EE 322 Advanced Analog Electronics, Spring 2012
Homework #10 solution

1. **Do H&H Exercise 9.5 in the body of the text.**

   The expression for the loop gain is

$$\begin{aligned} L =& K_P K_F \frac{K_{\text{VCO}}}{j\omega} K_{\text{div}} \\ =& 1.59 \times \frac{1 + j\omega R_4 C_2}{1 + j\omega\left(R_3 C_2 + R_4 C_2\right)} \times \frac{1.13 \times 10^5}{j\omega} \times \frac{1}{1024} \end{aligned}$$

   We need to verify that the choice of $R_4 = 330\,\text{k}\Omega$, $C_2 = 1\,\mu\text{F}$, and $R_3 = 4.3\,\text{M}\Omega$ results in a loop gain magnitude of unity at frequency $f = 2\,\text{Hz}$, or $\omega = 4\pi\,\text{s}^{-1}$. We can reduce to

$$L = 175.5 \times \frac{1 + j\omega R_4 C_2}{1 + j\omega\left(R_3 C_2 + R_4 C_2\right)} \frac{1}{j\omega}$$

   which has magnitude

$$\begin{aligned} |L| =& 175.5 \times \frac{\sqrt{1 + \omega^2 R_4^2 C_2^2}}{\sqrt{1 + \omega^2\left(R_3 C_2 + R_4 C_2\right)^2}} \frac{1}{\omega} \\ =& 175.5 \frac{\sqrt{1 + \left(4 \times \pi \times 330 \times 10^3 \times 1 \times 10^{-6}\right)^2}}{\sqrt{1 + \left(4 \times \pi \times \left(4.3 \times 10^6 \times 1 \times 10^{-6} + 330 \times 10^3 \times 1 \times 10^{-6}\right)\right)^2}} \frac{1}{4 \times \pi} \\ =& 1.02 \end{aligned}$$

   which is close enough to within rounding errors.

2. **Design a PLL for FM radio demodulation. Follow the example in H&H Figure 9.72 and accompanying text. Use a VCO with gain 100 MHz/V, and use a loop filter bandwidth of 20 kHz (to capture audio up to that frequency). Use a XOR phase detector. The carrier frequency is 90 MHz.**

   (a) **What should be the values of the components of the LP filter?**

   Begin with the expression for the loop gain on H&H page 649,

$$L = K_P K_F \frac{K_{\text{VCO}}}{j\omega} K_{\text{div}}$$

   We have the same phase detector, so $K_P = 1.59\,\frac{\text{V}}{\text{rad}}$. We are not doing any frequency multiplication, so $K_{\text{div}} = 1$. The response of the VCO is given as

$K_{\text{VCO}} = 100\,\text{MHz/V} = 6.28 \times 10^8\,\text{rad/s/V}$. Also, we use $f = 20\,\text{kHz}$, $\omega = 20 \times 10^3 \times 2 \times \pi = 126 \times 10^3\,\text{s}^{-1}$. We then get

$$K_P \frac{K_{VCO}}{\omega} K_{\text{div}} = 1.59 \times \frac{6.28 \times 10^8}{126 \times 10^3} \times 1 = 7.9 \times 10^3$$

Put the frequency of the zero at a quarter of the bandwidth, so $f_1 = 5\,\text{kHz}$.

$$R_4 C_2 = \frac{1}{2\pi f_1} = \frac{1}{2\pi \times 5 \times 10^5} = 3.183 \times 10^{-5}$$

If we select $R_4 = 1\,\text{k}\Omega$ then we get

$$C_2 = \frac{3.183 \times 10^{-5}}{1 \times 10^3} = 31.8 \times 10^{-9} = 31\,\text{nF}$$

Now we just find R_3 such that the loop gain magnitude is unity at $\omega = 125 \times 10^3\,\text{s}^{-1}$. Begin with an expression for $|K_F|$ as we have the rest.

$$|K_F| = \frac{\sqrt{1 + \omega^2 R_4^2 C_2^2}}{\sqrt{1 + \omega^2 \left(R_3 C_2 + R_4 C_2\right)^2}}$$

and then isolate R_3 in the expression for the loop gain

$$1 = |L| = 7.9 \times 10^3 \frac{\sqrt{1 + \omega^2 R_4^2 C_2^2}}{\sqrt{1 + \omega^2 \left(R_3 C_2 + R_4 C_2\right)^2}}$$

$$\sqrt{1 + \omega^2 \left(R_3 C_2 + R_4 C_2\right)^2} = 7.9 \times 10^3 \sqrt{1 + \omega^2 R_4^2 C_2^2}$$

$$\omega^2 \left(R_3 C_2 + R_4 C_2\right)^2 = (7.9 \times 10^3)^2 \left(1 + \omega^2 R_4^2 C_2^2\right) - 1$$

$$R_3 C_2 + R_4 C_2 = \frac{1}{\omega} \sqrt{(7.9 \times 10^3)^2 \left(1 + \omega^2 R_4^2 C_2^2\right) - 1}$$

$$\begin{aligned} R_3 =& \frac{1}{\omega C_2} \sqrt{(7.9 \times 10^3)^2 \left(1 + \omega^2 R_4^2 C_2^2\right) - 1} - R_4 \\ =& \frac{1}{125 \times 10^3 \times 31 \times 10^{-9}} \sqrt{(7.9 \times 10^3)^2 (1 + (125 \times 10^3 \times 10^3 \times 31 \times 10^{-9})^2) - 1} - 10^3 \\ =& 8.16 \times 10^6\,\Omega = 8.2\,\text{M}\Omega \end{aligned}$$

(b) Explain how the frequency and amplitude of the audio signal are encoded on the FM carrier.

The instantaneous frequency difference from the central 90 MHz carrier is the instantaneous amplitude of the wave. Larger amplitude is created by increasing the frequency difference. The frequency of the audio signal is a function of how quickly the signal frequency changes.

(c) **Where do you measure the output demodulated audio signal?**

At the output of the LP filter.

(d) **If the FM band is 50 kHz wide, what is the largest amplitude of the demodulated signal?**

$$V_{\max} = \frac{\Delta f}{2} \frac{1}{K_{\text{VCO}}} = \frac{25 \times 10^3}{2 \times 100 \times 10^6} = 125\,\mu\text{V}$$

(e) **If you need a maximum amplitude of 20 V to drive the audio speakers, what gain should you apply to the output signal?**

$$G = \frac{20\text{ V}}{V_{\max}} = 1.6 \times 10^5$$

(f) **Design the circuit, including values for components, which will couple the demodulated signal to make a AC signal of maximum amplitude of 20 V to drive the speakers.**

This will be a HP filter followed by a non-inverting amplifier. The break frequency of the HP filter was not given in the problem, so let's select it to be 50 Hz so that

$$f_0 = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_0} = 3.18 \times 10^{-3}$$

Pick $R = 100\,\text{k}\Omega$ and we get $C = \frac{3.18\times10^{-3}}{10^5} = 3.18 \times 10^{-8} = 31\,\text{nF}$. For the following amplifier we need a gain of 1.6×10^5 over a bandwidth of 20 kHz. That is a GBWP of 3.2×10^9. Better cascade two gain stages each with gain 400. In that case the GBWP of each must be 8 MHz which is more realistic. To get a gain of 400 we select $R_1 = 1\,\text{k}\Omega$, and $R_2 = 399\,\text{k}\Omega$ on both stages.