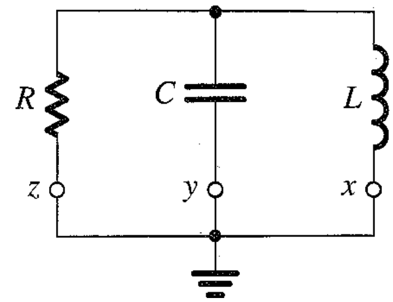
EE 322 Advanced Analog Electronics, Spring 2012
Homework #12 solution

SS 12.33. Design the LCR resonator of Fig. 12.17(a) to obtain natural modes with $\omega_0 = 10^4$ rad/s and $Q = 2$. Use $R = 10\,\text{k}\Omega$.



The expressions for ω_0 and Q in terms of circuit parameters are

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad\qquad Q = \omega CR$$

First find

$$C = \frac{Q}{\omega_0 R} = \frac{2}{10^4 \times 10 \times 10^3} = 20\,\text{nF}$$

Next,

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(10^4)^2 \times 20 \times 10^{-9}} = 0.5\,\text{H}$$

SS 12.34. For the LCR resonator of Fig. 12.17(a) find the change in ω_0 that results from: (a) increasing L by 1%, (b) increasing C by 1%, (c) increasing R by 1%.

For the LCR resonator the resonance frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

(a) If we replace L with αL, then

$$\omega_{0,L} = \frac{1}{\sqrt{\alpha LC}} = \frac{1}{\sqrt{\alpha}}\frac{1}{\sqrt{LC}} = \omega_0 \frac{1}{\sqrt{\alpha}}$$

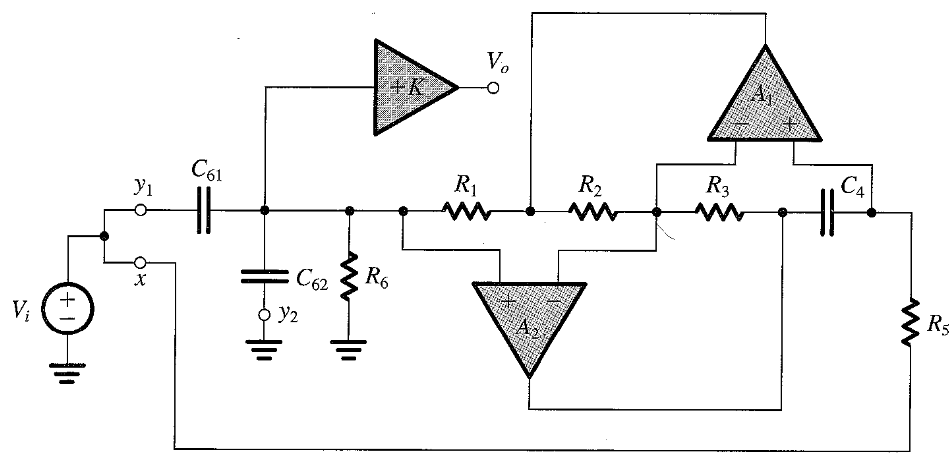
$$\frac{\omega_{0,L}}{\omega_0} = \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{1.01}} = 0.995$$

Thus if we increase the value of L by 1% we decrease the value of the resonance frequency by 0.5%.

(b) It is obvious that we obtain the same result when we increase the value of the capacitance by 1%. We get a reduction of the resonance frequency by 0.5%.

(c) The resistance does not enter into the equation for the resonance frequency. Thus, changing the value of the resistance does not change the resonance frequency - but it does change the value of Q.

SS 12.43. Design the circuit of Figure 12.22(e) to realize an LPN function with $f_0 = 4\,\mathrm{kHz}$, $f_n = 5\,\mathrm{kHz}$, $Q = 10$, and unity DC gain. Select $C_4 = 10\,\mathrm{nF}$.
The circuit looks like this



(e) LPN, $\omega_n \geq \omega_0$

For this circuit,

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{R_2}{C_4\left(C_{61}+C_{62}\right) R_1 R_3 R_5}}$$

$$Q = R_6\sqrt{\frac{C_{61}+C_{62}}{C_4}\frac{R_2}{R_1 R_3 R_5}}$$

First, assume that $\frac{R_1 R_3 R_5}{R_2} = (1\,\mathrm{k\Omega})^2$. Then we get

$$\begin{aligned} C_{61} =& \frac{1}{\left(2\pi f_n\right)^2 C_4}\frac{R_2}{R_1 R_3 R_5} \\ =& \frac{1}{\left(2\pi \times 5 \times 10^3\right)^2 \times 10 \times 10^{-9}}\frac{1}{(1 \times 10^3)^2} = 101\,\mathrm{nF} \end{aligned}$$

This is a reasonable value, so let us proceed. Next we use the relationship for f_0 to find C_{62}.

$$\begin{aligned} C_{62} =& \frac{1}{\left(2\pi f_0\right)^2 C_4}\frac{R_2}{R_1 R_3 R_5} - C_{61} \\ =& \frac{1}{\left(2\pi \times 4 \times 10^3\right)^2 \times 10 \times 10^{-9}}\frac{1}{\left(1 \times 10^3\right)^2} - 101 \times 10^{-9} = 57.3\,\mathrm{nF} \end{aligned}$$

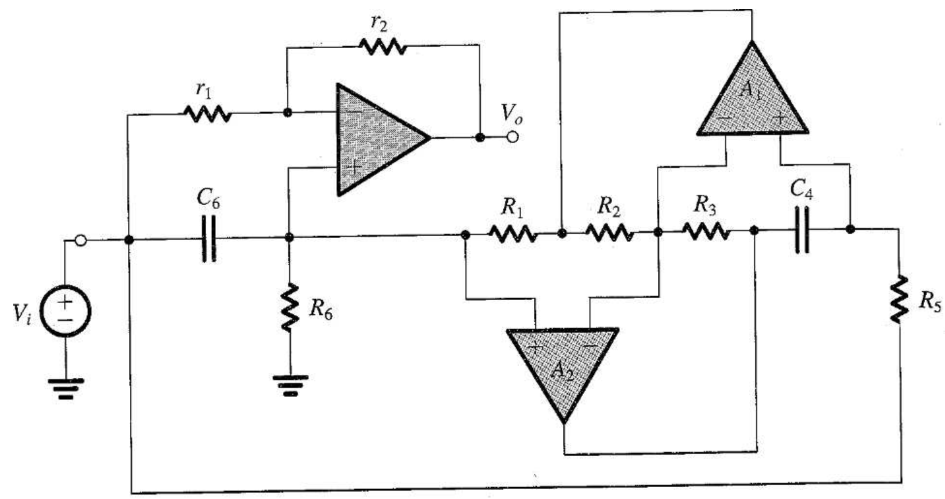
This is also a reasonable value, so we continue. The only thing we still need to determine is R_6, which we can find from the expression for Q,

$$\begin{aligned}R_6 =& Q\sqrt{\frac{C_4}{C_{61}+C_{62}}\frac{R_1R_3R_5}{R_2}} \\ =& 10\sqrt{\frac{10\times 10^{-9}}{(101+57.3)\times 10^{-9}}\left(1\times 10^3\right)^2} = 2.51\,\mathrm{k\Omega}\end{aligned}$$

This is also a reasonable value, so we are almost done. We just want to select R_1, R_3, R_5, and R_2 such that $\frac{R_1R_3R_5}{R_2} = (1\,\mathrm{k\Omega})^2$. A simple choice could be $R_1 = R_2 = R_3 = R_5 = 1\,\mathrm{k\Omega}$. Now we are done.

SS 12.44. Design the all-pass filter of Figure 12.22(g) to provide a phase shift of 180° at $f = 1\,\mathrm{kHz}$ and to have $Q = 1$. Use $1\,\mathrm{nF}$ capacitors.

The circuit looks like this



(g) All-pass

For this circuit,

$$T(s) = \frac{s^2 - s\frac{1}{C_6R_6}\frac{r_2}{r_1} + \frac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$$

We choose $\frac{r_2}{r_1} = 1$, and then note that the numerator and denominator are complex conjugates. If the numerator has a phase shift of -90°, then the denominator will have a phase shift $+90^\circ$, and the whole transfer function will have a phase shift of -180°. We thus want the real component of the numerator to be zero,

$$s^2 + \frac{R_2}{C_4C_6R_1R_3R_5} = 0$$

or

$$-\omega^2 + \frac{R_2}{C_4C_6R_1R_3R_5} = 0$$

or

$$\omega = \sqrt{\frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$$

Where $\omega = 2\pi \times 1\,\text{kHz}$. We also want $Q_z = Q = 1$,

$$1 = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$

We are also told to make all the capacitances equal to $C = 1\,\text{nF}$, so

$$\omega^2 C^2 = \frac{R_2}{R_1 R_3 R_5} \qquad 1 = \frac{R_6^2 R_2}{R_1 R_3 R_5}$$

Let's make $R_1 = R_3 = R_5 = R$. Then

$$R = \frac{1}{\omega C} = \frac{1}{2\pi \times 1 \times 10^3 \times 1 \times 10^{-9}} = 159\,\text{k}\Omega$$

In that case,

$$1 = \frac{R_6}{R^2}$$

So that $R_6 = R = 159\,\text{k}\Omega$.

SS 12.48. It is required to design a third-order low-pass filter whose $|\boldsymbol{T}|$ is equiripple in both the passband and the stopband (in the manner showin in Fig. 12.3, except that the response shown is for $N = 5$). The filter passband extends from $\omega = 0$ to $\omega = 1\,\text{rad/s}$ and the stopband edge is at $\omega = 1.2\,\text{rad/s}$. The following transfer function was obtained using filter design tables:

$$\boldsymbol{T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}}$$

The actual filter is to have $\omega_p = 10^4\,\text{rad/s}$.

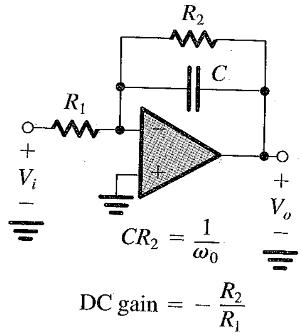
(a) Obtain the transfer function of the actual filter by replacing s by $s/10^4$.

(b) Realize this filter as the cascade connection of a first-order LP op-amp RC circuit of the type shown in Fig. 12.13(a) and a second-order LPN circuit of the type shown in Fig. 12.22(e). Each section is to have a dc gain of unity. Select appropriate component values. (Note: A filter with an equiripple response in both the passband and the stopband is known as an elliptic filter).

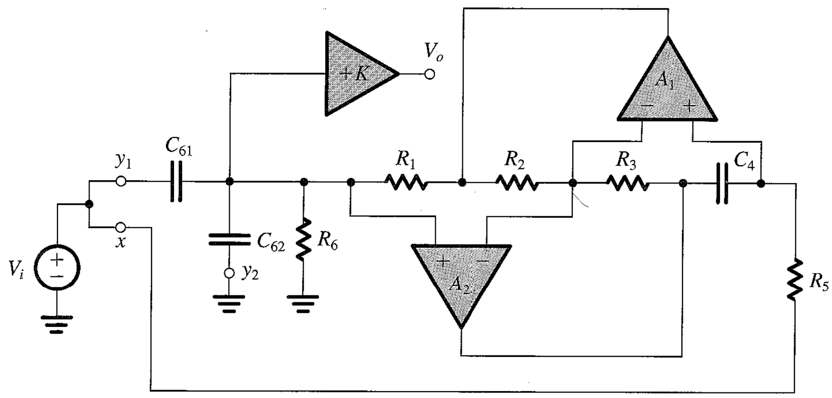
(a) Replacing s by $s/10^4$ we get

$$\begin{aligned} T(s) =& \frac{0.4508\left(\frac{s^2}{10^8} + 1.6996\right)}{\left(\frac{s}{10^4} + 0.7294\right)\left(\frac{s^2}{10^8} + \frac{s0.2786}{10^4} + 1.0504\right)} \\ =& \frac{4.508 \times 10^3\left(s^2 + 1.6996 \times 10^8\right)}{\left(s + 7.294 \times 10^3\right)\left(s^2 + s2.786 \times 10^3 + 1.0504 \times 10^8\right)} \end{aligned}$$

(b) Here are the two filters we are to cascade. The first-order LP filter



and the second-order LPN filter



(e) LPN, $\omega_n \geq \omega_0$

The DC gain of the third-order filter is 0.9778, but I will design each filter for unity DC gain as directed.

For the first-order filter we need

$$CR_2 = \frac{1}{\omega_0} \qquad\qquad \omega_0 = 7.294 \times 10^3\,\mathrm{s}^{-1}$$

Choose $C = 10\,\mathrm{nF}$ we get

$$R_2 = \frac{1}{\omega_0 C} = \frac{1}{7.294 \times 10^3 \times 10 \times 10^{-9}} = 13.7\,\mathrm{k}\Omega$$

For unity DC gain amplitude we need $R_1 = R_2 = 13.7\,\mathrm{k}\Omega$

Here are the properties of the 2nd-order LPN filter:

Low-pass notch (LPN) Fig. 12.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$	K = DC gain
	$\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$	$C_{61}+C_{62} = C_6 = C$
	$\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2}$	$C_{61} = C(\omega_0/\omega_n)^2$
	$Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\dfrac{R_2}{R_1R_3R_5}}$	$C_{62} = C - C_{61}$

Comparing this to the transfer function let's begin by selecting $R_6 = 10\,\text{k}\Omega$, and using the first-order term in the denominator. Then

$$C_{61} + C_{62} = \frac{1}{2.786 \times 10^3 \times R_6} = \frac{1}{2.786 \times 10^3 \times 10 \times 10^3} = 35.9\,\text{nF}$$

which is a reasonable value, so we proceed. Next, if we look at the constant term in the denominator and choose $R_1 = R_2 = R_3 = R_5 = 10\,\text{k}\Omega$, we get

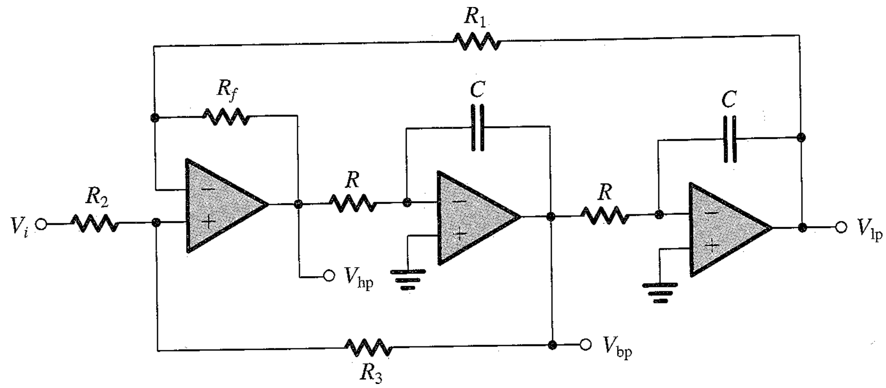
$$\begin{aligned} C_4 =& \frac{R_2}{1.0504 \times 10^8 \times (C_{61}+C_{62})\,R_1R_3R_5} \\ =& \frac{10^4}{1.0504 \times 10^8 \times 35.9 \times 10^{-9} \times (10^4)^3} \\ =& 2.65\,\text{nF} \end{aligned}$$

This is also an acceptable value, so we proceed. Next, we use the constant term in the parenthesis in numerator to determine C_{61}. We get

$$\begin{aligned} C_{61} =& \frac{R_2}{1.6996 \times 10^8 C_4R_1R_3R_5} \\ =& \frac{10^4}{1.6996 \times 10^8 \times 2.65 \times 10^{-9} \times (10^4)^3} \\ =& 22.2\,\text{nF} \end{aligned}$$

This is also a reasonable value, and fortunately less then $C_{61} + C_{62}$, so we can now determine $C_{62} = C_{61} + C_{62} - C_{61} = 35.9 - 22.2 = 13.7\,\text{nF}$

SS 12.49. Design the KHN circuit of Fig. 12.24(a) to realize a bandpass filter with a center frequency of 1 kHz and a 3 − dB bandwidth of 50 Hz. Use 10 nF capacitors. Give the complete circuit and specify all component values. What value of center-frequency gain is obtained?



The center frequency is $\omega = \frac{1}{RC}$. Given that $C = 10\,\text{nF}$, we get $R = \frac{1}{\omega C} = \frac{1}{2\pi\times 10^3\times 10\times 10^{-9}} = 15.9\,\text{k}\Omega$. We also know that $R_f = R_1$, so let's choose $R_f = R_1 = 100\,\text{k}\Omega$. Lastly we need to choose R_2 and R_3 which sets the bandwidth. First we find Q, which is the inverse of the fractional bandwidth,

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{10^3}{50} = 20$$

Next, choose $R_2 = 10\,\text{k}\Omega$, and we have
and we have

$$R_3 = R_2\,(2Q - 1) = 39\,R_2 = 390\,\text{k}\Omega$$

The K gain parameter is then

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{20} = 1.95$$

Finally, the gain function is

$$T_{\text{bp}}(s) = -\frac{K\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

For $s = j\omega_0$ we get

$$T_{\text{bp}}(j\omega_0) = -\frac{jK\omega_0^2}{-\omega_0^2 + j\frac{\omega_0^2}{Q} + \omega_0^2} = -KQ = -1.95\times 20 = -39$$

SS 12.50. (a) Using the KHN biquad with the output summing amplifier of Fig. 12.24(b) show tthat an all-pass function is realized by selecting $R_L = R_H = R_B/Q$. Also show that the flat gain obtained is KR_F/R_H. (b) Design the all-pass circuit to obtain $\omega_0 = 10^4\,\text{rad/s}$, $Q = 2$, and flat gain $= 10$. Select appropriate component values.

(a) Here is Figure 12.24.

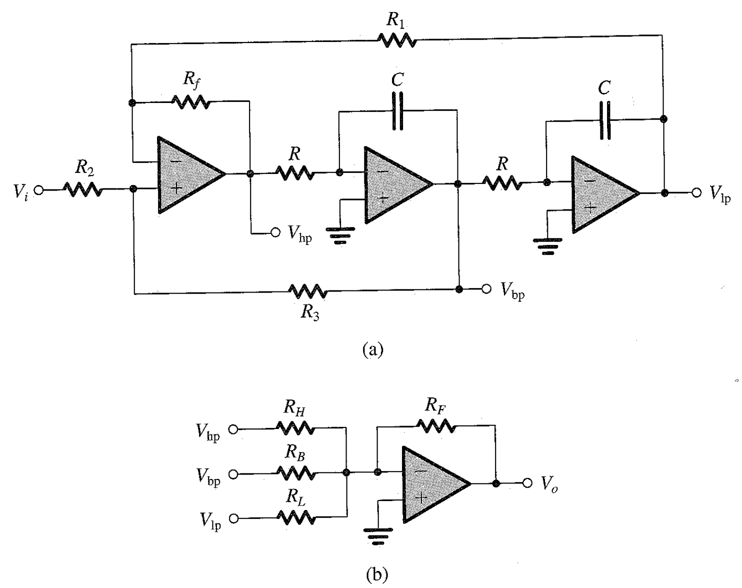


FIGURE 12.24 **(a)** The KHN biquad circuit, obtained as a direct implementation of the block diagram of Fig. 12.23(c). The three basic filtering functions, HP, BP, and LP, are simultaneously realized. **(b)** To obtain notch and all-pass functions, the three outputs are summed with appropriate weights using this op-amp summer.

The output is

$$\begin{aligned}
V_o &= -R_F\left[\frac{V_{hp}}{R_H}+\frac{V_{bp}}{R_B}+\frac{V_{lp}}{R_L}\right]\\
&= -R_F\left[\frac{V_{hp}}{R_H}-\frac{\omega_0}{s}\frac{V_{hp}}{R_B}+\frac{\omega_0^2}{s^2}\frac{V_{hp}}{R_L}\right]\\
&= -R_F\left[\frac{1}{R_H}-\frac{\omega_0}{s}\frac{1}{R_B}+\frac{\omega_0^2}{s^2}\frac{1}{R_L}\right]V_{hp}\\
&= -R_F\left[\frac{1}{R_H}-\frac{\omega_0}{s}\frac{1}{R_B}+\frac{\omega_0^2}{s^2}\frac{1}{R_L}\right]\frac{Ks^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}
\end{aligned}$$

Now inserting the values given in the problem statement.

$$\begin{aligned}
V_O &= -R_F\left[\frac{1}{R_H}-\frac{\omega_0}{s}\frac{1}{R_HQ}+\frac{\omega_0^2}{s^2}\frac{1}{R_H}\right]\frac{Ks^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}\\
&= -\frac{KR_F}{R_H}\frac{s^2-\frac{\omega_0 s}{Q}+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}
\end{aligned}$$

Notice that the real and imaginary components are of the same size in the numerator and denominator of the second fraction. That means that the amplitude of that fraction is unity. Also, the overall gain is

$$|T| = -\frac{KR_F}{R_H}$$

(b) First pick

$$CR = \frac{1}{\omega_0} = 10^{-4}$$

Let's select $C = 1\,\mathrm{nF}$, and then

$$R = \frac{10^{-4}}{C} = \frac{10^{-4}}{10^{-9}} = 10^5 = 100\,\mathrm{k\Omega}$$

Also, since $R_f/R_1 = 1$, I select

$$R_f = R_1 = 100\,\mathrm{k\Omega}$$

Next, $R_3/R_2 = 2Q - 1 = 4 - 1 = 3$, so I select

$$R_2 = 50\,\mathrm{k\Omega}$$

and

$$R_3 = 150\,\mathrm{k\Omega}$$

Next the gain of the filter is

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{2} = \frac{3}{2}$$

In order to get gain of -10, we need then to have $-K\frac{R_F}{R_H} = -10$, or $\frac{R_F}{R_H} = \frac{10}{\frac{3}{2}} = \frac{20}{3}$. So we select

$$R_H = 30\,\mathrm{k\Omega}$$

and thus

$$R_F = 200\,\mathrm{k\Omega}$$

Finally, $R_L = R_H = 30\,\mathrm{k\Omega}$, and

$$R_B = R_H Q = 2R_H = 60\,\mathrm{k\Omega}$$