EE 322 Analog Electronics, Spring 2011
Exam 4 May 10, 2011
solution

Rules: This is open book test. The exam will last 50 minutes and each numbered question counts equally toward your grade.

BJT differential pair

1. **Design a differential pair biased with a current source. It should have a differential input resistance of $R_{id} = 100\,\text{k}\Omega$ and a differential voltage gain of $v_{od}/v_{id} = 100$. Use $\beta = 100$ and $\pm 15\,\text{V}$ supplies.**

   The input differential resistance is

$$R_{id} = 2r_\pi = \frac{2\beta}{g_m} = \frac{2\beta V_T}{I_C} = \frac{4\beta V_T}{I}$$

   We get I from

$$I = \frac{4\beta V_T}{R_{id}} = \frac{4 \times 100 \times 25 \times 10^{-3}}{100 \times 10^3} = 0.1\,\text{mA}$$

   The differential voltage gain is

$$\frac{v_{od}}{v_{id}} = g_m R_C$$

   such that

$$R_C = \frac{1}{g_m}\frac{v_{od}}{v_{id}} = \frac{2V_T}{I}\frac{v_{od}}{v_{id}} = \frac{2 \times 25 \times 10^{-3}}{0.1 \times 10^{-3}} \times 100 = 50\,\text{k}\Omega$$

2. **If this circuit is used as a single-ended output with a load resistor $R_L = 10\,\text{k}\Omega$, what is the voltage gain v_2/v_{id}? What is the output bias voltage?**

   When the differential pair is used single-ended the gain is cut in half, which will produce a open-loop gain of 50. If we further add a load resistor we replace R_C by $R_C||R_L$ in the expression for the gain. Thus we have

$$\begin{aligned}\frac{v_2}{v_{id}} =& \frac{g_m R_C||R_L}{2} = \frac{I}{2V_T}\frac{R_C||R_L}{2} = \frac{IR_C||R_L}{4V_T}\\ =& \frac{0.1 \times 10^{-3} \times 50 \times 10^3||10 \times 10^3}{4 \times 25 \times 10^{-3}}\\ =& 8.3\end{aligned}$$

3. **What will change, and to what value, if you insert $100\,\Omega$ resistors in the two emitters of the BJT you designed in question 1?**

   The input resistance will be increased. Instead of being $(\beta+1)\,r_e$ it will be $(\beta+1)\,[r_e+R_E]$.

$$\begin{aligned} R_{id} =& 2\,(\beta+1)\left[R_E+\frac{\alpha V_T}{I_C}\right] = (\beta+1)\left[R_E+\frac{2\alpha V_T}{I}\right] \\ =& 2\times(100+1)\left[100+\frac{2\times 0.99\times 25\times 10^{-3}}{0.1\times 10^{-3}}\right] = 120\,\mathrm{k\Omega} \end{aligned}$$

Multi-stage amplifier

Consider an BJT differential pair similar that in question 1, biased with $I = 1\,\mathrm{mA}$, and $R_C = 10\,\mathrm{k\Omega}$. Assume $\pm 15\,\mathrm{V}$ supplies and $\beta = 100$.

4. **What is the open output differential to single-ended voltage gain? The output bias voltage?**

   The differential to single-ended gain is

$$\frac{v_2}{v_{id}} = \frac{g_m R_C}{2} = \frac{\alpha I R_C}{4V_T} = \frac{0.99\times 1\times 10^{-3}\times 10\times 10^3}{4\times 25\times 10^{-3}} = 100$$

   The output bias voltage is

$$V_2 = V_{CC} - \frac{I}{2}R_C = 15 - \frac{1\times 10^{-3}}{2}\times 10\times 10^3 = 10\,\mathrm{V}$$

5. **Design an output stage consisting of a PNP common-emitter and a NPN emitter follower. Bias it with a $1\,\mathrm{mA}$ current and zero output voltage. Make the emitter resistor on the output $10\,\mathrm{k\Omega}$.**

   The base of the PNP should be at $10\,\mathrm{V}$. This means that the emitter (at the top) should be at $10.7\,\mathrm{V}$. The emitter resistor is then

$$R_{E1} = \frac{V_{CC}-V_E}{I} = \frac{15-10.7}{1\times 10^{-3}} = 4.3\,\mathrm{k\Omega}$$

   The collector resistor should be such that the base is biased at $0.7\,\mathrm{V}$. Thus

$$V_B - V_{EE} = I R_C$$

$$R_C = \frac{V_B - V_{EE}}{I} = \frac{0.7+15}{1\times 10^{-3}} = 15.7\,\mathrm{k\Omega}$$

6. **What is the output resistance?**

$$R_o = R_E || \left(r_e + \frac{R_C}{\beta + 1} \right)$$

where $r_e = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$. And I_E is found from the emitter resistor,

$$I_E = \frac{-V_{EE}}{R_E} = \frac{15}{10 \times 10^3} = 1.5\,\text{mA}$$

and thus

$$r_e = 16.7\,\Omega$$

Putting it together we get

$$R_o = 10 \times 10^3 || \left(16.7 + \frac{15.7 \times 10^3}{101} \right) = 169\,\Omega$$

7. **What is the most effective way to further lower the output resistance? What simple change can you make to the circuit to roughly cut the output resistance in half?**

   The dominant resistance is the R_C. If we cut it in half we roughly cut the output resistance in half. But if we do so we also have to cut the emitter resistor in half on the PNP to keep the base of the PNP at the same voltage.

8. **Roughly how does this change affect the overall gain of the circuit.**

   Cutting R_E on the PNP in half will roughly cut the input resistance on that stage in half. Since the input resistance is already smaller than the output resistance on the differential pair, the effect is to roughly cut the gain of that coupling in half. The effect is to roughly cut the overall gain of the circuit in half.