EE 322 Advanced Analog Electronics, Spring 2010
Homework #7 solution

SS 13.5. In a particular oscillator characterized by the structure of Fig 13.1, the frequency-selective network exhibits a loss of 20 dB and a phase shift of 180° at ω_0. What is the minimum gain and the phase shift that the amplifier must havfe for oscillation to begin?

The amplifier must have a minimum gain of 20 dB (a factor of 10), and a phase shift of 180°.

SS 13.13. For the circuit in Fig P13.13 find $L(s)$, $L(j\omega)$, the frequency for zero phase loop phase, and R_2/R_1 for oscillation.

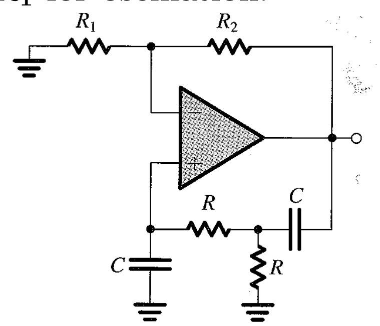

FIGURE P13.13

We have

$$\frac{V_+}{V_X} = \frac{Z_C}{Z_C + R}$$

$$\frac{V_X}{V_O} = \frac{(Z_C + R)\,||R}{(Z_C + R)\,||R + Z_C} = \frac{\frac{1}{\frac{1}{Z_C+R}+\frac{1}{R}}}{\frac{1}{\frac{1}{Z_C+R}+\frac{1}{R}} + Z_C} = \frac{1}{1 + \left(\frac{1}{Z_C+R} + \frac{1}{R}\right) Z_C}$$

$$\frac{V_O}{V_+} = 1 + \frac{R_2}{R_1}$$

The loop gain is the product of these three factors,

$$\begin{aligned}
L(s) =& \frac{V_X}{V_O}\frac{V_+}{V_O}\frac{V_O}{V_+} = \left(1 + \frac{R_2}{R_1}\right) \frac{Z_C}{Z_C + R} \frac{1}{1 + \left(\frac{1}{Z_C+R} + \frac{1}{R}\right) Z_C} \\
=& \left(1 + \frac{R_2}{R_1}\right) \frac{Z_C}{Z_C + R + \left(1 + \frac{Z_C+R}{R}\right) Z_C} \\
=& \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \frac{R}{Z_C} + 1 + \frac{Z_C+R}{R}} \\
=& \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3 + \frac{R}{Z_C} + \frac{Z_C}{R}} \\
=& \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3 + sRC + \frac{1}{sRC}}
\end{aligned}$$

and thus

$$L(j\omega) = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3 + j\omega RC + \frac{1}{j\omega RC}}$$

We want the complex portion to be zero to get zero phase, so we have

$$j\omega_0 RC = -\frac{1}{j\omega_0 RC} = \frac{j}{\omega_0 RC}$$

or

$$\omega_0 = \frac{1}{RC}$$

At that frequency we have

$$1 = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{3}$$

or

$$R_2 = 2R_1$$

SS 13.14. Repeat problem 13.13 for the circuit shown in Figure P13.14.

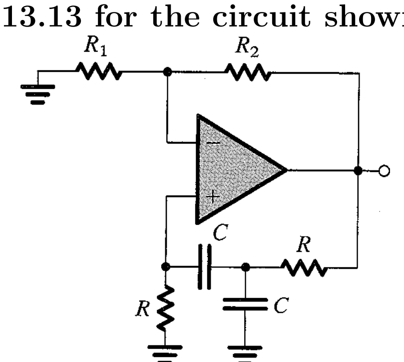

FIGURE P13.14

$$\frac{V_+}{V_X} = \frac{R}{R + Z_C}$$

$$\frac{V_X}{V_O} = \frac{(R + Z_C)\,||Z_C}{(R + Z_C)\,||Z_C + R} = \frac{\frac{1}{\frac{1}{R+Z_C} + \frac{1}{Z_C}}}{\frac{1}{\frac{1}{R+Z_C} + \frac{1}{Z_C}} + R} = \frac{1}{1 + \left(\frac{1}{R+Z_C} + \frac{1}{Z_C}\right) R}$$

$$\frac{V_O}{V_+} = 1 + \frac{R_2}{R_1}$$

The loop gain is

$$
\begin{aligned}
L(s) =& \frac{V_+}{V_X}\frac{V_X}{V_O}\frac{V_O}{V_+} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{R}{R+Z_C}\frac{1}{1+\left(\frac{1}{R+Z_C}+\frac{1}{Z_C}\right)R} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{1}{\frac{R+Z_C}{R}+1+\frac{R+Z_C}{Z_C}} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\frac{Z_C}{R}+1+\frac{R}{Z_C}+1} \\
=& \left(1+\frac{R_2}{R_1}\right)\frac{1}{3+\frac{Z_C}{R}+\frac{R}{Z_C}}
\end{aligned}
$$

Now it is clear that it is the same problem with the same solution,

$$
\omega_0 = \frac{1}{RC} \qquad R_2 = 2R_1
$$