# EE 322 Advanced Analog Electronics, Spring 2012
# Homework #9 solution

**SS 13.34. Figure P13.34 shows a monostable multivibrator circuit. In the stable state, $v_o = L_+$, $v_A = 0$, and $v_B = -V_{\text{ref}}$. the circuit can be triggered by applying a positive input pulse of height greater than $V_{\text{ref}}$. For normal operation, $C_1 R_1 \ll CR$. Show the resulting waveforms of $v_o$ and $v_A$. Also, show that the pulse generated at the output will have a width $T$ given by**

$$T = CR \ln\left(\frac{L_+ - L_-}{V_{\text{ref}}}\right)$$

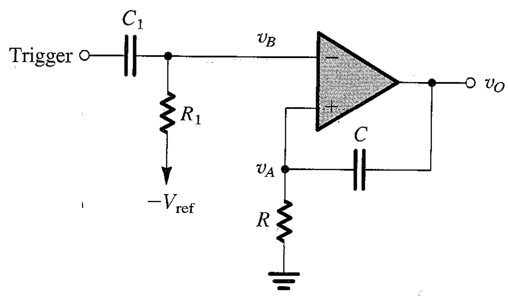
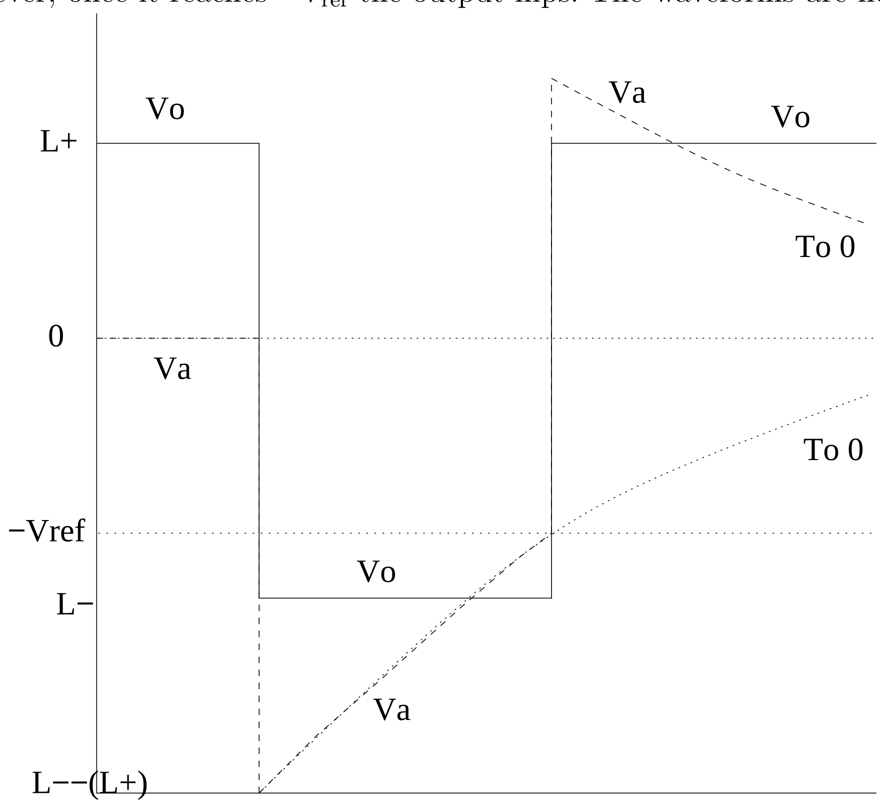



FIGURE P13.34

$v_A$ begins at ground because there is no current through $R$. As the positive pulse is applied to trigger, $v_B$ is pulled low which causes $v_o$ to go low. The voltage across the capacitor is still $L^+$, so the voltage $v_A = L_- - L_+$. The capacitor begins to charge from that voltage to ground. However, once it reaches $-V_{\text{ref}}$ the output flips. The waveforms are here



and here is the expression for determining $T$

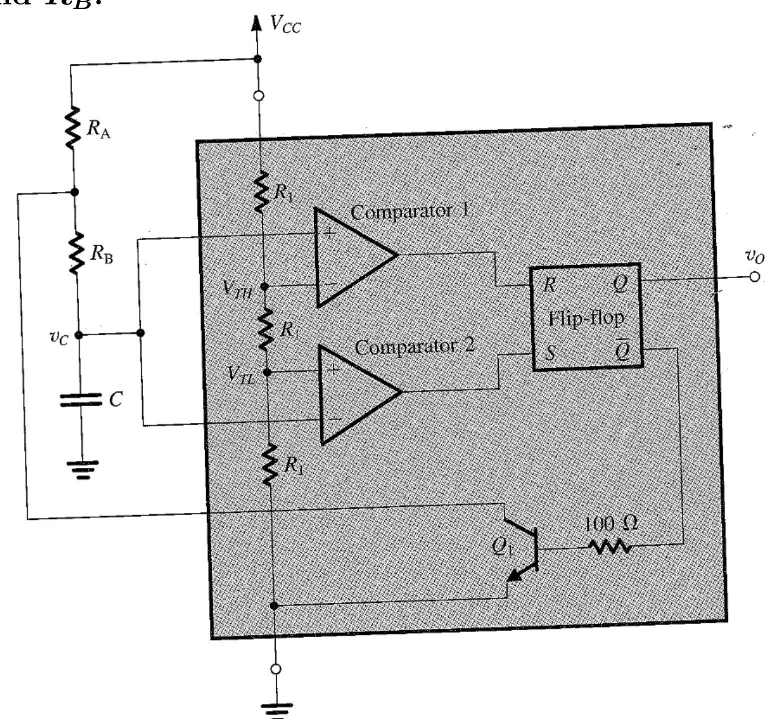
$$(L_- - L_+)\, e^{-\frac{T}{RC}} = -V_{\text{ref}}$$

which can be re-written as

$$\ln\left(\frac{-V_{\text{ref}}}{L_- - L_+}\right) = -\frac{T}{RC}$$

$$T = RC \ln\left(\frac{L_+ - L_-}{V_{\text{ref}}}\right)$$

**SS 13.39. Using a 680 pF capacitor, design the astable circuit of Fig. 13.29(a) to obtain a square wave with a 50 kHz frequency and a 75% duty cycle. Specify the values of $R_A$ and $R_B$.**




This is a straightforward application of formulas given in the text book,

$$T = \frac{1}{f} = 0.69C\,(R_A + 2R_B) \qquad \text{duty} = \frac{R_A + R_B}{R_A + 2R_B}$$

Begin by finding $R_A + 2R_B$,

$$R_A + 2R_B = \frac{1}{0.69\, f\, C} = \frac{1}{0.69 \times 50 \times 10^3 \times 680 \times 10^{-12}} = 42.63\,\text{k}\Omega$$

Next, find $R_A + R_B$ from the duty cycle formula,

$$R_A + R_B = \text{duty} \times (R_A + 2R_B) = 0.75 \times 42.63 = 31.97\,\text{k}\Omega$$

Next,

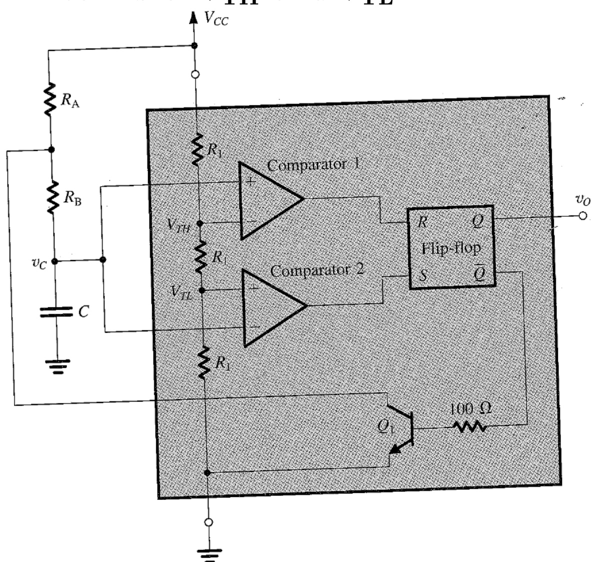
$$R_B = (R_A + 2R_B) - (R_A + R_B) = 42.63 - 31.97 = 10.66\,\mathrm{k\Omega}$$

and

$$R_A = (R_A + R_B) - R_B = 31.97 - 10.66 = 21.31\,\mathrm{k\Omega}$$

**SS 13.40 The node in the 555 timer at which the voltage is $V_{\mathbf{TH}}$ (i.e. the inverting input terminal fo the comparator 1) is usually connected to an external terminal. This allows the user to change $V_{\mathbf{TH}}$ externally (i.e., $V_{\mathbf{TH}}$ no longer remains at $\frac{2}{3}V_{CC}$). Note, however, that whatever the value of $V_{\mathbf{TH}}$ becomes, $V_{\mathbf{TL}}$ always remains $\frac{1}{2}V_{\mathbf{TH}}$.**

**(a) For the astable circuit of Fig. 13.29, rederive the expression for $T_{\mathbf{H}}$ and $T_{\mathbf{L}}$, expressing them in terms of $V_{\mathbf{TH}}$ and $V_{\mathbf{TL}}$.**




**(b) For the case $C = 1\,\mathrm{nF}$, $R_A = 7.2\,\mathrm{k\Omega}$, $R_B = 3.6\,\mathrm{k\Omega}$, and $V_{CC} = 5\,\mathrm{V}$, find the frequency of oscillation and the duty cycle of the resulting square wave when no external voltage is applied to the terminal $V_{\mathbf{TH}}$.**

**(c) For the design in (b), let a sine-wave signal of a much lower frequency than that found in (b) and of 1-V peak amplitude be capacitively coupled to the circuit node $V_{\mathbf{TH}}$. This signal will cause $V_{\mathbf{TH}}$ to change around its quiescent value of $\frac{2}{3}V_{CC}$, and thus $T_H$ will change correspondingly - a modulation process. Find $T_H$, and find the frequency of oscillation and the duty cycle at the two extreme values of $V_{\mathbf{TH}}$.**

(a) $T_H$ is the time when the output is high and the capacitor is charging up. It is charging from $V_{TL}$ towards $V_{CC}$ with time-constant $(R_A + R_B)C$ and gets interrupted at $V_{TH}$. Thus we have

$$\left(1 - e^{-\frac{T_H}{(R_A+R_B)C}}\right)(V_{CC} - V_{TL}) - V_{TL} = V_{TH}$$

$$1 - e^{-\frac{T_H}{(R_A+R_B)C}} = \frac{V_{TH} - V_{TL}}{V_{CC} - V_{TL}}$$

$$e^{-\frac{T_H}{(R_A+R_B)C}} = 1 - \frac{V_{TH} - V_{TL}}{V_{CC} - V_{TL}} = \frac{V_{CC} - V_{TL} - V_{TH} + V_{TL}}{V_{CC} - V_{TL}} = \frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}}$$

$$T_H = (R_A + R_B)\, C \ln \frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}}$$

$T_L$ is the time when the output is low and the capacitor is discharging. It discharges from $V_{TH}$ towards ground with time constant $R_B C$ and gets interrupted at $V_{TL}$. Thus we have

$$e^{-\frac{T_L}{R_B C}} V_{TH} = V_{TL}$$

$$T_L = R_B C \ln \frac{V_{TH}}{V_{TL}} = R_B C \ln 2$$

(not it is independent of $V_{TH}$)

(b) In this case we have $V_{TH} = \frac{2}{3} V_{CC}$ and $V_{TL} = \frac{1}{3} V_{CC}$. Then $T_H$ is

$$T_H = (7.2 + 3.6) \times 10^3 \times 1 \times 10^{-9} \ln \left( \frac{1 - \frac{1}{3}}{1 - \frac{2}{3}} \right) = 7.5 \times 10^{-6}\,\text{s} = 7.5\,\mu\text{s}$$

and $T_L$ is

$$T_L = 3.6 \times 10^3 \times 1 \times 10^{-9} \times \ln 2 = 2.5 \times 10^{-6} = 2.5\,\mu\text{s}$$

The period of the signal is then $T = T_H + T_L = 10\,\mu\text{s}$ and the frequency is $f = 100\,\text{kHz}$. The duty cycle for high output is 75%.

(c) We always have $V_{TL} = \frac{V_{TH}}{2}$. At the two extremese we have $V_{TH} = \frac{2}{3} V_{TH} \pm A$, where $A$ is the amplitude of the sine wave. When the the voltage is increased we have

$$V_{TH} = \frac{2}{3} V_{CC} + A = \frac{2}{3} \times 5 + 1 = 4.33\,\text{V}$$

and

$$V_{TL} = \frac{V_{TH}}{2} = 2.17\,\text{V}$$

so that

$$T_H = (7.2 + 3.6) \times 10^3 \times 1 \times 10^{-9} \ln \frac{5 - 2.17}{5 - 4.33} = 1.55 \times 10^{-5}\,\text{s} = 15.5\,\mu\text{s}$$

and the frequency is

$$f = \frac{1}{T_H + T_L} = \frac{10^6}{15.5 + 2.5} = 55.5\,\text{kHz}$$

and the duty

$$\frac{T_H}{T_H + T_L} = \frac{15.5}{15.5 + 2.5} = 86.1\%$$

when the voltage is one volt lower we get

$$V_{TH} = 2.33\,\text{V} \qquad\qquad V_{TL} = \frac{2.33}{2} = 1.17\,\text{V}$$

and

$$T_H = (7.2 + 3.6) \times 10^3 \times 1 \times 10^{-9} \ln \frac{5 - 1.17}{5 - 2.33} = 3.90\,\mu\text{s}$$

Then the frequency is

$$f = \frac{1}{T_H + T_L} = \frac{10^6}{3.90 + 2.5} = 156\,\text{kHz}$$

and the duty cycle is

$$\frac{T_H}{T_H + T_L} = 60.9\%$$