EE 333 Electricity and Magnetism, Fall 2009
Homework #1 solution

1.14. Two point charges, $Q_1 = 1\,\text{nC}$ and $Q_2 = 3\,\text{nC}$, are located at the Cartesian points (1,1,0) and (0,2,1), respectively. Find $\vec{E}$ at (3,5,5). Coordinates are given in meters and one nC = 10^{-9} C.
Use Coulomb's law,

$$\vec{E} = \sum_i \frac{q_i\,(\vec{r}' - \vec{r}_i)}{4\pi\epsilon_0\,|\vec{r}' - \vec{r}_i|^3}$$

$$\vec{r}' - \vec{r}_1 = (3,5,5) - (1,1,0) = (2,4,5) = 6.708$$

$$|\vec{r}' - \vec{r}_1| = \sqrt{2^2 + 4^2 + 5^2}$$

$$\vec{r}' - \vec{r}_2 = (3,5,5) - (0,2,1) = (3,3,2)$$

$$|\vec{r}' - \vec{r}_1| = \sqrt{3^2 + 3^2 + 2^2} = 4.690$$

Now sum,

$$\begin{aligned}\vec{E} =& \frac{1.000\times10^{-9}\times(2,4,5)}{4\times\pi\times8.854\times10^{-12}\times6.708^3} + \frac{3.000\times10^{-9}\times(3,3,2)}{4\times\pi\times8.854\times10^{-12}\times4.690^3}\\ =&(2.523, 2.582, 1.791)\,\text{V/m}\end{aligned}$$

1.15. What is the force of attraction between the electron and the nucleus of the hydrogen atom, which are spaced at approximately 10^{-10} m? The hydrogen atom has one electron of charge $e = -1.6\times10^{-19}$ C, and the nucleus has a charge equal but opposite in sign to that of the electron.
Use Coulomb's law in the form

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$F = \frac{(1.602\times10^{-19})^2}{4\times\pi\times8.854\times10^{-12}\times(10^{-10})^2} = 2.3\times10^{-8}\,\text{N}$$

1.21. Two conducting spheres of negligible diameter have masses 0.2 g each. Two nonconducting threads, each 1 m long and of negligible mass, are used to suspend the two sphere from a common support. After placing an equal charge on the spheres, it is found that they separate with an angle of 45° between the threads. (a) if the gravitational force is 980×10^{-5} N/g, find the charge on each sphere. (b) Find the angle between the threads if the charge on each sphere is 0.5 μC.
The Coulomb force and the force from the thread must balance the force of gravity. The sum of the force of gravity and the Coulomb force must point along the thread. Thus,

$$\tan\frac{\theta}{2} = \frac{F_C}{F_g}$$

where

$$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$F_c = \frac{q^2}{4\pi\epsilon_0 r^2} = \frac{q^2}{4\pi\epsilon_0(2l\sin^2\frac{\theta}{2})^2}$$

Inserting we get

$$\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{q^2}{\pi\epsilon_0 mgl^2\sin^2\frac{\theta}{2}}$$

For $\theta << 1$ we can simplify to $\cos\frac{\theta}{2} = 1$ and $\sin\frac{\theta}{2} = \frac{\theta}{2}$, so

$$\theta^3 = \frac{q^2}{2\pi\epsilon_0 mgl^2}$$

(a) Find the charge for $\theta = 45° = 0.785$,

$$\begin{aligned} q =& \sqrt{2\pi\epsilon_0 mgl^2\theta^3} \\ =& \sqrt{2\times\pi\times 8.854\times 10^{-12}\times 0.2\times 980\times 10^{-5}\times 0.785^3} \\ =& 0.23\,\mu\text{C} \end{aligned}$$

(b) Now, find the angle when $q = 0.5\,\mu$C. Because I expect this angle to be larger than 45°, its determination is going to be more approximated.

$$\begin{aligned} \theta =& \sqrt[3]{\frac{q^2}{2\pi\epsilon_0 mgl^2}} \\ =& \sqrt[3]{\frac{(0.5\times 10^{-6})^2}{2\times\pi\times 8.854\times 10^{-12}\times 0.2\times 980\times 10^{-5}\times 1^2}} \\ =& 1.3186 \end{aligned}$$

$$= 75.55°$$