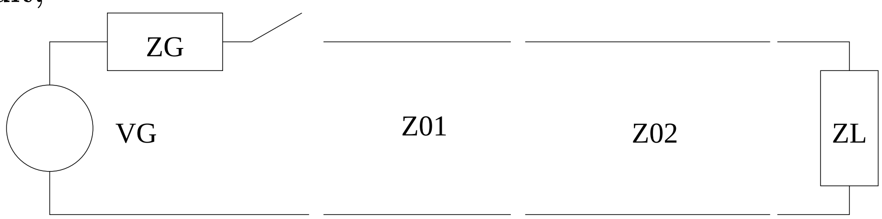
EE 333 Electricity and Magnetism, Fall 2009
Exam 4

Rules: This is a open book exam. You may use the textbook and your notes. The exam will last 50 minutes, and each of the following numbered questions count equally toward your grade. None of these problems require extensive calculations. If you find yourself in long complex computations your are likely on the wrong track.

Reflection diagram

In this circuit,



$\boldsymbol{V_G = 1\,\mathrm{V},\ Z_G = 50\,\Omega,\ Z_{01} = 50\,\Omega,\ Z_{02} = 100\,\Omega,\ Z_L = 50\,\Omega.}$

1. **Compute the reflection coefficients Γ_{11}, Γ_{22}, Γ_L (reflection off the load), and Γ_G (reflection off the generator), and the transmission coefficients τ_{12}, and τ_{21}.**

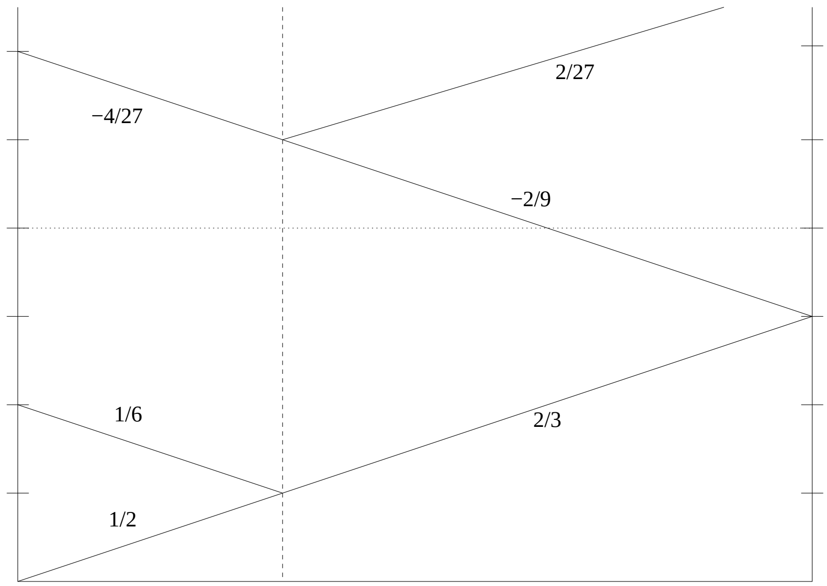
$$\Gamma_{11} = \frac{Z_{02} - Z_{01}}{Z_{01} + Z_{02}} = \frac{100 - 50}{50 + 100} = \frac{1}{3} \qquad \Gamma_{22} = \frac{Z_{01} - Z_{02}}{Z_{02} + Z_{01}} = -\Gamma_{11} = -\frac{1}{3}$$

$$\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} = \frac{50 - 100}{50 + 100} = -\frac{1}{3} \qquad \Gamma_G = \frac{Z_G - Z_{01}}{Z_G + Z_{02}} = 0$$

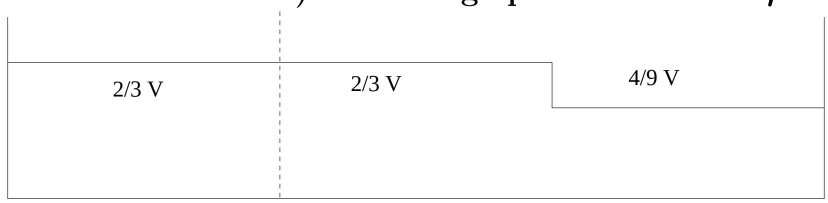
$$\tau_{12} = 1 + \Gamma_{11} = 1 + \frac{1}{3} = \frac{4}{3} \qquad \tau_{21} = 1 + \Gamma_{22} = 1 - \frac{1}{3} = \frac{2}{3}$$

2. **If line 1 is 300 m long, line 2 is 600 m long, the propagation speed is 3×10^8 m/s, draw the reflection diagram and label each wave with its magnitude, in volts, up to time 6 μs.**

   Note that no waves are reflected off the generator.



3. **Plot (and label with values) the voltage profile at time $4\,\mu$s.**



Active load

Consider a transmission line with characteristic impedance Z_0, supplied by a generator with output impedance $Z_G = Z_0$ and open-circuit voltage V_G, terminated by a capacitor C. As the switch to the generator is closed, the wave $v+$ is launched toward the capacitor. The wave arrives at the capacitor at time $t = 0$.

4. **What is the magnitude of v^+?**

$$v^+ = V_G \frac{Z_{01}}{Z_G + Z_{01}} = \frac{V_G}{2}$$

5. **What is the initial value of v_T, the total voltage across the capacitor, immediately after the arrival of v^+? What is the limiting value of v_T for $t \to \infty$?**

   Initially the capacitor is discharged, so the voltage across it immediately after the arrival of the wave is still zedro, so $v_T(0) = 0$. At infinite time the capacitor is charged to the voltage of the generator, so $v(t \to \infty) = V_G$.

6. **Derive the expression for v_T for $t \geq 0$. Assume the capacitor is discharged for $t < 0$. Begin with $v_T + Z_0 i_T = 2v^+$, insert the differential i-v relationship for the capacitor, solve the resulting differential equation and apply the boundary conditions to get an expression for v_T in terms of Z_0, C, and V_G.**

   The i-v relationship for the capacitor is

$$i_T = C\frac{dv_T}{dt}$$

Inserting it we get

$$v_T + Z_0 C\frac{dv_T}{dt} = V_G$$

The solution is

$$v_T = Ae^{-\frac{t}{\tau}} + B$$

Since $v_T(t=0) = 0$, $B = -A$, and since $v_T(t \to \infty) = V_G$, $A = -V_G$, and thus

$$v_T = V_G\left(1 - \frac{t}{\tau}\right)$$

To find τ, insert this into the differential equation

$$V_G\left(1 - e^{-\frac{t}{\tau}}\right) - \frac{Z_0 C}{\tau}Ae^{-\frac{t}{\tau}} = V_G$$

Let $t = 0$, and get

$$-\frac{Z_0 C}{\tau}A = V_G$$

$$\tau = -\frac{Z_0 C A}{V_G} = Z_0 C$$

Finally we get

$$v_T = V_G\left(1 - e^{-\frac{t}{Z_0 C}}\right)$$