[\left(\frac{R_2}{R_1}\right)^2 4kTR_1 + \left(\frac{R_2}{R_1} + 1\right)^2 e_n^2 + 4kTR_2\right] B$$

3. **Compute the value of the output mean-squared noise (in V^2) over a bandwidth of $1\,\mathbf{kHz}$ if the input resistance is $R_1 = 100\,\Omega$, feedback resistance is $R_2 = 10\,\mathbf{k}\Omega$, and the op-amp input noise is $e_n = 4\,\frac{\mu\mathbf{V}}{\sqrt{Hz}}$. Assume $k = 1.4\times10^{-23}\,\frac{\mathbf{J}}{\mathbf{K}}$ and $T = 293\,\mathbf{K}$.**

$$
\begin{aligned}
\langle v_{\text{out}}^2 \rangle =& 10^4 \times 1.4 \times 10^{-23} \times 293 \times 100 \times 10^3 \\
&+ \left(10^4 + 1\right) \times \left(4 \times 10^{-6}\right)^2 \times 10^3 \\
&+ 4 \times 1.4 \times 10^{-23} \times 293 \times 10^4 \times 10^3 \\
=& 1.6 \times 10^{-4}\,\mathrm{V}^2
\end{aligned}
$$

4. **What is the smallest detectable mean-squared input signal, assuming SNR $= 1$ is the detection limit.**

   The smallest detectable MS input is equal to the MS output noise divided by the square of the closed-loop gain. Thus

$$\langle V_{\text{in}}^2 \rangle_{\text{min}} = \frac{\langle v_{\text{out}}^2 \rangle}{10^4} = \frac{1.6 \times 10^{-4}}{10^4} = 1.6 \times 10^{-8}\,\mathrm{V}^2$$

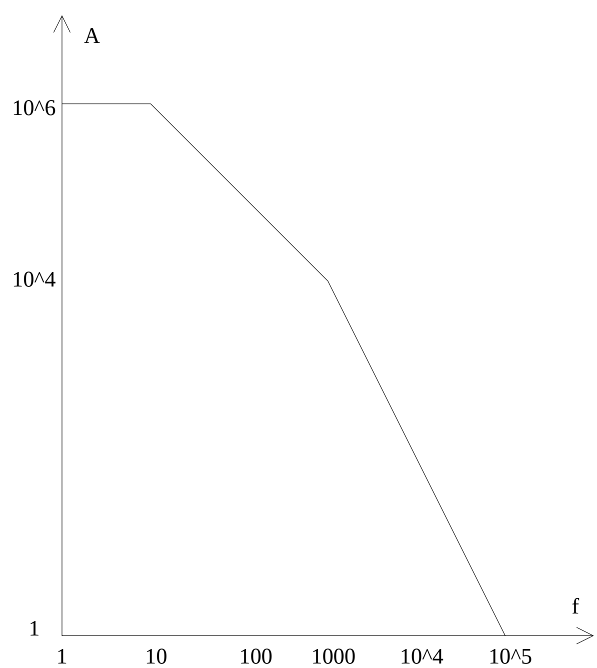
   or an amplitude:

$$\sqrt{\langle V_{\text{in}}^2 \rangle_{\text{min}}} = \sqrt{1.6 \times 10^{-8}}\,\mathrm{V} = 0.13\,\mathrm{mV}$$

Stability

An op-amp has DC open-loop gain of 10^6 and poles at $10\,\mathbf{Hz}$ and $10^3\,\mathbf{Hz}$.

5. **Plot the open-loop gain as a function of frequency. What is the unity gain bandwidth?**



The unity gain bandwidth is the frequency at which the gain drops to unity. That is $B = 10^5$ Hz.

6. **What value of β will make the amplifier stable at a 45° phase margin?**

   The second knee corresponds to 45° phase margin and is at $A = 10^4$. Thus the corresponding value of β to achieve unity loop gain at 45° phase margin is $\beta = 10^{-4}$.

7. **What bandwidth does this correspond to?**

   A quick estimate is that this corresponds to a bandwidth of 10^3 Hz, which is the frequency of the second pole. At that point the gain amplitude is

$$
\begin{aligned}
|G| =& \frac{A}{1+A\beta} = \frac{10^4}{|1+10^4 e^{-j135^\circ} \times 10^{-4}|} = \frac{10^4}{|1+e^{-j135^\circ}|} = \frac{10^4}{|1-\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}}|} \\
=& \frac{10^4}{\sqrt{0.29^2+0.5}} = 1.3 \times 10^4
\end{aligned}
$$

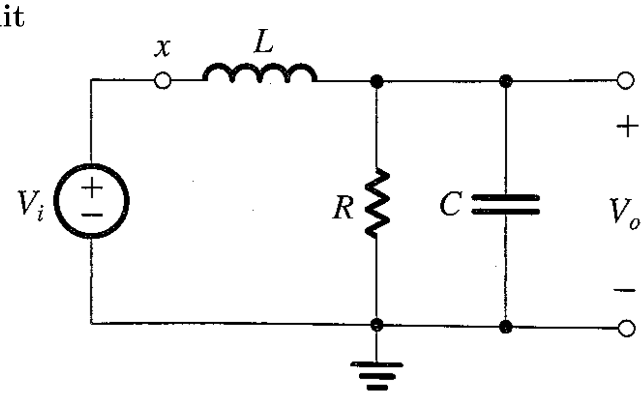
   If we want to work out the actual frequency (a little higher) at which the gain has dropped by 3dB compared with the DC gain that is more complicated.

8. **For both a inverting and non-inverting amplifier, what value of the closed loop gain does this value of β correspond to? Is this a minimum or maximum permitted gain for stability?**

   For a non-inverting amplifier we have $G = 1 + \frac{1}{\beta} = 10001$. For a inverting amplifier we have $G = -\frac{1}{\beta} = -10^4$. These are minimum gains for stability.

Second order filter

Consider this circuit



9. **Show that it is a second order low-pass filter.**

$$
\begin{aligned}
T =& \frac{R||\frac{1}{sC}}{sL + R||\frac{1}{sC}} = \frac{R\frac{1}{sC}}{\left(R+\frac{1}{sC}\right)sL + R\frac{1}{sC}} = \frac{R\frac{1}{sC}}{sLR + \frac{L}{C} + R\frac{1}{sC}} \\
=& \frac{1}{s^2LC + s\frac{L}{R} + 1} = \frac{\frac{1}{LC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}
\end{aligned}
$$

10. Assume $Q = 5$. What is the DC gain? What is the gain magnitude at $\omega = \omega_0$?

At $\omega = 0$ we can reduce to $T = 1$. At $\omega = \omega_0$ we get

$$T_{\omega=\omega_0} = \frac{\omega_0^2}{-\omega_0^2 + j\frac{\omega_0^2}{Q} + \omega_0^2} = \frac{\omega_0^2}{j\frac{\omega_0^2}{Q}} = -jQ$$

and

$$|T|_{\omega=\omega_0} = Q$$