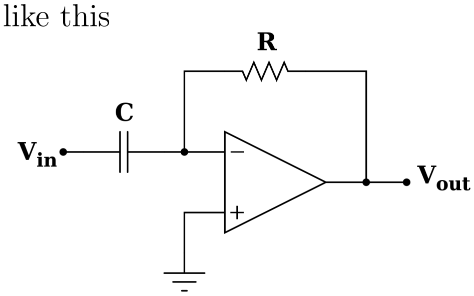
Homework 6, due March 19, 2012
Solution

1. **SS 8.74. A two-pole amplifier for which $A_o = 10^3$ and having poles at 1 MHz and 10 MHz is to be connected as a differentiator. On the basis of the rate-of-closure rule, what is the smallest differentiator time constant for which operation is stable? What are the corresponding gain and phase margins?**

   A differentiator looks like this



For this circuit β is the voltage division ratio between the resistor and the capacitor,

$$\beta = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

and

$$\frac{1}{\beta} = 1 + j\omega RC \approx j\omega RC \approx j\omega\tau$$

This has a slope of +20 dB/decade. Thus following the rate-of-closure rule, $1/\beta$ must cross A where A has zero slope. In other words

$$`\omega_1\tau \geq A(\omega_1)$$

where ω_1 is the frequency of the first pole.

$$\tau \geq \frac{A_0}{\omega_1} = \frac{10^3}{2 \times \pi \times 10^6\sqrt{2}} = 1.6 \times 10^{-4}\,\mathrm{s}$$

The phase at that point is $-135°$, $-90°$ from β and $-45°$ from the first pole. So the phase margin is $45°$.

The gain margin is the amount that $\frac{1}{\beta}$ is above A at the point where the phase of $A\beta$ is $-180°$. That point is halfway between the two poles, where β is up 10 dB and A is down 10 dB. So the gain margin is approximately 20 dB