EE 322 Advanced Electronics, Spring 2012
Homework #3 solution

1. **Plot, to scale, the voltage across, and current through the inductor of a step-down switching voltage regulator with $V_{in} = 10\,\text{V}$ and $V_{out} = 3\,\text{V}$**

   Here is a step-down regulator

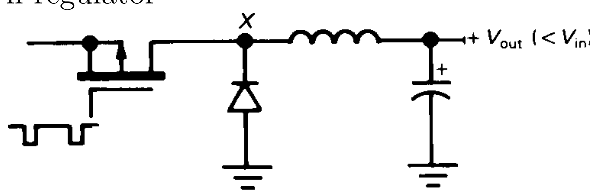


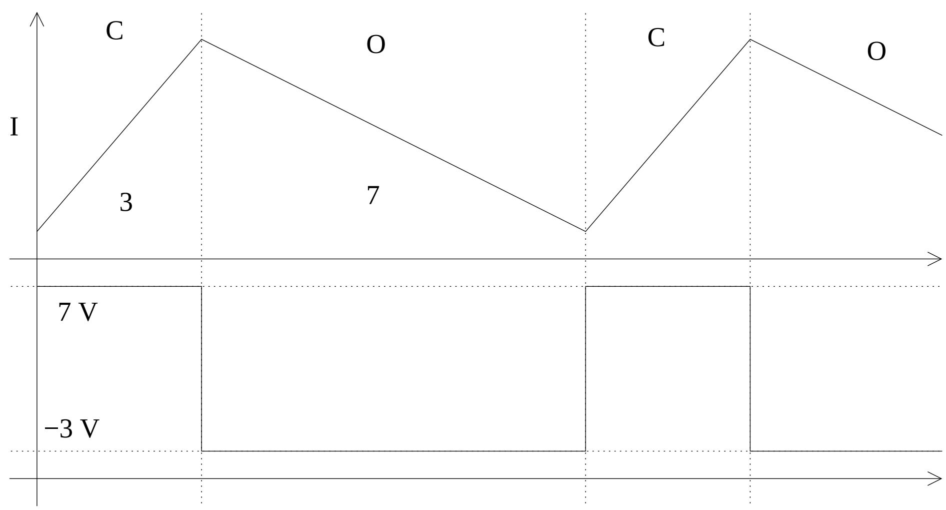
Figure 6.40. Step-down switcher.

   First the voltage across the inductor. When the switch is closed it is $V_L = V_{in} - V_{out} = 10 - 3 = 7\,\text{V}$. When the switch is open it is $V_L = -V_{out} = -3\,\text{V}$.

   First find the relationship between T_O and T_C.

$$T_C\left(V_{in} - V_{out}\right) - T_O V_{out} = 0$$

$$\frac{T_C}{T_O} = \frac{V_{out}}{V_{in} - V_{out}} = \frac{3}{10-3} = \frac{3}{7} \approx 0.43$$



2. **If the cycle-averaged input current is 1 mA, what is the cycle-averaged output current?**

   A switching power supply is lossless. Thus

$$V_{in} I_{in} = V_{out} I_{out}$$

   and thus

$$I_{\text{out}} = I_{\text{in}} \frac{V_{\text{in}}}{V_{\text{out}}} = 1 \times \frac{10}{3} = 3.3\,\text{mA}$$

3. **If the cycle frequency is 10 kHz what are T_C and T_O in continuous mode?**

   We have $T_C + T_O = T$, and $T = 0.1\,\text{ms}$. Then

$$T_C + \frac{7}{3} T_C = T$$

$$T_C = \frac{T}{1 + \frac{7}{3}} = \frac{0.1}{3.33} = 0.03\,\text{ms} = 30\,\mu\text{s}$$

$$T_O = T - T_C = 0.1 - 0.03 = 0.07\,\text{ms} = 70\,\mu\text{s}$$

4. **What size inductor current will cause the current to vary by 100 mA during a cycle?**

   Begin with

$$V = L \frac{dI}{dt}$$

   which we can convert to

$$V \Delta T = L \Delta I$$

   or

$$L = \frac{V \Delta T}{\Delta I}$$

   Apply this to the closed-time $V = 7\,\text{V}$, $\Delta T = 0.03\,\text{ms}$, and $\Delta I = 100\,\text{mA}$.

$$L = \frac{7 \times 0.03 \times 10^{-3}}{100 \times 10^{-3}} = 0.0021\,\text{H} = 2.1\,\text{mH}$$

5. **To obtain continuous mode operation to a load current of 10 mA, what size inductor is required? Is this a minimum or maximum inductor size for that operation?**

   Because the inductor is connected continuously to the output, the mean inductor current must equal the load current, and thus at the edge between continuous and discontinuous mode the minimum inductor current is 0 and the maximum is 20 mA. Then we use the same equation as before

$$L = \frac{V \Delta T}{\Delta I}$$

but now with $\Delta I = 20\,\text{mA}$.

$$L = \frac{7 \times 0.03 \times 10^{-3}}{20 \times 10^{-3}} = 0.0105\,\text{H} = 10.5\,\text{mH}$$

Now on the question of whether this is a minimum or maximum inductor size for continuous mode operation at 10 mA note that smaller inductors means larger current variation for same voltage, and larger inductors result in smaller current variation for same voltage. Since we want a small current variation we must have a large inductor. This is a minnimum inductor size.

6. **For that same inductor what is T_C which results in a 1 mA current?**

   The amount of charge transferred per cycle is $\Delta Q = T\, I_{\text{out}}$, and is also the integratl under the inductor current curve, which in turn can be written as

$$\Delta Q = \frac{T_Q I_{\max}}{2}$$

   where T_Q is the amount of time that current is flowing, and we found in a earlier question that it is $T_Q = 3.33\,T_C$.

   And $I_{\max}$ can be written as

$$I_{\max} = V\frac{T_C}{L}$$

   Inserting we get

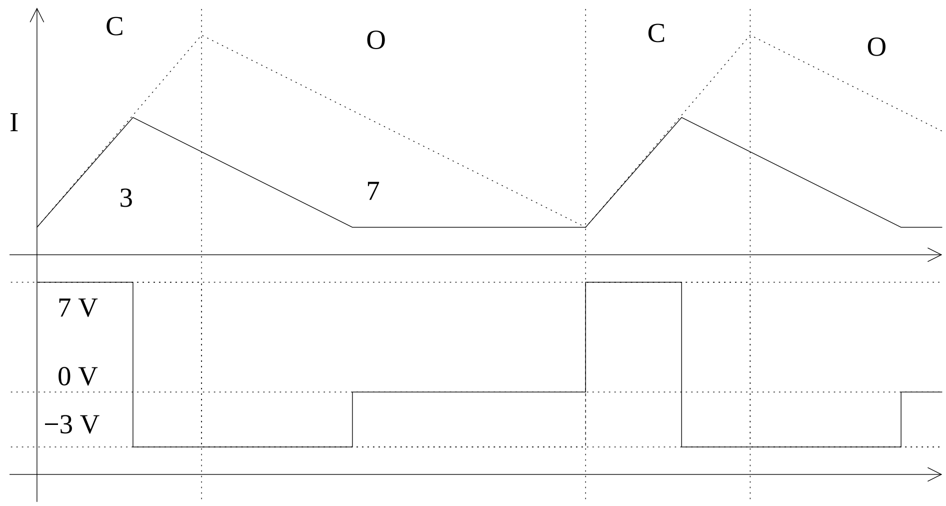
$$T\, I_{\text{out}} = \frac{3.33 V T_c^2}{2L}$$

   or

$$T_C = \sqrt{\frac{2LTI_{\text{out}}}{3.33}} = \sqrt{\frac{2 \times 10.5 \times 10^{-3} \times 0.1 \times 10^{-3} \times 1 \times 10^{-3}}{3.33}} = 2.5 \times 10^{-5}\,\text{s} = 25\,\mu\text{s}$$

7. **Plot, to scale, inductor voltage and inductor current which results in one third the output current of the minimum continuous mode output current.**

   We saw in the previous question that the output current varies with the square root of T_C. Thus to obtain one third the output current we need to divide T_C by a factor of $\sqrt{3} \approx 1.73$. That results in $T_c = 30\,\mu\text{s}/\sqrt{3} = 17\,\mu\text{s}$. We can add that to the plot from question 1.



C
O
C
O
I
3
7
7 V
0 V
−3 V