EE 322 Advanced Electronics, Spring 2012
Exam 4
solution
Friday May 4, 2012

Rules: This is a closed-book exam. You may use only your brain, a calculator and pen/paper. Each numbered question counts equally toward your grade.

Note: The questions are designed to test your conceptual understanding, not your ability to do many pages of math. If you find yourself doing long calculations there is a high probability that you are doing something wrong.

Stability

Consider a amplifier with DC gain of 10^6, and poles at 10^4, 10^5, and 10^6 Hz.

1. **At what gain ($1/\beta$) is the amplifier stable with 45° phase-margin? Is that a minimum or a maximum stable gain?**

   That would be at the second pole. At the second pole $A = 10^6/10 = 10^5$. So $1/\beta = 10^5$. This is the minimum stable gain, the amplifier is stable for closed-loop gains larger than or equal to this.

2. **If you could remove one pole, which one would you remove to extend the stable gain range the most (still for 45° phase margin)?**

   The goal is to make the second pole appear at the smallest possible value for A. I would then remove the second pole which makes the new second pole appear at $A = 10^4$. The amplifier is now stable down to a gain of 10^4.

Relaxation oscillator

Consider a relaxation oscillator which uses a bistable element. The saturation levels of the op-amp are $L_\pm$, and the trigger levels are $V_\pm$.

3. **If $V_\pm = \frac{L_\pm}{2}$, derive a relationship between the oscillator period and the relaxation time-constant $\tau = RC$. HINT: Remember for this type of problem a good approach is: (1) how many volts away from the final value does the exponential decay begin, (2) how many volts away from the final value is the exponential decay stopped, (3) write an exponential that decays to that ratio and solve for the time in terms of the time-constant, τ.**

   In this case the relaxation will start at $\frac{3}{2}V_\pm$ and go toward zero, but be stopped at $\frac{1}{2}V_\pm$. It will be stopped at 1/3 of its starting value. We can thus write, for the half period

$$\frac{1}{3} = \exp\left(-\frac{T/2}{\tau}\right)$$

$$\frac{T}{2\tau} = \ln 3$$

$$T = 2\tau \ln 3$$

4. **Repeat, but for the case where $V_\pm = \alpha L_\pm$.**

In this case the relaxation starts at $V_\pm + \alpha V_\pm$, relaxes toward zero, but is stopped at $(1-\alpha)\,V_\pm$. So now we have

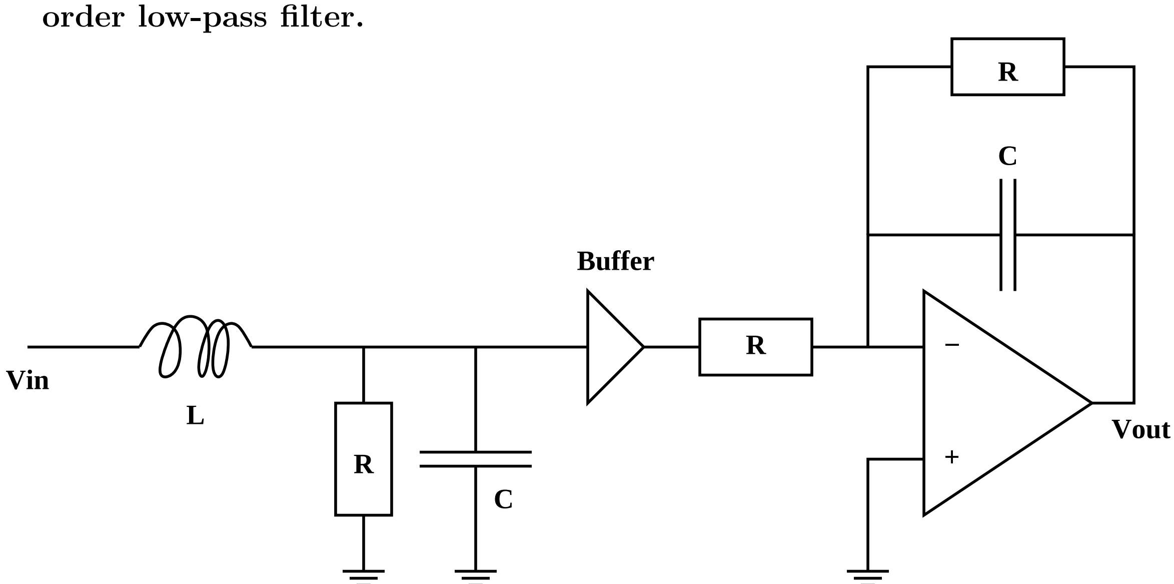
$$\frac{1-\alpha}{1+\alpha} = \exp\left(-\frac{T/2}{\tau}\right)$$

$$\frac{T}{2\tau} = \ln\frac{1+\alpha}{1-\alpha}$$

$$T = 2\tau\ln\frac{1+\alpha}{1-\alpha}$$

Active filters

5. **Show that this circuit is a third order LP filter by writing the transfer function as the product of a second-order and a first-order transfer function each in the standard fraction-of-polynomials form. Explain why it is a 3rd order low-pass filter.**



What goes into the non-inverting input is a second-order low-pass filter. The op-amp forms a integrator which is a first-order low-pass filter.

$$\frac{V_B}{V_{\rm in}} = \frac{R||\frac{1}{sC}}{sL + R||\frac{1}{sC}} = \frac{\frac{R}{sC}\frac{1}{R+\frac{1}{sC}}}{sL+\frac{R}{sC}\frac{1}{R+\frac{1}{sC}}} = \frac{\frac{R}{sC}}{sL\left(R+\frac{1}{sC}\right)+\frac{R}{sC}} = \frac{\frac{R}{sC}}{sLR+\frac{L}{C}+\frac{R}{sC}}$$

$$= \frac{R}{s^2LRC + sL + R} = \frac{1}{s^2LC + s\frac{L}{R}+1} = \frac{\frac{1}{LC}}{s^2 + s\frac{1}{RC}+\frac{1}{LC}}$$

For the first-order term we have

$$I = \frac{V_B}{R} = -\frac{V_{\text{out}}}{R||\frac{1}{sC}}$$

$$\frac{V_{\text{out}}}{V_B} = \frac{R||\frac{1}{sC}}{R} = \frac{1}{R}\frac{\frac{R}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+sRC}$$

For the whole filter we then have

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\frac{1}{LC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}} \frac{1}{1+sRC}$$

6. What is Q for the second-order component?

$$\frac{1}{RC} = \frac{\omega_0}{Q} = \frac{1}{Q\sqrt{LC}}$$

$$Q = \frac{RC}{\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$