EE 333 Electricity and Magnetism, Fall 2009
Exam 2

Rules: This is a closed-book test. You may use calculator, and the attached help sheets. The exam will last 50 minutes, and each of the following numbered questions count equally toward your grade. None of these problems require extensive calculations. If you find yourself in long complex computations your are likely on the wrong track.
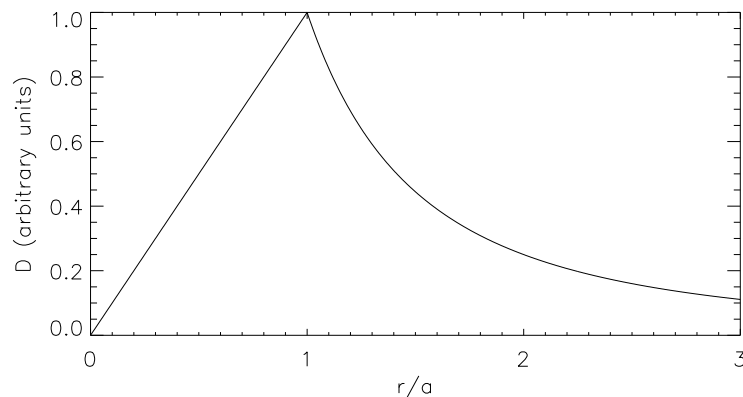
Charge distribution
Consider the spherical charge distribution

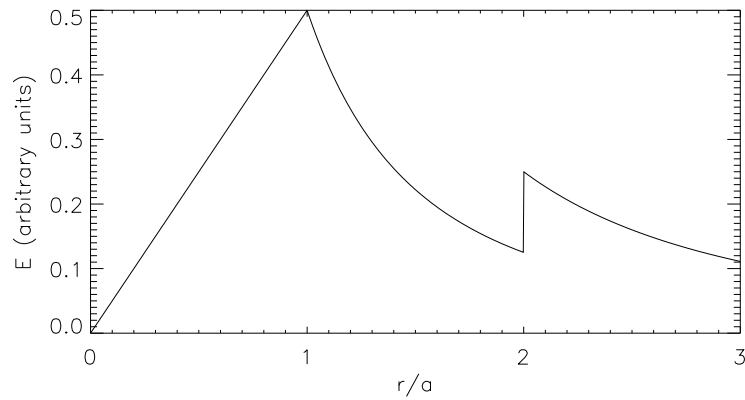$$\rho_v = \begin{cases} \rho_0 & r \leq a \\ 0 & a < r \end{cases}$$

and the permittivity distribution

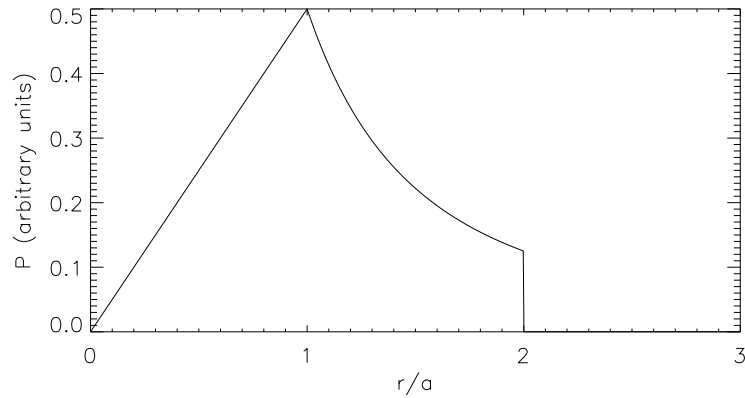$$\epsilon_r = \begin{cases} 2 & r \leq 2a \\ 1 & 2a < r \end{cases}$$

1. **Carefully sketch and label $\vec{D}$ as a function of r. You should be able to do this with almost no derivation. With a long derivation you will run out of time.**

2. **Carefully sketch and label $\vec{E}$ as a function of r. Same comment as above.**

3. Carefully sketch and label $\vec{P}$ as a function of r. Same comment as above.

4. **What is the polarization surface charge density at $r = 2a$?**

Since $\nabla \cdot \vec{P} = -\rho_v$, we can infer that

$$P_2 - P_1 = -\rho_s$$

or since $P_2 = 0$, $\rho_s = P_1$. Now we just need to find $P_1(2a-)$. We know that $P = \chi_e \epsilon_0 \vec{E} = (\epsilon_r - 1) \epsilon_0 \vec{E} = (\epsilon_r - 1) \frac{D}{\epsilon_r \epsilon_0}$. And that

$$4\pi (2a)^2 D = \frac{4\pi}{3} a^3 \rho_0$$

or

$$D = \frac{\rho_0 a}{24}$$

so

$$\rho_s = (\epsilon_r - 1) \frac{\rho_0 a}{24 \epsilon_r \epsilon_0}$$

Wave in conductor

5. **How deep into a sheet of iron can a cell phone signal penetrate before its electric field amplitude is reduced to 1% of its initial value? Assume $f = 850\,\text{MHz}$, $\sigma_{\text{iron}} = 1.0 \times 10^7\,\Omega^{-1}\text{m}^{-1}$, and $\epsilon_r = \mu_r = 1$. What is the wavelength, and how does it compare to the wavelength in a vacuum?**

Compute α and solve for z in $0.01 = e^{-\alpha z}$,

$$z = \frac{1}{\alpha}\ln 100$$

Because $\frac{\sigma}{\omega\epsilon} \gg 1$, we can simplify to

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$
$$= \sqrt{\frac{2 \times \pi \times 850 \times 10^6 \times 4 \times \pi}{2}}$$
$$= 1.83 \times 10^5\,\text{m}^{-1}$$

Inserting we get

$$z = \frac{1}{\alpha}\ln 100 = \frac{1}{1.83 \times 10^5}\ln 100 = 25.2\,\mu\text{m}$$

Next, note that $\beta \approx \alpha$ here so

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\beta} = \frac{2\pi}{1.83 \times 10^5} = 34.3\,\mu\text{m}$$

In a vacuum we would have

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{850 \times 10^6} = 0.35\,\text{m}$$

MUCH longer wavelength in a vacuum.

Capacitance

6. **Compute the capacitance per unit length of two co-axial cylindrical conductors. The outer radius of the inner cylinder is a, and the inner radius of the outer cylinder is b. The space between the two cylinders is filled with a dielectric of relative permittivity ϵ_r. Hint: $\frac{d}{dx}\ln x = \frac{1}{x}$.**

Here we need to compute the electric field and then integrate it from a to b. Place Q coulomb/length on the inner and $-Q$ on the outer conductor. Then the field between the conductors is

$$2\pi\rho D_\rho = Q$$

or

$$E_\rho = \frac{Q}{2\pi\epsilon_r\epsilon_0\rho}$$

Next integrate to get the voltage difference

$$V = \int_a^b E_\rho\, d\rho = \frac{Q}{2\pi\epsilon_r\epsilon_0} \int_a^b \frac{1}{\rho}d\rho = \frac{Q}{2\pi\epsilon_r\epsilon_0}\left[\ln\rho\right]_a^b = \frac{Q}{2\pi\epsilon_r\epsilon_0}\ln\frac{b}{a}$$

The capacitance per unit length is

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_r\epsilon_0}{\ln\frac{b}{a}}$$