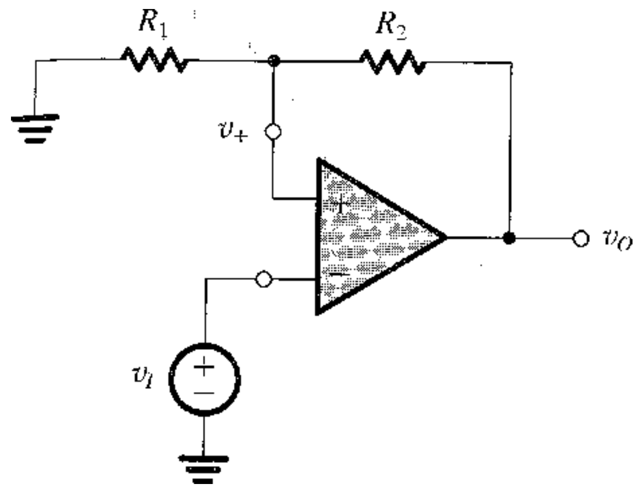
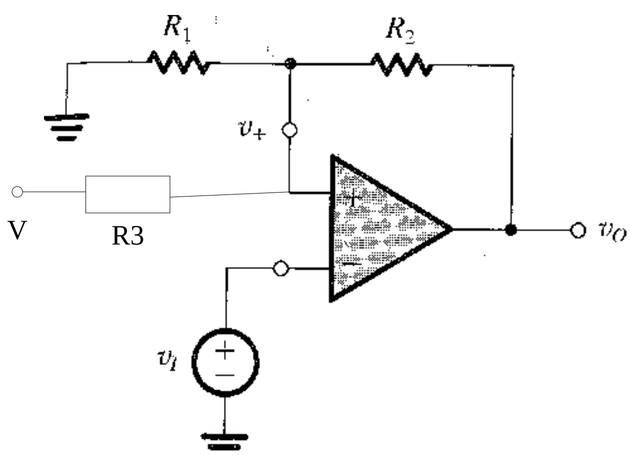
EE 322 Advanced Analog Electronics, Spring 2012
Homework #8 solution

SS 13.24 Consider the bistable circuit of Figure 13.19(a) with the op-amp's positive-input terminal connected to a positive voltage source V through a resistor R_3



The circuit we will consider looks like this



(a) Derive expressions for the threshold voltage V_{TL} and V_{TH} as a function of the op-amp's saturation levels L_+ and L_-, R_1, R_2, R_3, and V.

To solve this we consider that the output can be either L_+ or L_-. Then it is a simple matter of computing V_+ for those two conditions. For the condition when $V_o = L_+$, we are computing V_{TH}, whereas when $V_o = L_-$, we are computing V_{TL}. First the generic expression for V_+. We use current balance,

$$\frac{V - V_+}{R_3} + \frac{0 - V_+}{R_1} + \frac{V_o - V_+}{R_2} = 0$$

$$\frac{V}{R_3} + \frac{V_o}{R_2} = V_+ \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$
\begin{aligned}
V_+ =& \frac{\frac{V}{R_3} + \frac{V_o}{R_2}}{\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2}} \\
=& \frac{V R_1 R_2 + V_o R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}
\end{aligned}
$$

Now as mentioned before, for V_{TL} we insert $V_o = L_-$,

$$V_{TL} = \frac{V R_1 R_2 + L_- R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

and to find V_{TH} we insert $V_o = L_+$,

$$V_{TH} = \frac{V R_1 R_2 + L_+ R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

(b) Let $L_+ = L_- = 13\,\mathbf{V}$, $V = 15\,\mathbf{V}$, and $R_1 = 10\,\mathbf{k\Omega}$. Find the values of R_2 and R_3 that result in $V_{TL} = 4.9\,\mathbf{V}$ and $V_{TH} = 5.1\,\mathbf{V}$.
Manipulate the two equations with two unknowns R_2 and R_3.

$$
\begin{aligned}
V_{TL} \left(R_1 R_2 + R_1 R_3 + R_2 R_3\right) =& V R_1 R_2 + L_- R_1 R_3 \\
V_{TH} \left(R_1 R_2 + R_1 R_3 + R_2 R_3\right) =& V R_1 R_2 + L_+ R_1 R_3
\end{aligned}
$$

First, isolate R_3 in both equations,

$$
\begin{aligned}
R_3 =& \frac{\left(V - V_{TL}\right) R_1 R_2}{V_{TL} R_1 + V_{TL} R_2 - L_- R_1} \\
R_3 =& \frac{\left(V - V_{TH}\right) R_1 R_2}{V_{TH} R_1 + V_{TH} R_2 - L_+ R_1}
\end{aligned}
$$

Next take the ratio

$$1 = \frac{\left(V - V_{TL}\right) \left[V_{TH} \left(R_1 + R_2\right) - L_+ R_1\right]}{\left(V - V_{TH}\right) \left[V_{TL} \left(R_1 + R_2\right) - L_- R_1\right]}$$

$$\left(V - V_{TH}\right) \left[V_{TL} \left(R_1 + R_2\right) - L_- R_1\right] = \left(V - V_{TL}\right) \left[V_{TH} \left(R_1 + R_2\right) - L_+ R_1\right]$$

Now isolate R_2,

$$
\begin{aligned}
&\left(V - V_{TL}\right) V_{TH} R_2 - \left(V - V_{TH}\right) V_{TL} R_2 = \\
&\qquad \left(V - V_{TH}\right) V_{TL} R_1 - \left(V - V_{TL}\right) V_{TH} R_1 + \left(L_+ \left(V - V_{TL}\right) - L_- \left(V - V_{TH}\right)\right) R_1
\end{aligned}
$$

$$\left(V_{TH} - V_{TL}\right) V R_2 = \left(V_{TL} - V_{TH}\right) V R_1 + \left(L_+ \left(V - V_{TL}\right) - L_- \left(V - V_{TH}\right)\right) R_1$$

$$R_2 = \left[\frac{L_+ (V - V_{TL}) - L_- (V - V_{TH})}{(V_{TH} - V_{TL}) V} - 1\right] R_1$$

Inserting values we get

$$R_2 = \left[\frac{13 (15 - 4.9) + 13 (15 - 5.1)}{(5.1 - 4.9)\, 15} - 1\right] 10\,\mathrm{k}\Omega$$
$$= 856\,\mathrm{k}\Omega$$

Now returning to one of the expressions for R_3 we get

$$\begin{aligned} R_3 =& \frac{(V - V_{TL}) R_1 R_2}{V_{TL} (R_1 + R_2) - L_- R_1} \\ =& \frac{(15 - 4.9)\, 10 \times 10^3 \times 856 \times 10^3}{4.9 (10 \times 10^3 + 856 \times 10^3) - 4.9 \times 10 \times 10^3} \\ =& 20.6\,\mathrm{k}\Omega \end{aligned}$$

SS 13.26 For the circuit in Figure P13.26 sketch the transfer characteristics $v_o - v_i$. The diodes are assumed to have a constant $0.7\,\mathrm{V}$ drop when conducting, and the op-amp saturates at $\pm 12\,\mathrm{V}$. What is the maximum diode current?

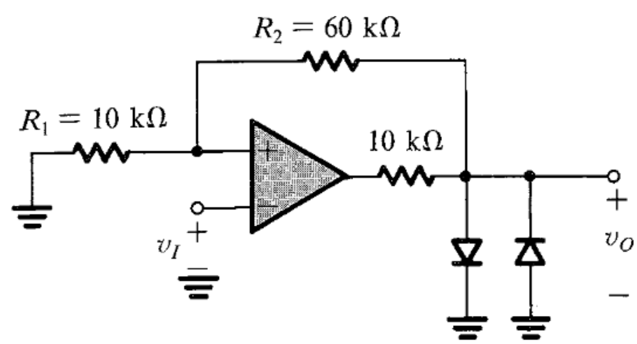


FIGURE P13.26

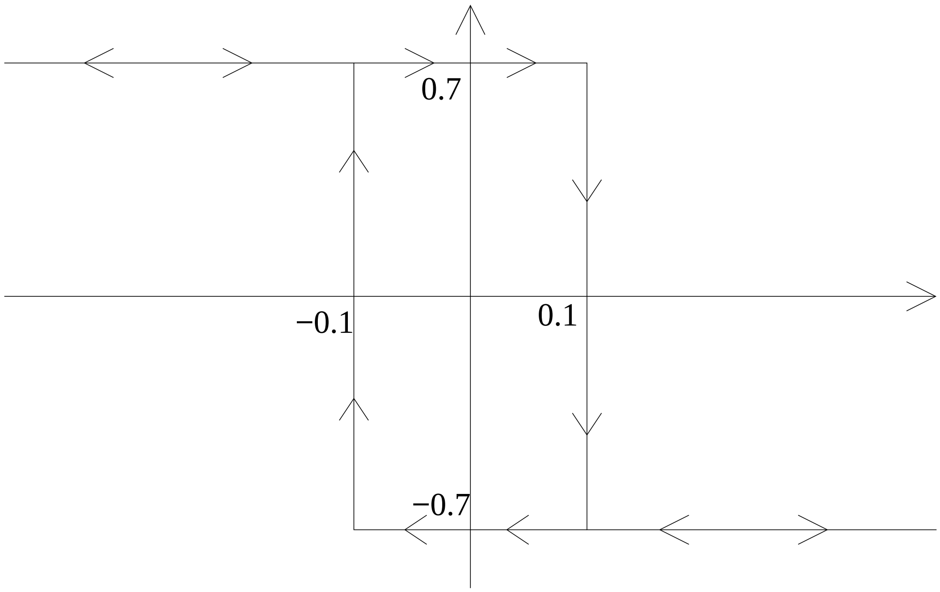
When the output of the op-amp is high, the output is $0.7\,\mathrm{V}$. When the output of the op-amp is low, the output is $-0.7\,\mathrm{V}$. When the output is high (input is low), V_+ equals the high threshold voltage, V_{TH},

$$V_{TH} = V_o \frac{R_1}{R_1 + R_2} = 0.7 \frac{10}{10 + 60} = 0.1\,\mathrm{V}$$

When the output is low (input is high), V_+ equals the low threshold voltage, V_{TL},

$$V_{TL} = V_o \frac{R_1}{R_1 + R_2} = -0.7 \frac{10}{10 + 60} = -0.1\,\mathrm{V}$$

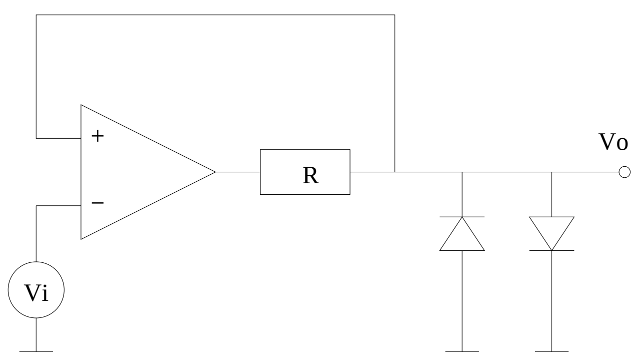
The transfer function thus looks like this



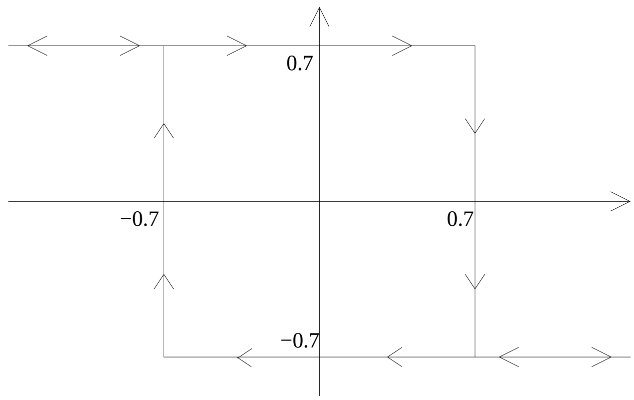
The maximum diode current is the output current from the op-amp minus the current diverted in the feedback loop.

$$
\begin{aligned}
I_{D,\max} =& \left(V_{12} - V_D\right) 10\,\mathrm{k}\Omega - \frac{V_D}{R_2 + R_1} \\
=& \frac{12 - 0.7}{10 \times 10^3} - \frac{0.7}{70 \times 10^3} \\
=& 1.12\,\mathrm{mA}
\end{aligned}
$$

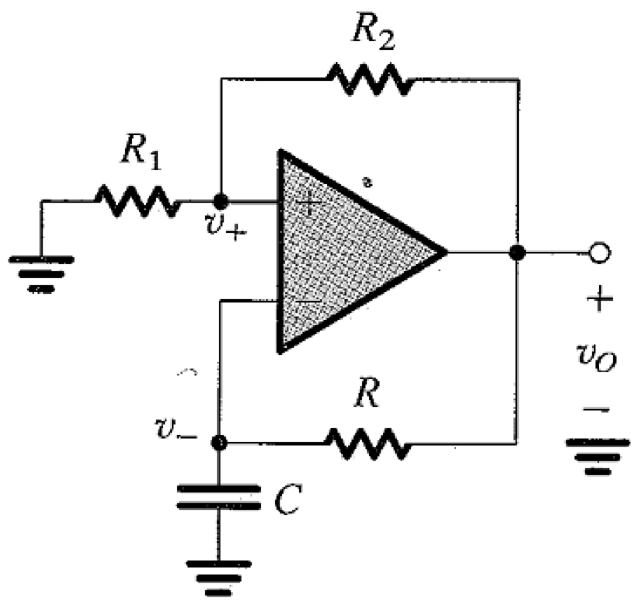
SS 13.27 Consier the circuit in Figure P13.26 with R_1 eliminated and R_2 short-circuited. Sketch and label the transfer characteristics $v_o - v_i$. Assume the diodes have a constant $0.7\,\mathbf{V}$ drop when conducting, and the op-amp saturates at $\pm 12\,\mathbf{V}$.
The circuit looks like this



In this case, when the output is high (input is low), $V_+ = 0.7\,\mathrm{V}$, so $V_{TH} = 0.7\,\mathrm{V}$. When the output is low (input is high), $V_+ = -0.7\,\mathrm{V}$, so $V_{TL} = -0.7\,\mathrm{V}$. the transfer function looks like this



SS 13.30 Find the frequency of oscillation of the circuit in Figure 13.24b for the case $R_1 = 10\,\mathbf{k\Omega}$, $R_2 = 16\,\mathbf{k\Omega}$, $C = 10\,\mathbf{nF}$, and $R = 62\,\mathbf{k\Omega}$.



In order to solve this problem we either need to know what the values of L_+ and L_- are, or we need to know that $L_+ = -L_-$. I will assume the latter. In that case we can use SS equation 13.33,

$$T = 2\tau \ln \frac{1+\beta}{1-\beta},$$

where $\tau = RC = 0.62\,\text{ms}$, and $\beta = \frac{R_1}{R_1+R_2} = 0.3846$. We get

$$T = 2 \times 0.62 \times \ln \frac{1.3846}{0.6154} = 1.01\,\text{ms}$$