EE 434 Electricity and Magnetism, Spring 2009
Homework #10 extra credit and exam preparation

1. Derive the equation

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \omega^2 \mu\epsilon + \gamma^2 = 0$$

   (which is applied to $\vec{E}$ and $\vec{H}$) for harmonic waves. Begin with the harmonic wave expressions for Faraday's and Ampere's law.

2. Explain (using math) why it is sufficient to solve for E_z and H_z in the above equations

3. Show why E_z is expressed as a product of sines in the waveguide coordinate system we have been using.

4. Show why H_z is expressed as a product of cosines.

5. Explain what the critical frequency is and how it is derived.

6. Given the dispersion relation

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

   show that the group velocity does not exceed the speed of light.

7. In a empty waveguide with $a = 2\,\text{cm}$, $b = 3\,\text{cm}$, the TE_{11} mode with $\omega = 10^{11}\,\text{rad/s}$, has electric field amplitude $E_{x0} = 1\,\text{V/m}$. Give the amplitude E_{y0}, H_{x0}, and H_{y0}.

8. Sketch the magnetic field in the transverse plane in the TM_{32} mode in a rectangular waveguide. Sketch the electric field in the transverse plane in the TE_{32} mode in a rectangular waveguide.

9. A rectangular waveguide with $a = 2\,\text{cm}$, and $b = 3\,\text{cm}$ is excited with a signal with $\omega = 10^{11}\,\text{rad/s}$. What modes are expected to be excited?

10. What are the wave impedances for each of the modes in the previous question?

11. What are the wavelengths for each of the modes in the previous question?

12. What are the group velocities for each of the modes in the previous question?