EE 322 Advanced Electronics, Spring 2012
Homework #3 solution

1. **SS 8.1. A negative-feedback amplifier has a closed-loop gain $A_f = 100$ and a open-loop gain $A = 10^5$. What is the feedback factor β? If a manufacturing error results in a reduction of A to 10^3, what closed-loop gain results? What is the percentage change in A_f corresponding to this factor of 100 reduction of A?**

   Start with

   $$A_f = \frac{A}{1 + A\beta}$$

   and get

   $$A_f + A_f A \beta = A$$

   $$\beta = \frac{A - A_f}{A_f A} = \frac{10^5 - 100}{100 \times 10^5} = 0.01$$

   If a manufacturing error results in $A = 10^3$ then we get

   $$A_f = \frac{A}{1 + A\beta} = \frac{10^3}{1 + 10^3 \times 0.01} = 91$$

   The percentage reduction is 9% due to the factor 100 reduction in gain.

2. **SS 8.4. The noninverting buffer op-amp configuration shown in Figure P8.4 provides a direct implementation of the feedback loop of Fig 8.1. Assuming that the op-amp has infinite input resistance and zero output resistance, what is β? If $A = 100$, what is the closed-loop gain? What is the amount of feedback (in dB)? For $V_s = 1\,\text{V}$, find V_o and V_i. If A decreases by 10%, what is the corresponding decrease in A_f?**
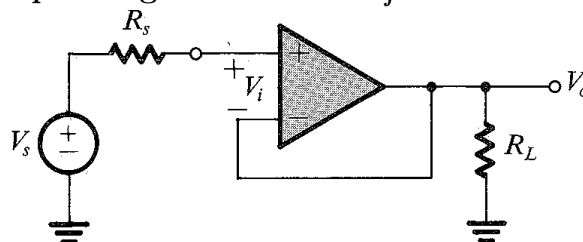

   FIGURE P8.4

   What is β?

   $$\beta = 1$$

If $A = 100$, what is the closed-loop gain?

$$A_f = \frac{A}{1 + A\beta} = \frac{100}{1 + 100} = 0.99$$

What is the amount of feedback (in dB)?

$$20 \log (1 + A\beta) = 20 \log(1 + 100 \times 1) = 40 \,\text{dB}$$

For $V_s = 1\,\text{V}$, find V_o and V_i.

$$V_o = A_f V_s = 0.99 \times 1 = 0.99 \,\text{V}$$

$$V_i = \frac{V_o}{A} = \frac{0.99}{100} = 0.0099 \,\text{V}$$

If A is decreased by 10% what is the decrease in A_f? Do this numerically, using $A = 90$

$$A_f = \frac{A}{1 + A\beta} = \frac{90}{1 + 90 \times 1} = 0.989$$

The decrease is approximately 0.1%.

3. **SS 8.8. A newly constructed feedback amplifier undergoes a performance test with the following results: With the feedback connection removed, a source signal of 2 mV is required to provide a 10 V output to the load; with the feedback connected, a 10 V output requires a 200 mV source signal. For this amplifier, identify values of A, β, $A\beta$, the closed-loop gain, and the amount of feedback (in dB).**

Withe the feedback removed we have

$$A = \frac{10 \,\text{V}}{2 \,\text{mV}} = 5000$$

The closed-loop gain is

$$A_f = \frac{10 \,\text{V}}{200 \,\text{mV}} = 50$$

$A\beta$ can then be found from

$$A_f = \frac{A}{1 + A\beta}$$

$$A\beta = \frac{A - A_f}{A_f} = \frac{5000 - 50}{50} = 99$$

which makes β

$$\beta = \frac{A\beta}{A} = \frac{99}{5000} = 0.020$$

The amount of feedback in dB

$$1 + A\beta = 20 \log(1 + A\beta) = 20 \log(1 + 5000 \times 0.02) = 40 \, \text{dB}$$

4. **SS 8.10. It is required to design an amplifier with a gain of 100 that is accurate to within $\pm 1\%$. You have available amplifier stages with a gain of 1000 that is accurate to within $\pm 30\%$. Provide a design that uses a number of these gain stages in cascade, with each stage employing negative feedback of an appropriate amount. Obviously, your design should use the lowest possible number of stages while meeting specification.**

When cascading N amplifiers with gain, B, and uncertainty σ_B, the combined gain will be

$$C = NB$$

and the combined uncertainty will be

$$\sigma_C = \sqrt{N} \sigma_B$$

The relative uncertainy on C is then

$$\frac{\sigma_C}{C} = \frac{\sqrt{N} \sigma_B}{C} = \frac{\sqrt{N} \sigma_B}{NB} = \frac{1}{\sqrt{N}} \frac{\sigma_B}{B}$$

Next we need to find the relationship beween the relative uncertainty of the individual feedback amplifiers and the relative uncertainty of the open-loop amplifier. We have

$$\frac{dA_f}{dA} = \frac{d}{dA} \frac{A}{1 + A\beta} = \frac{1 + A\beta - A\beta}{(1 + A\beta)^2} = \frac{1}{(1 + A\beta)^2}$$

which shows that

$$\sigma_B = \frac{\sigma_A}{(1 + A\beta)^2} = \frac{\sigma_A}{A} \frac{A}{1 + A\beta} \frac{1}{1 + A\beta} = \frac{\sigma_A}{A} \frac{B}{1 + A\beta}$$

$$\frac{\sigma_B}{B} = \frac{\sigma_A}{A} \frac{1}{1 + A\beta}$$

Now we just need to do something about the $\frac{1}{1+A\beta}$ term. Let's assume that $A\beta \gg 1$ which is most likely reasonable. Then we can write

$$\frac{\sigma_B}{B} = \frac{\sigma_A}{A}\frac{1}{A\beta}$$

and remember that $\beta = \frac{1}{B}$ under that assumption

$$\frac{\sigma_B}{B} = \frac{\sigma_A}{A}\frac{B}{A} = \frac{\sigma_A}{A}\frac{C}{NA}$$

Now substitute this in for σ_B/B above and we get

$$\frac{\sigma_C}{C} = \frac{1}{\sqrt{N}}\frac{\sigma_B}{B} = \frac{1}{\sqrt{N}}\frac{\sigma_A}{A}\frac{C}{NA} = \frac{C}{N^{\frac{3}{2}}A}\frac{\sigma_A}{A}$$

Now finally we can find N

$$N = \left(\frac{\frac{\sigma_A}{A}}{\frac{\sigma_C}{C}}\frac{C}{A}\right)^{\frac{2}{3}}$$

Inserting values we get

$$N = \left(\frac{0.3}{0.01}\frac{100}{1000}\right)^{\frac{2}{3}} = 2.08$$

So it appears we will need 3, although 2 will bring it very close.